DIFFUSION APPROXIMATION FOR BILLIARDS II

L. Dumas

Universit'e Paris 6, Laboratoire d'Analyse Num'erique
4 place Jussieu, 75230 Paris Cedex 05

Abstract: This article generalizes the previous results obtained in [1] about diffusion approximation for billiards. The latter consist here in a periodic array of circular obstacles, without any finite horizon hypothesis as it was assumed in [1]. Define

$$\mathcal{L} = \boldsymbol{Z}(a;0) \bigoplus \boldsymbol{Z}(0;a) \quad (a > 0)$$

and

$$\boldsymbol{X} = \{x \in \boldsymbol{R}^2 / d(x;\mathcal{L}) > r\}$$

with $0 < r < \frac{a}{2}$ so that the scatterers do not overlap.

The particles moving in $\boldsymbol{X}$ through the circular obstacles, are punctual and not interacting each other. Their constant velocity is taken equal to one. Denote $f \equiv f(t,x,v)$, $x \in \partial\boldsymbol{X}$, $v \in \boldsymbol{S}^1$ their number density at time $t > 0$ and suppose that

$$f(0;x;v) = \Phi(x;v) \quad x \in X, \quad v \in \boldsymbol{S}^1. \tag{0.1}$$

On the whole domain, f satisfies the free transport equation:

$$\partial_t f + v \cdot \Delta_x f = 0, \quad t \geq 0, \quad x \in X, \quad v \in \boldsymbol{S}^1. \tag{0.2}$$

The particles reflects upon the obstacles on a mixed manner: their reflexion is whether specular with probability α or diffuse with probability $1-\alpha$. The probability coefficient $1-\alpha$ is usually named the accomodation coefficient and ranges from 0 to 1. The case of Sinai billiards where $\alpha = 1$ is excluded in any strong diffusion approximation theorem for obvious considerations on energy. The boundary condition expresses as (with the usual notations of kinetic theory):

$$f_+(t,x,v) = \alpha f_-(t,x,v^*) + (1-\alpha) \int_{v'\cdot n_x < 0} k(x,v,v') f_-(x,x,v') dv', \quad (x,v) \in \Gamma_+. \tag{0.3}$$

The more general admissible form of k than in [1] allows a king of "memory effect" in the collisions: indeed, the particles can be reemitted differently according to their incoming velocity. The aim of the present article is to prove that their density is still approximated by the solution of a diffusion equation in the long time and large scale regimes.

R e f e r e n c e s

[1] Bardos, C., Dumas, L., Golse, F. Diffusion approximation for Billiards. Journal of Statistical Physics, 86, No. 1-2, 1997, 351-375