Fourier solution of canonical problem in starlike domains

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ABSTRACT

Many applications of Mathematical Physics and Engineering are connected with the Laplacian, however, the most part of BVP relevant to the Laplacian are solved in explicit form only for domains with a very special shape, namely intervals, cylinders or domains with particular (circular or spherical) symmetries [1].

In recent articles (see [2], [3], [4], [5]) a solution of the Dirichlet problem for the Laplace equation in two and three-dimensional starlike domains (i.e. domains which are normal with respect to a suitable polar or spherical co-ordinate system) was achieved by using classical methods.

Different techniques was also used for solving this problem, both by theoretical or numerical approaches. The relevant literature is quite extensive, mainly in Russian language (see e.g. [6], [7], [8], [9], [10], [11], [12]), however, none of the above mentioned articles is connected with the approach we have considered, which traces back to the original Fourier method.

In this survey we show how to solve several canonical problems, including the Dirichlet problem for the Laplace and Helmhotlz equations, and the wave or heat equations in starlike domains.

The boundary of the domains we have considered in all our applications are defined by using a generalization of the so called "superformula" due to J. Gielis [13].

Many numerical applications, we have performed by using the Computer Algebra system Mathematica©, confirm, even in this more general case, the theoretical results of L. Carlson [14], since we have found point-wise convergence in all regular points of the boundary, with possible oscillation usually occurring only in singular points.

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