THE FOURTH INTERNATIONAL CONFERENCE
„MODERN PROBLEMS IN APPLIED MATHEMATICS“ (MPAM2023)

Dedicated to the 105th Anniversary of
I. Javakhishvili Tbilisi State University &
55th Anniversary of I. Vekua Institute of Applied Mathematics

September 13-15, 2023

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I. Javakhishvili Tbilisi State University (TSU)
I. Vekua Institute of Applied Mathematics (VIAM) of TSU
Tbilisi International Centre of Mathematics and Informatics
Georgian National Committee of Theoretical and Applied Mechanics

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მეოთხე საერთაშორისო კონფერენცია
"გამოყენებითი მათემატიკის თანამედროვე პრობლემები"

ეძღვნება ი. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის 105
და
თსუ ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტის 55 წლისთავს

2023 წლის 13-15 სექტემბერი

ორგანიზატორები

ი. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი
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Wednesday, September 13

TSU, Main Building, 1 Chavchavadze ave.

Assembly Hall

TIME	
9:00-10:00	Registration
10:00-11:00	**Opening of the International Conferences** MPAM2023 & "Logic Algebra and Truth Degrees" **Jaba Samushia** - Rector of TSU **Erekle Astakhishvili** – Director General of Shota Rustaveli National Science Foundation **George Jaiani** - Director of VIAM *Ilia Vekua Institute of Applied Mathematics of TSU - 55 Years History* **Omar Purtukhia** - Head of Department of Mathematics of the Faculty of Exact and Natural Sciences *Mathematics at the Ivane Javakhishvili Tbilisi State University*
11:00-11:20	Concert
	Opening Lecture
11:20-12:00	**Ayech Benjeddou** (France) Nonlinear Frequency- and Field-Dependent Operational Non-Linearities of Soft Piezoceramics d15 Shear Response-Mode
12:00-12:20	Coffee Break
12:20-12:55	**David Natroshvili** (Georgia) Application of the Potential Method in the Theory of Elasticity

12:55-13:30	**Holm Altenbach,** Lidiia Nazarenko (Germany) Variational Principles in Coupled Strain Gradient Elasticity
13:30-13:55	**Giorgi Buzghulashvili**, Tamaz Vashakmadze(Georgia) To the Approximate Solution of Boundary Value Problems for Ordinary Differential Equation
13:55-14:55	Lunch
14:55-15:30	**Paolo Emilio Ricci** (Italy), Diego Caratelli (The Netherlands), Pierpaolo Natalini (Italy) Analytic Solution to Functional Differential Equations Via Bell's Polynomials
15:30-16:05	**Vakhtang Kvaratskhelia**, Giorgi Giorgobiani, Vazha Tarieladze (Georgia) Subgaussian Random Elements in Infinite Dimensional Spaces
16:05-16:30	**George Rukhaia**, Teimuraz Davitashvili (Georgia) Exploring the Possibilities of Using Renewable Energy in Georgia in the Context of Climate Change Issues
16:30-16:50	Coffee Break
16:50-17:25	**Reinhold Kienzler** (Germany) Dimension Reduction in Elasticity
17:25-18:00	**Gia Avalishvili**, Mariam Avalishvili (Georgia) On Nonclassical Dynamic Two-dimensional Models for Thermoelastic Shells with Three Phase-lags
18:00-18:25	**Zurab Vashakidze**, Jemal Rogava (Georgia) On Convergence of a Three-Layer Semi-Discrete Scheme for the Nonlinear Dynamic String Equation of Kirchhoff-Type with Time-Dependent Coefficients

Thursday, September 14

VIAM TSU, 11 University street

Ilia Vekua Lecture Hall

TIME	
11:00-11:25	**Ani Ozbetelashvili** (Georgia) Hardy-Littlewood Maximal Function on Locally Compact Abelian Groups
11:25-11:50	**Archil Papukashvili** (Georgia) About the Numerical Solutions of Two Nonlinear Integro-Differential Equations
11:50-12:15	**Roman Koplatadze** (Georgia) On Asymptotic Behavior of Solutions of Higher Order Emden-Fowler Type Difference Equations with Deviating Argument
12:15-12:40	**Bakur Gulua**, Roman Janjgava, Guranda Charkseliani (Georgia) On Construction of General Solutions of Equations of the Plane Theory of Elasticity in the Coupled Theory of Double-Porosity Materials
12:40-13:05	**Gvantsa Shevardenidze** (Georgia) On the Convergence of Cesáro Means of Negative Order of Vilenkin-Fourier Series
13:05-13:25	Coffee Break
13:25-13:50	**Mariam Kokhreidze** (Georgia) Construction of Mathematical Models for Reiner Elastic Materials
13:50-14:15	**Giorgi Geladze**, Archil Papukashvili, Meri Sharikadze (Georgia) Further Exploration of the Mesoscale Atmospheric Boundary Layer

14:15-14:40	**VardenTsutskiridze** (GTU, Georgia), M. Tsutskiridze (Grenoble University, France) Heat Transfer with the Flow of Conducting Fluid in Circular Pipes with Finite Conductivity under Uniform Transverse Magnetic field
14:40-15:05	**Besik Tabatadze**, Mikheil Gagoshidze, Temur Jangveladze (Georgia), Zurab Kiguradze (USA) Two Methods of the Numerical Solution of One System of Nonlinear Partial Differential Equations
15:05-15:30	**Gogi Kezheradze** (Georgia). Some Issues on the Summability of the General Dirichlet's Integrals

Excursion with Conference Dinner

Friday, September 15

TSU, Main Building, 1 Chavchavadze ave.

TIME	Room №201
10:00-10:35	**Wolfgang H. Müller** (Germany) Electrodynamics and Rational Mechanics
10:35-11:10	**Lucian Beznea** (Romania) Multiplicative L^p-Semigroups and Continuous Flows
	Parallel Sessions
	Session 1, Room №201
11:15-11:40	**Tea Shavadze,** Tamaz Tadumadze (Georgia) On the Representation Formula of Solution for a Class of Perturbed Controlled Neutral Functional-Differential Equation
11:40-12:05	**Lasha Baramidze**, Ushangi Goginava (Georgia) Summability of Tkebuchava's Means of One and Two Dimensional Trigonometric Fourier Series

