JACOBI NONLINEAR ITERATION METHOD FOR A DISCRETE KIRCHHOFF SYSTEM

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As is well-known, the equation $w_{tt} = w_{xx}$ models string vibration. This equation should be considered as the first approximation of the vibration process. The equation

$$w_{tt} - \left(\lambda + \frac{2}{\pi}\int_0^\pi w_x^2 dx\right) w_{xx} = 0, \tag{1}$$

$$0 < x < \pi, \quad 0 < t \le T, \quad \lambda > 0,$$

proposed by Kirchhoff [1] in 1883 is regarded as the best model as it takes into account an increase in tension resulting from the extension of the string.

Equation (1) was first studied by S. Bernstein [2] in 1940. In the subsequent years, many authors (see, for example, [3], [4] and the references cited therein) devoted their studies to this equation and its natural generalization

$$w_{tt} - g\left(\int_\Omega |\nabla w|^2 dx\right) \triangle w = 0,$$

$$x \in R^n, \quad g(u) > 0, \quad n > 1.$$

In most cases these studies were concerned with the problem of the existence of a solution. Problems of controllability and stabilization were also considered, and computations were carried out (see, for example, [5], [6] and the references cited therein).

We know only one work [7] in which an approximate algorithm is applied to an initial boundary value problem for equation (1) and the convergence of the algorithm is proved. Speaking more exactly, in [7], under somewhat specific conditions with given

$$w_t(x,0), \quad w_x(x,0)$$

and

$$w_t(0,t) = w_t(\pi,t) = 0,$$

the solution of equation (1) is semidiscretized with respect to the argument x using the finite element method and the error is estimated.

Here we consider equation (1) when

$$w(x,0) = w^{(0)}(x), \quad w_t(x,0) = w^{(1)}(x), \quad w(0,t) = w(\pi,t) = 0,$$

where $w^{(p)}(x)$ are the given functions, $p = 0, 1$.

Using the Galerkin method and a Crank-Nicolson type scheme, the solution is discretized with respect to a spatial and a time variable. Thus the problem is reduced to a system of nonlinear algebraic equations which is solved by the iteration method. The convergence of the method is proved.

References

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