ON THE PRANDTL EQUATION ON A FINITE INTERVAL

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The purpose of this paper is the investigation of the basic integro-differential equation of a finite span aeroplane wing

$$-\Gamma(z)=\pi Vb(z)\left[\frac{\alpha(z)}{2}+Q(z)+\frac{1}{4\pi V}\int_{-l}^{+l}\frac{\Gamma'(t)}{z-t}dt\right] \tag{1}$$

which is known as the Prandtl equation on a finite interval. Equation (1) relates the wing's geometrical characteristics to its aerodynamic properties. The unknown function $\Gamma(z)$ represents circulation of the wing, while functions $b(z), \alpha(z), Q(z)$ are defined by the construction and represent a chord, curvature of a profile and twist of the wing (see [2]). V=const is the speed of a stream at infinity and is usually detected by the experiments. The circulation satisfies the following boundary conditions

$$\Gamma(-l)=\Gamma(+l)=0. \tag{2}$$

The Prandtl equation was investigated mostly by the numerical methods (see [1], [6]). Our purpose is to give precise theory of the Prandtl equation in the Bessel potential spaces.

We shall treat equation (1) as a pseudodifferantial equation in the Bessel potential spaces $\mathbb{H}_p^s$, $s\in\mathbb{R}$, $1<p<+\infty$, namely $\Gamma\in\tilde{\mathbb{H}}_p^s([-l,+l])$ and $b[\alpha/2+Q]\in\mathbb{H}_p^{s-1}([-l,+l])$ (For the standard notation see [5]). The investigation is based on the results of R. Duduchava on convolution equations with discontinuous symbols (see [4]) and the local principle from [3].

Fredholm criteria and the index formula for equation (1) are found.

Theorem 1. *Let $s\in\mathbb{R}$ and*

$$s=[s]+\{s\},\quad [s]=0,\pm1,\pm2,\ \ldots,\quad 0\le\{s\}<1$$

be the decomposition of s into integer and fractional parts. The operator $\boldsymbol{A}$

$$A\varphi(x)=c_0(x)\varphi(x)+c_1\frac{1}{\pi i}\int_{-l}^{+l}\frac{\varphi'(t)}{t-x}dt,\quad c_1=const$$

$$A:\tilde{\mathbb{H}}_p^s([-l,+l])\mapsto\mathbb{H}_p^{s-1}([-l,+l])$$

is Fredholm if and only if

$$|\{s\}-\frac{1}{p}|\neq\frac{1}{2}.$$

*If this condition holds, the operator **A** is invertible, is invertible from the left or is invertible from the right provided σ iz zero, positive or negative, respectorly. Here*

$$\sigma = [s] \quad if \quad \mid \{s\} - \frac{1}{p} \mid < \frac{1}{2},$$

$$\sigma = [s] + 1 \quad if \quad \{s\} - \frac{1}{p} > \frac{1}{2},$$

$$\sigma = [s] - 1 \quad if \quad \{s\} - \frac{1}{p} < -\frac{1}{2},$$

and

$$IndA = -\sigma.$$

References

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