	Session 2, №201A
11:15-11:40	**Petre Babilua,** Elizbar Nadaraia (Georgia) On the Testing Hypothesis of Equality of Two Bernoulli Regression Functions
11:40-12:05	**Nino Manjavidze**, Giorgi Makatsaria (Georgia) Functional Properties of the Solutions of Some Classes of Two-Dimensional Elliptic Systems
12:05-12:20	Coffee Break
	Room №201
12:20-12:55	**Flavia Lanzara** (Italy) Approximation of Multidimensional Fractional Laplacian Based on Gaussians
12:55-13:30	**Merab Svanadze** (Georgia) On the Coupled Linear Theory of Thermoelastic Nanoporous Materials with Triple Porosity
13:30-14:05	**Vladimir Mityushev** (Poland) Effective Properties of Dispersed Regular and Random Composites
14:05-15:00	Lunch
	Parallel Sessions
	Session 1, №201
15:00-15:25	**Maia Svanadze** (Georgia) Problems of Steady Vibrations in the Linear Coupled Theory of Thermoviscoelasticity of Porous Materials
15:25-15:50	**David Kaladze** (Georgia) On the Exact Solutions of the Gardner Equations Via Tanh-Coth Method
15:50-16:15	**Omar Kikvidze** (Georgia) Numerical Analysis of the Equation of Motion of a Thick-Walled Pipe During Axisymmetric Deformation
	Session 2, №201A
15:00-15:25	**Giorgi Kakulashvili** (Georgia) On Schwarz-Christoffel Mapping of n-gon

15:25-15:50	**Irakli Sikharulidze** (Georgia) Sheaf-theoretic Approach to the Theory of Pseudoanalytic Functions
15:50-16:15	**Mariam Chakhoiantsi** (Georgia) Riemann-Hilbert Boundary Value Problem with a Shift for Generalized Analytic Functions
16:15-16:30	Coffee Break
	Room №201
16:30-16:55	**George Jaiani**, Natalia Chinchaladze (Georgia) Zero Approximation of Hierarchical Models for Fluids in Pipes of Angular Cross-Sections
16:55-17:30	**Alberto Cialdea** (Italy). The Functional Dissipativity of Certain Systems of Partial Differential Equations
	Closing Lecture
17:30-18:10	**Alexander Meskhi** (Georgia) Duality, Interpolation and Extrapolation for Weighted Grand Morrey Spaces
18:10-19:30	**Discussion over the talks presented at the conference; Closing**

Ilia Vekua Institute of Applied Mathematics of TSU – 55 Years History

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The talk concerns 55 years long history of Ilia Vekua Institute of Applied Mathematics of Ivane Javaxishvili Tbilisi State University [1-4]. The Institute was founded by Georgian mathematician and mechanist Ilia Vekua on October 29, 1968. The aim of the Institute was to carry out research on important problems of applied mathematics, to involve University professors, teachers and students in research activities on topical problems of applied mathematics in order to integrate mathematics into the study processes and research, and to implement mathematical methodologies and calculating technology in the non-mathematical fields of the University. In 1978, the Institute was named after its founder and first director Ilia Vekua. In December, 2006 - May, 2009 the Institute was acting at the Faculty of the Exact and Natural Sciences. In June, 2009 - September, 2016 the Institute was directly subordinated to the University Administration. Since the end of September, 2016 the Institute has a status of the Independent Scientific-Research Institute. At present, the Institute successfully continues and develops activities launched by his founder in the following four main scientific directions:

- Mathematical problems of mechanics of continua and related problems of analysis;

- Mathematical modelling and numerical mathematics;
- Discrete mathematics and theory of algorithms;
- Probability Theory and mathematical Statistics.

The institute sees its mission as threefold:

- Carrying out fundamental and practical scientific research in applied mathematics, mathematical and technical mechanics, industrial mathematics and informatics, undertaking state and private sector contracts to provide expert services;
- Offering the university a high-level computer technology base for University professors and teachers, research employees and students undertaking their scientific research activities;
- Supporting PhD and post-graduate students to attain scientific grants, as well as through employment within the Institute and participation in scientific conferences.

References

[1] Ilia Vekua Insitute of Applied Mathematics of TSU. TSU Science. Vol. 2 (2012), 89-91.

[2] VAIM-45. Tbilisi University Press, 2013.

[3] VIAM-50. Tbilisi University Press, 2018.

[4] VIAM-55. Tbilisi University Press, 2023.

Mathematics at the Ivane Javakhishvili Tbilisi State University

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The talk deals with the history of mathematics at the Ivane Javakhishvili Tbilisi State University (TSU). To a group of scientists who founded TSU, under the leadership Ivane Javakhishvili, belonged Andrea Razmadze – the first Georgian scientist-mathematician. Along with A. Razmadze a crucial role in the development of the mathematical science and education at TSU was played by N. Muskhelishvili, G. Nikoladze, A. Kharadze, A. Benashvili and later I. Vekua and V. Kupradze. In the presented talk, in the brief, will be described a contribution of those scientists who are founders and organizers of scientific and pedagogical activity on mathematics in TSU. Presently, these traditions are continuing on successfully in the Department of Mathematics of the Faculty of Exact and Natural Sciences of TSU, as well as in the Ilia Vekua Institute of Applied Mathematics and Andrea Razmadze Mathematical Institute.

Variational Principles in Coupled Strain Gradient Elasticity

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Strain gradient elasticity is a special case of high-gradient theories where the potential energy density depends on the first and second gradients of displacements. The presence of a coupling term in the equation for the stain and strain gradient energy density leads to a non-diagonal quadratic form of the stored energy and makes it difficult for the derivation of fundamental theorems. In this presentation, two variational principles of the minimum of potential and complementary energies are argued in the context of the coupled strain gradient elasticity theory. The basis of the proofs of both variational principles is the equivalent transformation of the stain and strain gradient energy density that allows to avoid the complication related to the presence of the fifth-rank coupling tensor C5 in the equation for the potential energy density and leads to diagonalization of the quadratic form of the stored energy. This transformation enables to inverse Hooke's law, to determine compliance tensors, and to obtain closed-form relation for complementary energy. After that, the proofs of both principles of a minimum of potential and complementary energies are provided in the usual manner adopted in the classical theory of elasticity.

On Nonclassical Dynamic Two-dimensional Models for Thermoelastic Shells with Three Phase-lags

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In the present paper, we consider the dynamic three-dimensional model for general thermoelastic shells with variable thickness, which may vanish on a part of the lateral boundary, within the framework of the nonclassical theory of thermoelasticity with three phase-lags that was proposed by Roy Choudhuri [1], where the classical Fourier law of heat conduction is replaced by an approximation of a generalization that depends on three relaxation times, which are phase-lags of heat flux, temperature gradient, and thermal displacement gradient. By applying the variational approach we investigate the initial-boundary value problem with mixed boundary conditions corresponding to the dynamic model for thermoelastic shells consisting of inhomogeneous anisotropic material in curvilinear coordinates, where the density of surface force and heat flux along the outward normal vector of the boundary are given on certain parts of the boundary, and on the corresponding remaining parts of the boundary displacement and temperature vanish. We construct nonclassical dynamic two-dimensional models for the thermoelastic shells by using the variational approach for implementation of the dimensional reduction idea proposed by I. Vekua [2] in the classical theory of elasticity for plates with variable thickness. For the initial-boundary value problems corresponding to the constructed two-dimensional models, we investigate the existence and uniqueness of solutions in suitable spaces of vector-valued distribution with respect to the time variable with values in weighted Sobolev spaces. Moreover, we study the relationship between the obtained two-dimensional and the original three-dimensional models and prove that the sequence of vector-functions of three space variables restored from the solutions of the two-dimensional problems converges in the corresponding spaces to the exact solution of the three-dimensional initial-boundary value problem and under additional regularity conditions we estimate the rate of convergence.

