DIFFERENCE SCHEMES FOR PLURIPARABOLIC EQUATIONS

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During the calculation process of diffusion of various mixtures in the water streams important problem for algorythmization is to construct correct and convenient mathematical model, which is adequate to the considered process within the desired precision.

So, by the following equation V.M.Makaveev [4] described the process of sewage and mixtures diffusion in case of small value of cross-section components of water stream velocity and in case of dilution process invariability in time as well as under the other suggestions :

$$v_{cp}\frac{\partial S}{\partial x}=\frac{g}{\gamma}\left(\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}\right),\qquad (x,y,z)\in\Omega=(0,l)\times D, \tag{1}$$

where v_{cp} is an average value of velocity of water streams along the Ox axis, g is a gravitation constant, γ is a specific weight, A_{cp} is an average value of turbulent viscosity, $S\equiv S(x,y,z)$ is a concentration of substance, Ω - area occupied by the considered part of sewage in certain orthogonal system of coordinates $Oxyz$, D cross-section area of sewage. Initial boundary conditions were stated as follows:

$$S(0,y,z)=S_0(y,z),\qquad (y,z)\in D, \tag{2}$$

$$\frac{\partial S}{\partial n}=0,\qquad (x,y,z)\in\Gamma, \tag{3}$$

where $S_0(y,z)$is a given value of concentration on Γ_1 upper sewage of water stream, n is an external normal to Γ, $\Gamma=\Gamma_2\cup\Gamma_3\cup\Gamma_4$, Γ_2 is surface of bottom, Γ_3 is a free surface of the water stream, Γ_4 is a sides. Condition (3) means, that transformation of diluted substance through Γ is equal to zero.

Model $(1)-(3)$ has many positive sides, it is convenient for calculation of dilution of enough mix and, for obtaining complete description of the pollution concentration.

In the present work we consider non-stationary case of the problem $(1)-(3)$ ([12]):

$$\frac{\partial S}{\partial t}+v_{cp}\frac{\partial S}{\partial x}=\frac{g}{\gamma}\left(\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}\right), \tag{4}$$

with the following initial and boundary conditions:

$$\begin{aligned} S(x,y,z,0) &= S_0(x,y,z), \qquad (x,y,z)\in\Omega, \\ S(0,y,z,t) &= S_0(y,z,t), \qquad 0\le t\le T, \quad (y,z)\in D, \end{aligned} \tag{5}$$

$$\frac{\partial S}{\partial n}=0, \qquad 0\le t\le T, \quad (x,y,z)\in\Gamma. \tag{6}$$

For the problem $(4)-(6)$ in case of Dirichlet type boundary conditions, Laplace operators in the right side and under the following notations: $t\equiv t_1, x\equiv t_2$,there is constructed the following regularized scheme (see [3]):

$$u_{t_1}+u_{t_t}+\tau_1\tau_2R_{12}u_{t_1t_2}+\tau_1R_1u_{t_1}+\tau_2R_2u_{t_2+}L_hu_{t_1t_2}=F_i^{j,k,}, \tag{7}$$

where R_{12}, R_1, R_2 are operators:

$$\begin{aligned} R_{12} &= \frac{\theta_1}{\tau_1}+\frac{\theta_2}{\tau_2}+\theta_1\theta_2L_h, \\ R_2 &= (\theta_5+\theta_3(\theta_4-\theta_5))\,L_h, \\ R_1 &= \theta_3L_h, \\ 0 &\le \theta_i\le 1, \quad i=1,...,5; \end{aligned}$$

$u_i^{j,k}$ is difference function corresponding to $S(t_1,t_2,y,z)$, L_h is a second order difference operator corresponding to Laplace operator, $\tau_1$and τ_2 are grid steps along the Ot_1, Ot_2 axis..

. It should be emphasized, that (7) difference scheme approximates the problem $(4)-(6)$ in case of Dirichlet type boundary conditions for sufficiently smooth $S(t_1,t_2,y,z)$ with precision $O(\tau_1+\tau_2+\|h\|^2)$. When $\theta_1=\theta_2=\theta_3=\theta_4=\theta_5=\frac{1}{2}$ the order of precision increases to $O(\tau_1^2+\tau_2^2+\|h\|^2)$.

Afterwards the following explicit and implicit schemes are investigated:

$$\tau_1=\tau_2=\tau, \quad y_i^{j+1,k+1}=y_i^{j,k}-L_hy_i^{j,k}+\tau F_i^{j,k};$$

$$\frac{y_i^{j+1,k+1}-y_i^{j,k+1}}{\tau_1}+\frac{y_i^{j+1,k+1}-y_i^{j+1,k}}{\tau_2}+L_hy_i^{j+1,k}+L_hy_i^{j,k+1}=F_i^{j,k};$$

$$\frac{y_i^{j+1,k+1}-y_i^{j,k+1}}{\tau_1}+\frac{y_i^{j+1,k+1}-y_i^{j+1,k}}{\tau_2}+L_hy_i^{j+1,k+1}=F_i^{j,k};$$

There also are proved the theorems of convergence and stability.

R e f e r e n c e s

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