ERROR INDICATORS AND MESH ADAPTIVITY

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The finite element method is a technique for the discretization of partial differential equations that relies on the approximation of the solution by functions which are polynomial on each element of a triangulation of the domain. For linear elliptic problems, it is easy to prove the convergence of the method when the maximal diameter of these elements tends to zero. An error estimate between the exact and discrete solutions called "a priori", is also derived, it depends on the maximal diameter and the global regularity of the exact solution. However a new improvement, which seems now unavoidable in the finite element framework, is that the mesh must be adapted to the solution. This allows either to increase the accuracy of the approximation with the same number of degrees of freedom or to highly reduce the number of degrees of freedom while keeping the same accuracy. Mesh adaptivity relies on the combination of two different tools: computing some error indicators and building meshes that satisfy the criteria given by these indicators.

Indeed, in most cases, the solution that must be approximated is singular in some parts of the domain, and numerical experiments prove that the error becomes smaller when the triangulation is finer in these parts. However the locations of these singularites is not known in the general case. So one aim of this work is to present and analyze some tools, called error indicators, that allow for identifying them during the computation. For some model problems, we propose residual type indicators associated with all elements of the triangulation. We refer to the book of R. Verfürth for their analysis in the case of more general problems and also for the description of other types of indicators. Next, we prove that they are equivalent to the error: the global error is bounded by the Hilbertian sum of the indicators (this is called "a posteriori" estimate) but also each indicator is bounded by the local error, which proves the efficiency of the technique for mesh adaptivity.

The mortar element method, due to C. Bernardi, Y. Maday and A.T. Patera, is a domain decomposition technique which allows for working with independent discretizations on the subdomains of a partition of the domain without overlapping. So it seems ideally suited for mesh adaptivity. Indeed, non matching grids can be used on the different subdomains of a partition, and this leads to highly reduce the number of degrees of freedom in order to satisfy the adaptivity criteria since no further node must be added for conformity reasons. Also, the regularity properties of the initial grid are preserved. So we extend both the definition and the analysis of the previous error indicators to this framework, and present some numerical experiments which are in good agreement with the analysis and prove the interest of this new technique.