References

[1] S.K. Roy Choudhuri. On a thermoelastic three-phase-lag model. *J. Thermal Stresses*, vol. 30 (2007), 231–238.

[2] I.N. Vekua. On one method of calculating prismatic shells. *Proc. A. Razmadze Math. Inst. Georg. Acad. Sci.*, vol. 21 (1955), 191–259 (in Russian).

On the Testing Hypothesis of Equality of Two Bernoulli Regression Functions

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We establish the limit distribution of the square-integrable deviation of two nonparametric kernel-type estimations for the Bernoulli regression functions. The criterion of testing the hypothesis of two Bernoulli regression functions. The question as to its consistency is studied. The power asymptotics of the constructed criterion is also studied for certain types of close alternatives.

Summability of Tkebuchava's Means of One and Two Dimensional Trigonometric Fourier Series

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The talk is devoted to characterize the set of convergence of the general logarithmic means of trigonometric Fourier series and also establish a condition that guarantees convergence in the measure of logarithmic means of the two-dimensional Fourier series. We also consider the summability of two-dimensional Tkebuchava means.

References

[1] L. Baramidze, Pointwise convergence of logarithmic means of Fourier series, *Acta Math. Acad. Paed. Nyreg.*, 32, 2 (2016), 225232.

[2] L. Baramidze and U. Goginava, Convergence in measure of logarithmic means of double Fourier series, *Bulletin of TICMI*, 41 (2015).

Nonlinear Frequency- and Field-Dependent Operational Non-Linearities of Soft Piezoceramics d15 Shear Response-Mode

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This plenary lecture investigates the non-resonant driving frequency- and AC electric field-dependent operational non-linearities of the thickness-shear strain piezoelectric coupling coefficient (d15) of poled soft piezoceramic rectangular patches. Therefore, after an introduction on the operational field-dependent non-linearity (FDNL) and the transverse shear actuation mechanism (SAM), an experimental database is analyzed for varying driving frequency (10Hz-1kHz) under fixed input (actuation) voltages in order to find a threshold frequency from which there is no frequency-dependence and, for varying input voltages (20V-400V) under a fixed driving frequency, a threshold voltage from which there is no field-dependence is searched. Then, the Levenberg-Marquardt-Fletcher algorithm is adapted and implemented in order to optimize two-parameter additive and multiplicative power laws for modelling the FDNL of soft piezoceramics. It is found that, while the additive power law is slower than the multiplicative one, they perform similarly for wide ranges of driving frequency (200Hz-1kHz) and actuation voltage (100V-400V). Besides, their two parameters are found to be frequency-dependent.

Multiplicative L^p-Semigroups and Continuous Flows

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We prove a continuity property for a right continuous flow in a convenient topology. We show that a Markovian multiplicative semigroup on an L^p space is generated by a continuous flow. The talk is based on a joint work with Mounir Bezzarga (Tunis) and Iulian Cîmpean (Bucharest).

Riemann-Hilbert Boundary Value Problem with a Shift for Generalized Analytic Functions

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In the talk we consider the dependence of the solvability of the Riemann-Hilbert boundary value problem containing the shift operator on the complex structure induced by the shift operator and hence on the class of pseudoanalytic functions. In particular,we consider a complex structure defined by the shift operator on a manifold representing a Riemann sphere marked by a circle and homeomorphism of this circle [1]. This structure is associated with a pseudoanalytic function of the second kind, which, in turn, uniquely and up to equivalence, determines the pseudoanalytic function of the first kind. The following result is derived from this: the Riemann-Hilbert problem with a shift operator is solvable if and only if the classes of pseudoanalytic functions of the first kind, for which the problem was posed and which are induced by the shift operator, are equivalent to each other [2].

References

[1] Akhalaia, G., Giorgadze G., Makatsaria G., and Manjavidze N. Deformation of complex structures and boundary value problem with shift. Analysis as a Life, (2019), 1-17.

[2] Chakhoiantsi M. Riemann-Hilbert boundary value problem with shift for generalized analytic functions. Proc.VIAM, vol. 72, (2022), 12-15

On Construction of General Solutions of Equations of the Plane Theory of Elasticity in the Coupled Theory of Double-Porosity Materials

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In the present talk, the linear coupled model of elastic double-porosity materials is proposed in which the coupled phenomenon of the concepts of Darcy's extended law and the volume fractions is considered. A twodimensional system of equations of plane deformation is written in the complex form and its general solution is represented by means of analytic functions of a complex variable and solutions of Helmholtz equations. The constructed general solution enables one to solve analytically a sufficiently wide class of plane boundary value problems of the elastic equilibrium of the coupled theory of elasticity for double-porous bodies.

Zero Approximation of Hierarchical Models for Fluids in Pipes of Angular Cross-Sections

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In the $N = 0$ approximation of hierarchical models [1], constructed by Ilia Vekua's dimension reduction method, it is established that the peculiarities of setting the Dirichlet and the Keldysh type boundary conditions (problems) for fluids in pipes of angular cross-sections and the results of the corresponding experiments carried out by Johan (Ivane) Nikuradze (see [2] and also [3]) at the Göttingen University in Prandtl's laboratory are in full consent (G. Jaiani).

We investigate dynamical problem of zero approximation of hierarchical models for fluids. Applying the Laplace transform technique, we reduce the dynamical problem to the elliptic problem which depends on a complex parameter and prove the corresponding uniqueness and existence results. Further, we establish uniform estimates for solutions and their partial derivatives with respect to the parameter at infinity and via the inverse Laplace transform show that the original dynamical problem is uniquely solvable (N. Chinchaladze).

References

[1] G. Jaiani. Mathematical Hierarchical models for fluids. *Book of Abstracts of XIII Annual International Meeting of the Georgian Mechanical Union*, 2022, p. 151.

[2] J. Nikuradze. *Untersuchungen über turbulente*. Strömungen in nichtkreisförmigen Rohren. Ing. Arch. B. I. 1930. S306.

[3] R. Kavtaradze. *Ivane (Johan) Nikuradze. Myth and Reality*. Tbilisi, 2023, pp. 154-155 (in Georgian).

