ON A CLASS OF BOUNDARY VALUE PROBLEMS OF THERMOELASTICITY FOR AN ORTHOTROPIC BODY IN THE RECTANQULAR CARTESIAN COORDINATE SYSTEM

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The basic equations system describing the thermoelastic equilibrium of a plane-stressed, orthotropic homogeneous body has the form [1]

$$\Delta_{21}\varphi_1 = \frac{g}{\gamma_4}\frac{\partial^2 \tilde{T}}{\partial z^2},$$
$$\Delta_{22}\Phi_1 = -\frac{\gamma_2}{\gamma_4}\frac{\partial^2 \varphi_1}{\partial z^2} + \frac{\gamma_5\gamma_2}{\gamma_4}\frac{\partial^2 \tilde{T}}{\partial z^2} \tag{1}$$

Further we use the following notation

$$a = \sqrt{\gamma_1\gamma_4 - \gamma_2^2}\,i,\ if\ \gamma_2^2 - \gamma_1\gamma_4 < 0,\ \ i = \sqrt{-1};\ \ a = \sqrt{\gamma_2^2 - \gamma_1\gamma_4},\ if\ \gamma_2^2 - \gamma_1\gamma_4 \geq 0,$$

$$a_1 = a - \frac{1}{2\mu},\ \ a_2 = a + \frac{1}{2\mu},\ \ g = \gamma_2\gamma_5 - a\gamma_5 - \gamma_3\gamma_4,$$

$$\Delta_{21} = \frac{\gamma_2 - a}{\gamma_4}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2},\ \ \Delta_{22} = \frac{\gamma_2 + a}{\gamma_4}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2},\ \ \Delta_{23} = \frac{\lambda_3}{\lambda_1}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2},$$

$$\gamma_1 = \frac{E_1 - E_2\nu_{12}^2}{E_1^2},\ \ \gamma_2 = \frac{E_1 - 2\mu\left(\nu_{13} + \nu_{23}\nu_{12}\right)}{2\mu E_1}, \gamma_3 = -\left(\frac{E_2\nu_{12}k_2}{E_1} + k_1\right),$$

$$\gamma_4 = \frac{E_2 - E_3\nu_{23}^2}{E_3E_2},\ \ \gamma_5 = -\left(\nu_{23}k_2 + k_3\right),$$

where E_1, E_2, E_3 are Young's modulus along Ox, Oy, Oz; ν_{12} is Poisson's ratio in the plane Oxy; ν_{13} is Poisson's ratio in the plane Oxz; ν_{23} is Poisson's ratio in the plane Oyz; k_1, k_2, k_3 are temperature linear expansion coefficients along Ox, Oy, Oz respectively; λ_1, λ_2, λ_3 are heat conductivity coefficients along Ox, Oy, Oz respectively and $\frac{\partial^2\tilde{T}}{\partial z^2} = T$ is a temperature in the elastic body $\left(\Delta_{23}T = 0,\ \Delta_{23}\tilde{T} = 0\right)$.

For the overwhelming majority of orthotropic bodies

$$\mu = \frac{E_1E_2}{E_1\left(1 + 2\nu_{12}\right) + E_2},$$

where μ is a shift modulus in the plane Ozx;

The following dependences

$$E_1\nu_{21} = E_2\nu_{12},\ E_2\nu_{32} = E_3\nu_{23},\ E_3\nu_{13} = E_1\nu_{31}.$$

If $a \neq 0$, $\gamma_3 = \dfrac{\lambda_3}{\lambda_1}\gamma_5$, then solution of the system (1) will take the form

$$\varphi_1 = \tilde{\varphi} + \gamma_5 \tilde{T}, \quad \Phi_1 = \tilde{\Phi} + G_2 \tilde{\varphi} \tag{2}$$

where $G_2 = -\dfrac{\gamma_2}{2a}$, $\Delta_{21}\tilde{\varphi} = 0$, $\Delta_{22}\tilde{\Phi} = 0$.

Taking into consideration (1) , (2), components of the displacement vector $\vec{U}$ and components of the stress tensor could be presented by means of the functions $\tilde{\Phi}$, $\tilde{\varphi}$, $\tilde{T}$ in the following form

$$\begin{aligned} u &= \left(1 + \frac{a_1 G_2}{\gamma_2}\right)\frac{\partial \tilde{\varphi}}{\partial x} + \frac{a_1}{\gamma_2}\frac{\partial \tilde{\Phi}}{\partial x} + \gamma_5 \frac{\partial \tilde{T}}{\partial x}, \\ w &= -\left(1 + \frac{a_2 G_2}{\gamma_2}\right)\frac{\partial \tilde{\varphi}}{\partial z} - \frac{a_2}{\gamma_2}\frac{\partial \tilde{\Phi}}{\partial z} - \gamma_5 \frac{\partial \tilde{T}}{\partial z}, \\ Y_y &= k_r E_r \frac{\partial^2 \tilde{T}}{\partial z^2}, \\ Z_z &= \frac{1}{\gamma_2}\frac{\partial^2 \tilde{\Phi}}{\partial x^2} + \frac{G_2}{\gamma_2}\frac{\partial^2 \tilde{\varphi}}{\partial x^2}, \\ X_x &= \frac{1}{\gamma_2}\frac{\partial^2 \tilde{\Phi}}{\partial z^2} + \frac{G_2}{\gamma_2}\frac{\partial^2 \tilde{\varphi}}{\partial z^2}, \\ Z_x &= -\frac{1}{\gamma_2}\frac{\partial^2 \tilde{\Phi}}{\partial x \partial z} - \frac{G_2}{\gamma_2}\frac{\partial^2 \tilde{\varphi}}{\partial x \partial z}. \end{aligned} \tag{3}$$

We consider thermoelastic equilibrium of a plane stressed body which occupies the domain $\Omega_p = \{0 < x < x_1,\, 0 < z < z_1\}$.

Let us consider the following boundary value conditions

$$\begin{array}{l} \text{If } x = x_0 = 0 \text{ then } X_x = 0,\, Z_x = 0,\, T = f_1(z), \\ \text{If } x = x_1 \text{ then } X_x = 0,\, Z_x = 0,\, T = f_2(z), \\ \text{If } z = z_0 = 0 \text{ then } Z_z = 0,\, Z_x = 0,\, T = f_3(x), \\ \text{If } z = z_1 \text{ then } Z_z = 0,\, Z_x = 0,\, T = f_4(x). \end{array} \tag{4}$$

The solution of problem (1), (4) (solution for temperature problem in the domain Ω_p is trivial and can be easily constructed using the method of separation of variables) could be presented assuming in (3) $\tilde{\varphi} = 0$, $\tilde{\Phi} = 0$. But in this case everywhere in the domain the components of the stress tensor

$$\begin{aligned} Z_z &= 0, \\ X_x &= 0, \\ Z_x &= 0, \end{aligned}$$

regardless of temperature field, but Y_y, u, w depend on the function T, and this is the generalization for orthotropic bodies of temperature effect obtained by N.Muskhelishvili [2].

References

[1] Khomasuridze N., Khomasuridze I., Two-dimensional Problems of Thermoelasticity in Cylindricall and Rectangular Coordinates. Proceedings of I.Vekua Institute of Applied Mathematics, 48, 1998.

[2] Muskhelishvili N., Some Basic Problems of the Mathematicals Theory of Elasticity. Nauka, Moscow, 1966, (Russian).