The Functional Dissipativity of Certain Systems of Partial Differential Equations

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In this talk I present some recent results obtained with Vladimir Maz'ya concerning the functional dissipativity of some partial differential operators. This is a new concept of dissipativity which extends the notion of L^p-dissipativity. We remark that the class of operators whose principal part is strictly L^p-dissipative coincides with the class of the so called p-elliptic operators, which was recently considered by several authors. We shall focus in particular on systems, where different concept of ellipticity arise

Exploring the Possibilities of Using Renewable Energy in Georgia in the Context of Climate Change Issues

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The territory of Georgia is rich in solar, wind, hydro, tidal, geothermal and biomass renewable energy sources (can be used for electricity generation, space heating and cooling and water, and for transport), but at present Georgia properly uses only hydro energy. In the context of current regional climate change challenges, Georgia needs cleaner energy from sources that naturally replenish rather than deplete. Wind, thermal and hydrogen energy are among the possible solutions as they are currently considered one of the most promising fuels of the future. In this article, based on a three-dimensional hydrostatic mesoscale model, an air flow over the complex relief of the South Caucasus (Georgia) is studied.

Numerical experiments have shown a strong influence of the orographic effects of the Caucasus (the Likhi Ridge) on the movement of air in the troposphere. Besides, the study of the wind regime and statistical characteristics of the Kolkhinsky region for the period 1960-2021 showed that the wind speeds were significant and important for the development of wind farms in Western Georgia.

This study discusses also one mathematical model that describes the flow of a mixture of natural gas and hydrogen substances in a pipeline. The distribution of pressure and gas flow through a branched gas pipeline has been studied. In addition, ways to reduce transportation costs are being studied, that is, the economic aspect of various methods of transporting hydrogen using hydrogen gas trailers, liquid hydrogen tanks and hydrogen pipelines of various technical levels is being studied.

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Further Exploration of the Mesoscale Atmospheric Boundary Layer

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We continue the study of thermohydrodynamics and humidity processes based on the numerical model of the mesoscale atmospheric boundary layer (MBLA) developed by us.

A new classification of foehns (warm downward wind) is given.

An attempt is made to study foehns based on our numerical model.

A certain opinion is expressed about the movement of a fluid against a gravitational field in different media.

Subgaussian Random Elements in Infinite Dimensional Spaces

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The concept of Subgaussian random variable first appeared in a paper [1] of the well-known French mathematician J.P. Kahane. As a motivation for introducing this concept Kahane cited the cycle of works of Paley and Zygmund in the early 1930s. Later, Subgaussian random variables and processes were discussed and studied by many authors. In our presentation, different definitions of Subgaussian random elements (weakly, T- and F-Subgaussian) in infinite-dimensional spaces are discussed and compared with each other. One of the problems considered will be the problem of characterization of T-Subgaussian random elements in an infinite-dimensional Banach space X. A solution of this problem in the case (which includes the case of an infinite dimensional Hilbert space) when X is a reflexive Banach space of type 2 will be discussed too.

Acknowledgment. The work was partially supported by European Commission HORIZON EUROPE WIDERA-2021-ACCESS03, Grant Project (GAIN), grant agreement no. 101078950.

References

[1] J. P. Kahane, Proprietes locales des fonctions a series de Fourier aleatoires. Studia Math. 19 (1960), 1-25

On Schwarz-Christoffel Mapping of n-gon

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In the talk we provide a comprehensive demonstration of theorems that accentuate the inherent geometric characteristics of a polygon consisting of n sides. These theorems serve as the foundation for constructing the Schwarz-Christoffel mapping, a mathematical technique employed to establish a mapping between the lower half plane and the aforementioned polygon (see [1]). By dealing with into the intricate details of these theorems, we gain a deeper understanding of the relationship between geometry and mapping, enabling us to navigate and comprehend the complex interplay between these two mathematical concepts.

Acknowledgment. The research is partially supported by the GNSF project titled "Problem of factorization and invariants of holomorphic bundles on Riemann surfaces", under grant agreement number $22_3 54$.

References

[1] Giorgadze G., Kakulashvili G. On the Parameter Problem of the SchwarzChristoffel. Mapping and Moduli of Quadrilaterals. Computational Methods and Function Theory. DOI:10.1007/s40315-022-00476-y

On the Exact Solutions of the Gardner Equations Via Tanh-Coth Method

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Using the tanh-coth method the traveling wave special exact solutions of $(1 + 1)$ and $(2 + 1)D$ nonlinear Gardner partial differential equations are represented. The results are expressed through hyperbolic functions and have spatially isolated structural forms.

Some Issues on the Summability of the General Dirichlets Integrals

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In 1974 R. Taberski introduced the special type of generalized Dirichlet's integrals and in the later he established some statements on the approximation properties of $(C, 1)$-means of this integrals.

In this report I am going to introduce (C, α) -means of this integrals and formulate statements on uniform convergence of this means. In the report, also, an equivalent form of (C, α)-kernel, corresponding to the general Dirichlet's integrals, is obtained. This allows us to apply the second mean value theorem for a finite segment, which is one of the decisive moment in the proof of the main theorem.

Dimension Reduction in Elasticity

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Consistent two- and one-dimensional theories are derived by the combination of the uniform-approximation method with the pseudo-reduction technique. Well-known theories are obtained from the three-dimensional theory of elasticity without any a priori assumptions or correction factors.

Numerical Analysis of the Equation of Motion of a Thick-Walled Pipe During Axisymmetric Deformation

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The axisymmetric deformation of a thick-walled pipe under dynamic loading within the limits of elasticity is considered. The pipe is assumed to be infinite in length, loaded with constant pressure along the axis. The radial displacement is considered to be a function of the pipe radius and time. A differential equation of motion with respect to radial displacement is obtained. Which is solved under static and kinematic boundaries and initial conditions.. The numerical solution of the partial differential equation is performed in the mathematical editor Mathcad. An application program has been developed.

The case when the change in the internal pressure gradient is significant over a short period of time is considered as well. Such a case takes place at the stage of unsteady mass flow in the pipe under the action of a given flow velocity at the initial moment. The change in the radial displacement with time is obtained. The dynamic coefficient is determined by comparing the amplitude value of the displacement with the displacement obtained by solving the static problem. The change in radial and circumferential stresses over time is obtained and the reliability of the pipeline is measured.

Construction of Mathematical Models for Reiner Elastic Materials

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The present talk is devoted to construction, using I.Vekuas dimension reduction method, for the Reiner elastic bodies, in the some sense linearized case. The shell-like domain $\Omega \subseteq R^3$ with both Lipschitz and Non-Lipschitz boundaries is occupied by the Reiner elastic material. In the N-th approximation initial-boundary value problems are posed. The peculiarities of settling the boundary conditions for cusped prismatic shells is discussed in the projection $\omega \subseteq R^2$ of Ω.

On Asymptotic Behavior of Solutions of Higher Order Emden-Fowler Type Difference Equations with Deviating Argument

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Consider the Emden-Fowler type difference equation

$$\Delta^{(n)}u(k) + p(k)\left|u(\tau(k))\right|^{\lambda}\operatorname{sign} u\big(\sigma(k)\big) = 0, \tag{1}$$

where $n \geq 2$, $p : \mathbb{N} \to \mathbb{R}$, $\sigma : \mathbb{N} \to \mathbb{N}$, where $\Delta^{(i)}u(k) = \Delta^{(1)} \circ \Delta^{(i-1)}u(k)$ $(i = 1, \dots, n)$, with $\Delta^{(0)}u(k) = u(k), \Delta^{(1)}u(k) = u(k+1) - u(k)$.

The oscillatory properties of the solutions of equation (1) is studied in the case when one of the following conditions are fulfilled

$$\sigma(k) \geq k+1, \quad k \in \mathbb{N}, \quad p : \mathbb{N} \to \mathbb{R}_+ \quad (p : \mathbb{N} \to \mathbb{R}_-), \quad 0 < \lambda < 1;$$

$$\sigma(k) \leq k, \ k \in \mathbb{N}, \ \lim_{k\to+\infty} \sigma(k) = +\infty, \ p : \mathbb{N} \to \mathbb{R}_+ \quad (p : \mathbb{N} \to \mathbb{R}_-), \ \lambda > 1;$$

$$\sigma(k) \leq k, \ k \in \mathbb{N}, \ \lim_{k\to+\infty} \sigma(k) = +\infty, \ p : \mathbb{N} \to \mathbb{R}_+ \, (p : \mathbb{N} \to \mathbb{R}_-), \ 0 < \lambda < 1.$$

The asymptotic behavior of solutions for various classes difference equations is investigated in [1-4].

References

[1] R. Koplatadze, N. Khachidze, Asymptotic behavior of solutions of n-th order Emden-Fowler difference equations with advanced argument. *J. Contemp. Math. Anal.* **56**, 4, 2021, 58–72.

[2] R. Koplatadze, On higher order nonlinear difference equations with Property B. *Semin. I. Vekua Inst. Appl. Math., Rep.* **47** (2021), 42–56.

[3] R. Koplatadze, On asymptotic behavior of solutions of n-th order Emden-Fowler type difference equations with deviating argument. *Semin. I. Vekua Inst. Appl. Math., Rep.* **48** (2022), 15–27.

[4] R. Koplatadze, Oscillatiion criteria for higher order Emden-Fowler type difference equations with delay argument. *Semin. I. Vekua Inst. Appl. Math., Rep.* **49** (2023) (accepted).

Approximation of Multidimensional Fractional Laplacian Based on Gaussians

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In this talk we discuss approximation formulas for the fractional Laplacian $(-\Delta)^{\alpha/2}$, $0 < \alpha < 2$, in the framework of the method approximate approximations. The fractional Laplacian appears in different fields of mathematics (PDE, harmonic analysis, semi- group theory, probabilistic theory) as well as in many applications (optimization, finance, materials science, water waves). If we introduce the convolution

$$\mathcal{N}_\alpha(f)(\mathbf{x}) = c_{n,\alpha} \int_{\mathbb{R}^n} \frac{f(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^{n-2+\alpha}} d\mathbf{y}, \quad c_{n,\alpha} = \frac{2^{\alpha-2}}{\pi^{n/2}} \frac{\Gamma(\frac{n-2+\alpha}{2})}{\Gamma(\frac{2-\alpha}{2})}, \qquad (2)$$

then the fractional Laplacian can be represented as the ordinary Laplacian of the volume potential $\mathcal{N}_\alpha f$,

$$(-\Delta)^{\alpha/2} f(\mathbf{x}) = -\Delta \mathcal{N}_\alpha(f)(\mathbf{x})\,. \qquad (3)$$

We propose a method of an arbitrary high order for the approximation of $\mathcal{N}_\alpha f$ and $(-\Delta)^{\alpha/2} f$, $n \geq 3$, which is based on the approximation of the function f via the basis functions introduced by approximate approximations (cf. [2]), which are product of Gaussians and special polynomials. Then the n-dimensional integral (2) applied to the basis functions is represented by means of a one-dimensional integral where the integrand has a separated representation, i.e., it is a product of functions depending only on one of the variables.

This construction enables to obtain one-dimensional integral representations with separated integrand also for the fractional Laplacian (3), when applied to the basis functions. An accurate quadrature rule and a separated representation of the density f provide a separated representation for $\mathcal{N}_\alpha f$ and $(-\Delta)^{\alpha/2} f$. Thus, only one-dimensional operations are used and the resulting approximation procedure is fast and effective also in high-dimensional cases, and provides approximations of high order, up to a small saturation error. We prove error estimates and report on numerical results illustrating that our formulas are accurate and provide the predicted convergence rate $2, 4, 6, 8$ (cf. [1]).

This is a joint work with Vladimir Maz'ya (Linköping University, Sweden) and Gunther Schmidt (Wias, Berlin, Germany).

References

[1] F. Lanzara, V. Mazya, G. Schmidt, *Fast computation of the multidimensional fractional Laplacian*,Applicable Analysis 101 (2022).

[2] V. Mazya, G. Schmidt, *Approximate approximations*, AMS, Providence (RI), 2007.

Functional Properties of the Solutions of Some Classes of Two-Dimensional Elliptic Systems

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The special classes of the solutions of Carleman-Bers-Vekua equations are introduced. We call them generalized meromorphic functions, and study them both from the point of view of the pure theory of functions, but also from the point of view of the analysis of boundary value problems of the theory of generalized analytic functions. Sufficiently important information about the functional properties of the solutions of two-dimensional elliptic systems are obtained. Due to these results, natural boundary value problems for generalized meromorphic functions are correctly posed and, in some sense, their complete analysis is carried out.

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Duality, Interpolation and Extrapolation for Weighted Grand Morrey Spaces

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Complex interpolation and duality problems for two-weighted grand Morrey spaces are studied. The derived interpolation statements are applied to obtain appropriate boundedness of linear operators of Harmonic Analysis in the aforementioned spaces with Muckenhoupt A_p weights. Further, Rubio de Francía's extrapolation theorem for new weighted grand Morrey spaces $M_w^{p),\lambda,\theta}(X)$ with weights w beyond the Muckenhoupt A_p classes is established. This result, in particular, implies the one-weight inequality for operators of Harmonic Analysis in these spaces for appropriate weights.

The talk is based on the papers [1], [2], [3].

References

[1] E. Gordadze, A. Meskhi and M. A. Ragusa, On some extrapolation in generalized grand Morrey spaces and applications to PDEs, *Electronic Research Archive*, 2023 (accepted).

[2] A.Meskhi, Extrapolation in new weighted grand Morrey spaces beyond the Muckenhoupt classes, *Journal of Mathematical Analysis and Applications*, 2023, https://doi.org/10.1016/j.jmaa.2023.127181.

[3] A. Meskhi, H. Rafeiro and T. Tsanava, Duality and interpolation for weighted grand Morrey spaces, *Trans. A. Razmadze Math. Inst.* **177** (2023), no. 1, 149-155.

Effective Properties of Dispersed Regular and Random Composites

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The study of structurally disordered dispersed patterns and the hidden relationships between the geometric random characteristics of composites and their physical properties is a common focus in various branches of mechanics, mathematics, and physics. Our objective is to address the challenge of providing a constructive quantitative description of the chaos/regularity exhibited by composites. We specifically investigate the macroscopic properties of dispersed regular and random composites with a qualitative analysis of the degree of randomness, anisotropy, and clustering. Our analytical theory of Representative Volume Element (RVE) based on the Riemann-Hilbert problem for a multiply connected domain enables us to investigate random media precisely.

The notion of dispersed random composites is frequently oversimplified in engineering studies. Some engineers take a picture of a random composite from their mind, perform computations with the prescribed physical constants and say that the deal is done. However, various random structures need advanced analysis based on the vast number of observations and computational experiments. The simulation of a class of random composites requires the rules fulfilled for the random number simulation in the comprehensive content of random geometrical objects. It is demonstrated [1, 2] that many universal approximations (Effective medium and Self-consistent approximations, Mori-Tanaka approach, etc.) are methodologically misleading and, at most, asymptotically equivalent to the famous lower-order dilute approximations by Clausius-Mossotti and by Maxwell.

We propose the computationally effective method of structural sums coinciding with the lattice sums for regular composites. In particular, the results yield new high-order analytical exact and asymptotic justified formulas for the effective conductivity and elasticity tensors of dispersed composites with isotropic phases. We introduce and justify quantitative characteristics of randomness, clustering, degree of anisotropy for random composites, degree of interaction of two different components, etc.

References

[1] V. Mityushev, and N. Rylko. Effective properties of two-dimensional dispersed composites. Part I. Schwarz's alternating method. *Comput. Math. Appl.*, 111, pages 50–60, 2022.

[2] V. Mityushev. Effective properties of two-dimensional dispersed composites. Part II. Revision of self-consistent methods. *Comput. Math. Appl.*, 121, pages 74–84, 2022.

Electrodynamics and Rational Mechanics

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This paper wants to draw attention to several issues in electrodynamic field theory of materials and to make way for a rational continuum approach to the subject. To this end the history of Maxwell's equations will be studied to find that, in the end, the starting point are the balances for magnetic flux and electric charge, both in a very general formulation for volumes and for open surfaces, all of which can deform and be immaterial or material. The spatial point-of-view for the description of fields is favored and its advantages in comparison to the concept of material particles is explained. A straightforward answer to the question of how to choose units for the electromagnetic fields most suitably is also presented. The transformation properties of the electromagnetic fields are addressed by rewriting the balances in space-time notation. Special attention is paid to the connection between the two sets of electromagnetic fields through the so-called Maxwell-Lorentz-æther relations. The paper ends with an outlook into constitutive theory of matter under the influence of electromagnetic fields and a discussion on curious developments in context with Maxwell's equations.

Application of the Potential Method in the Theory of Elasticity

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The presentation is devoted to the application of the potential and integral equations methods to the basic and mixed three-dimensional boundary value and boundary-transmission problems of the theory of elasticity. The main goals of the talk are an overview of the results obtained by representatives of the Georgian Mathematical School in the second part of the last century and new developments and extensions of the method to refined mathematical models of solid mechanics taking into account thermal and electro-magnetic physical fields. We treat problems of statics, steady state elastic oscillations and general dynamics for isotropic and anisotropic elastic solids.

Hardy-Littlewood Maximal Function on Locally Compact Abelian Groups

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In this paper we investigate a boundedness of the Hardy-Littlewood maximal operator M in the variable Lebesgue spaces in the context locally compact abelian group. We show that the local Muckenhoupt condition implies the local boundedness of M.

Acknowledgment. The research was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), grant no: FR-22 17770.

References

[1] D.Cruz-Uribe and A. Fiorenza, Variable Lebesgue Spaces, Foundations and Harmonic Analysis. Birkhäuser, Basel (2013).

[2] L. Diening, P. Harjulehto, P. Hästö, M. Rika, Lebesgue and Sobolev spaces with variable exponents. Lecture Notes in Mathematics, 2017. Springer, Heidelberg (2011).

[3] J. Duoandikoetxea, F.J. Martín-Reyes and S. Ombrosi, On the A∞ conditions for general bases, Math. Z. 282 (2016), no. 3-4, 955972.

About the Numerical Solutions of Two Nonlinear Integro-Differential Equations

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In this work we consider the issues of the approximate solutions and the results of numerical computations for the following two practical problems: 1. Non-linear initial-boundary value problem for the J. Ball dynamic beam. 2. Non-linear initial-boundary value problem for the Kirchhoff dynamic string.

The presented article is a direct continuation of the articles [1]-[5] that consider the construction of algorithms and their corresponding numerical computations for the approximate solution of nonlinear integro-differential equations for the J. Ball dynamic beam (see [1]-[3]) and for the Kirchhoff dynamic string (see [4]-[5]).

References

[1] A. Papukashvili, G. Papukashvili, M. Sharikadze. Numerical calculations of the J. Ball nonlinear dynamic beam. Rep. Enlarged Sess. Semin. I. Vekua Appl. Math., 32 (2018), 47-50.

[2] A. Papukashvili, G. Papukashvili, M. Sharikadze. On a numerical realization for a Timoshenko type nonlinear beam equation. Rep. Enlarged Sess. Semin. I. Vekua Appl. Math., 33 (2019), 51-54.

[3] A. Papukashvili, G. Geladze, Z. Vashakidze, M. Sharikadze. On the Algorithm of an Approximate Solution and Numerical Computations for J. Ball Nonlinear Integro-Differential Equation. Rep. Enlarged Sess. Semin. I. Vekua Appl. Math., 36 (2022), 75-78.

[4] G. Papukashvili, J. Peradze. A numerical solution of a string oscillation equation. Rep. Enlarged Sess. Semin. I. Vekua Inst. Appl. Math., 23 (2009), 80-83.

[5] J. Peradze. A numerical algorithm for the non-linear Kirchhoff string equation. Numer. Math., 102 (2005), 311-342.

Analytic Solution to Functional Differential Equations Via Bells Polynomials

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Bells polynomials have been already used in many different fields, ranging from number to operator theory.

These particular polynomials, having combinatorial character by their connection with partitions of integers via Fa' di Brunos formula, have applications in even seemingly distant fields, as:

- the Blissard problem,

- the representation of Lucas polynomials of the first or second kind in two variables,

- the construction of representation formulas for symmetric functions of a sequence of numbers, thus generalizing the classical Newton-Girard formulas,

- the approximation of the Laplace transforms of the composed function.

Generalizations to the multidimensional case have also be obtained.

Bivariate and higher order multi-variate Bells polynomials are applied in this presentation in order to construct approximate solutions to non-linear functional equations.

All the numerical results have been obtained using the computer algebra program Mathematicaľ.

References

[1] Bell, E.T. Exponential polynomials. Annals of Mathematics 1934, 35, 258277.

[2] Bernardini, A., Natalini, P., Ricci, P.E. (2005) Multidimensional Bell polynomials of Higher Order. Comput. Math. Appl., 50, no. 10-12, 16971708.

[3] Cassisa, C., Ricci, P.E. (2000), Orthogonal invariants and the Bell polynomials, Rend. Mat. Appl., (7) 20, 293303.

[4] Comtet, L. (1974) Advanced Combinatorics: The Art of Finite and Infinite Expansions. D. Reidel Publishing Co.; doi.org/10.1007/978-94-010-2196-8.

[5] Hale, J.K., Verduyn, L.S.M. (1993) Introduction to Functional Differential Equations, Springer Verlag: Berlin, Heidelberg, New York.

[6] Natalini, P., Ricci, P.E. (2004) An Extension of the Bell Polynomials, Comput. Math. Appl., 47, 719725.

[7] Noschese, S., Ricci, P.E. (2003) Differentiation of multivariable composite functions and Bell polynomials, J. Comput. Anal. and Appl., 5, 333340.

[8] Riordan, J. (1958) An Introduction to Combinatorial Analysis. J. Wiley and Sons, Chichester.

On Convergence of a Three-Layer Semi-Discrete Scheme for the Nonlinear Dynamic String Equation of Kirchhoff-Type with Time-Dependent Coefficients

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In this talk, we shall investigate an initial-boundary value problem associated with the Kirchhoff-type nonlinear dynamic string equation featuring time-varying coefficients, as discussed in the paper [1]. Our objective is to devise a temporal discretization algorithm capable of approximating the solution to the initial-boundary value problem. To accomplish this, we employ a symmetric three-layer semi-discrete scheme with respect to the temporal variable, where the nonlinear term is evaluated at the midpoint node. This approach facilitates the computation of numerical solutions at each temporal step by inverting linear operators, resulting in a system of second-order linear ordinary differential equations. We have established the local convergence of the proposed scheme, which reveals quadratic convergence with respect to the step size of the time discretization within the local temporal interval. Finally, we have conducted several numerical experiments using the proposed algorithm for various test problems, and the obtained numerical results are in accordance with the theoretical findings.

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References

[1] J. Rogava, Vashakidze, On Convergence of a Three-layer Semi-discrete Scheme for the Nonlinear Dynamic String Equation of Kirchhoff-type with Time-dependent Coefficients. arXiv preprint arXiv:2303.10350, 2023. doi 10.48550/arXiv.2303.10350, URL https://doi.org/10.48550/arXiv.2303.10350.

On the Representation Formula of Solution for a Class of Perturbed Controlled Neutral Functional-Differential Equation

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The neutral functional-differential equation is a mathematical model of a system whose behavior at a given moment depends on the velocity of the system in the past. Many real processes are described by neutral functional-differential equations [1,2]. In the paper the following neutral controlled function-al-differential equation

$$\dot{x}(t) = A(t, x(t))\dot{x}(t-\sigma) + f(t, x(t), x(t-\tau_0), u_0(t)), t \in [t_0, t_1] \qquad (4)$$

with the discontinuous initial condition

$$x(t) = \varrho_0(t), t \in [t_0 - \tau_0, t_0), x(t_0) = x_{00} \ \ (\varrho_0(t_0) \neq x_{00}) \qquad (5)$$

is considered. Let $x_0(t)$ be a solution of problem (1)-(2) and let $x(t)$ be a solution of the corresponding perturbed problem. In the paper, the analytic relation between solutions $x_0(t)$ and $x(t)$ is established. In the representation formula the effects of the discontinuous initial condition and perturbation of the initial data are revealed. Such analytic relation plays an important role in proving the necessary conditions of optimality in the neutral optimization problems and to carry out a sensitivity analysis of mathematical models. The case when $A(t, x(t)) \equiv A(t)$ is considered in [2].

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References

[1] J. Hale, Theory of functional differential equations. Springer-Verlag New York, Heidelberg Berlin, 1977

[2] T. Tadumadze, N. Gorgodze, Variation formulas of a solution and initial data optimization problems for quasi-linear neutral functional differential equat-ions with discontinuous initial condition. Mem. Differential Equations Math.Phys. 63, 2014, 3-97.

On the Convergence of Cesàro Means of Negative Order of Vilenkin-Fourier Series

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In their work [1], Onnewer and Waterman established a sufficient condition guaranteeing the uniform convergence of the Vilenkin-Fourier series for continuous functions. The focus of my talk lies in the investigation of distinct classes of functions characterized by generalized bounded oscillation. Within the context of these classes, we derive conditions that guarantee the uniform convergence of Cesàro means of negative order [2].

References

[1] C. W. Onneweer and D. Waterman. Uniform convergence of Fourier series on groups. I. Michigan Math. J., 18:265273, 1971. ISSN 0026-2285. URL http://projecteuclid.org/euclid.mmj/1029000689.

[2] G. Shavardenidze. On the convergence of Cesáro means of negative order of Vilenkin-Fourier series. Studia Sci. Math. Hungar., 56(1):2244, 2019. ISSN 0081-6906. doi: 10.1556/012.2019.56.1.1422. URL https://doi.org/10.1556/012.2019.56.1.1422

Sheaf-theoretic Approach to the Theory of Pseudoanalytic Functions

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Sheaves of the pseudo-analytic functions on a complex manifold with values in a complex vector space are introduced and the case where the domain is a Riemann surface is considered more closely for which the sheaf of pseudo-analytic differential forms is also defined and related to that of pseudo-analytic functions several propositions regarding these sheaves are proven,

their Čech cohomology groups are characterized. A proof of a Serre-type duality theorem relating zero'th and first cohomology groups of the sheaf of pseudo-analytic functions and differential forms on a compact Riemann surface is given along with a proof of an analogue of the Riemann-Roch theorem [1].

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References

[1] Sikharulidze I. Sheaves of pseudo-holomorphic functions and differential 1-forms. Proc.VIAM, vol. 71, (2021), 50-62

Problems of Steady Vibrations in the Linear Coupled Theory of Thermoviscoelasticity of Porous Materials

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In this talk, a linear mathematical model of thermoviscoelasticity for porous materials is proposed in which the coupled effect of Darcys law and the concept the volume fraction of pores is considered. The basic internal and external boundary value problems (BVPs) of steady vibrations of this model are investigated. Indeed, the fundamental solution of the system of steady vibration equations is constructed. The uniqueness theorems for the regular (classical) solutions of the BVPs of steady vibrations are proved. The single-layer and double-layer potentials are constructed and the basic properties of these potentials are given. Finally, the existence theorems for classical solutions of the BVPs of steady vibrations are proved by means of the potential method and the theory of singular integral equations.

On the Coupled Linear Theory of Thermoelastic Nanoporous Materials with Triple Porosity

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An important class of materials with a new generation of nanostructures are nanoporous solids, which have specific mechanical and physical properties. Many of the engineering problems have coupled physical nature, and it is required to consider several coupled mechanical concepts simultaneously in the models of such materials. Recently, Svanadze [1, 2, 3, 4] introduced the linear models of elastic and thermoelastic materials with single and double porosity in which the coupled phenomenon of Darcys law and the volume fraction concept of pore network is considered.

In this work, the 3D linear coupled mathematical model of thermoelastic nanomaterials with triple porosity is presented in which the coupled effect of Darcy's law and the volume fraction concept of three levels of pores (macro-, meso- and micropores) is proposed (for details, see [5]) and the following results are obtained: The fundamental solution of the governing system of steady vibration equations is constructed explicitly by using elementary functions. Greens formulae are obtained and the uniqueness theorems for classical solutions of the boundary value problems of steady vibrations are proved. The basic properties of surface and volume potentials are established. The existence theorems for classical solutions of above mentioned boundary value problems are proved by using the potential method and the theory of singular integral equations.

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References

[1] M. Svanadze, Boundary integral equations method in the coupled theory of thermoelasticity for porous materials, *Proceedings of ASME, IMECE2019, Vol. 9: Mechanics of Solids, Structures, and Fluids*, V009T11A033, November 1114, 2019. DOI: 10.1115/IMECE2019-10367.

[2] M. Svanadze, Potential method in the coupled linear theory of porous elastic solids, *Math. Mech. Solids*, 25(3), 768-790, 2020.

[3] M. Svanadze, Potential method in the coupled theory of elastic double-porosity materials, *Acta Mechanica*, 232(6), 2307-2329, 2021.

[4] M. Svanadze, On the coupled theory of thermoelastic double-porosity materials, *J. Thermal Stresses*, 45(7), 576-596, 2022.

[5] M. Svanadze, On the coupled linear theory of thermoelasticity for nanomaterials which triple porosity, *Mech. Res. Commun.,* 2023 (in press).

Two Methods of the Numerical Solution of One System of Nonlinear Partial Differential Equations

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Two different approaches were used to construct approximate solutions of the initial-boundary problem for the system of equations corresponding to a two-dimensional nonlinear model [1]. Such type models are studied in many works (see, for example, [2]-[7] and references therein). The first approach used a decomposition method based on an averaged model [3]. An appropriate scheme and necessary algorithms for computer implementation were built. The program was compiled and calculations were made for various tests. The second approach used a variable direction difference scheme [4]. Necessary algorithms for computer implementation were also built for this method. The number of operations was determined for both methods. The time required for the realization of the algorithms and the accuracy of the numerical experiments were compared with each other. An analysis of the obtained results was carried out, and appropriate conclusions were drawn.

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References

[1] G. I. Mitchison, A model for vein formation in higher plants. Proc. R. Soc. Lond. B. volume 207, pages 79-109, 1980.

[2] J. Bell, C. Cosner, W. Bertiger, Solution for a flux-dependent diffusion model. SIAM J. Math. Anal. volume 13, pages 758-769, 1982.

[3] T. A. Dzhangveladze, Averaged model of sum approximation for a system of nonlinear partial differential equations (Russian). Proc. I.Vekua Inst. Appl. Math. volume 19, pages 60-73, 1987.

[4] T. A. Jangveladze, The difference scheme of the type of variable directions for one system of nonlinear partial differential equations. Proc. I.Vekua Inst. Appl. Math. volume 42, pages 45-66, 1992.

[5] T. Jangveladze, M. Nikolishvili, B. Tabatadze, On one nonlinear two-dimensional diffusion system. Proc. 15th WSEAS Int. Conf. Applied Math. (MATH 10), pages 105-108, 2010.

[6] T. Jangveladze, Z. Kiguradze, B. Tabatadze, M. Gagoshidze, Comparison of two methods of numerical solution of Mitchison biological system of nonlinear partial differential equations. International Journal of Mathematics and Computers in Simulation, volume 11, pages 25–31. 2017.

[7] T. Jangveladze, Z. Kiguradze, M. Gagoshidze, Economical difference scheme for one multi-dimensional nonlinear system. Acta Mathematica Scientia. volume 39, no. 4, pages 971-988, 2019.

Heat Transfer with the Flow of Conducting Fluid in Circular Pipes with Finite Conductivity under Uniform Transverse Magnetic field

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The flow of conducting, viscous fluids in circular pipes under the transverse magnetic field is studied theoretically.The correlation of Hartman's figure, Poisales figure, Reynold's figure and conductivity of walls are considered.

Hartmann carried out the pioneer work in the study of steady magnetohydrodynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. The magnetohydrodynamic interaction under constant uniform pressure gradient is clearly demonstrated. Later Chang and Lundgren solved the same problem with the channel wall of different conductivity. It is important to the basic understanding of magnetohydrodynamics to extend this problem to include the effect of transient pressure gradient. This paper carries out such work.

It should be pointed out that the ordinary hydrodynamic channel flow under time-dependent pressure gradient is not a vailable in the literature except for Uchidas treatment of pipe flow under oscillatory pressure gradient. His work is related to a limiting case of the present work if the fluid is assumed to be electrically nonconductive, or if there is no applied magnetic field.

The problem is simplified by assuming fully developed laminar flow and a perfectly conducting channel wall, which leads to linear partial differential equations. Thus the equations are amenable to the methods of the Laplace transformation.

To the Approximate Solution of Boundary Value Problems for Ordinary Differential Equation

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Using a method developed in [1] we construct numerical schemes for the generalized solutions of boundary value problems for ordinary differential equations of the second-order and by the iterative process find the corresponding solution. We also consider the problem of solving the same boundary value problems when the right-hand side of the equation represents oscillatory functions. In this case, usually, the iteration process presents non-stable behavior. In the present talk we show how these difficulties could be overcomed.

References

[1] Vashakmadze, T.S. The Theory of Anisotropic Elastic Plates, Kluwer Academic Publishers, Dordrecht/ Boston/ London, 1999, xv+240p. https://doi.org/10.1007/978-94- 017-3479-0, Publisher Springer, Hardcover ISBN 978-0-7923-5695-0, Softcover ISBN978- 90-481-5215.

Speaker's Index

C O N T E N T S

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