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Dedicated to the

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ON
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Dedicated to the 90th Anniversary of the Iv. Javakhishvili Tbilisi State University
and
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გამოყენებითი მათემატიკის თანამედროვე პრობლემები

ეძღვნება

ივ. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის 90 წლისთავს

და

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თეზისების კრებული

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ორგანიზატორები

ივ. ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი
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Contents

SOME APPLICATIONS OF ALMOST DISJOINT FAMILIES OF SETS

A. B. Kharazishvili
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It is well known that families of sets with certain combinatorial properties play an important role in many fields of contemporary mathematics. Among such families, almost disjoint families of sets are of special interest and have numerous applications (see, e.g., [1]-[3]).

Let E be an infinite set, $\{X_i : i \in I\}$ be a family of subsets of E. This family is almost disjoint if $card(X_i) = card(E)$ for each $i \in I$ and $card(X_i \cap X_j) < card(E)$ for any two distinct indices $i \in I$ and $j \in I$.

The notion of an almost disjoint family of sets was first introduced and investigated by Sierpiński. He observed that there exists (within **ZF** theory) an almost disjoint family $\{X_i : i \in I\}$ of subsets of ω such that $card(I)$ is equal to the cardinality of the continuum. Moreover, Sierpiński proved that the following statement holds true in **ZFC**: for every infinite set E, there exists an almost disjoint family $\{X_i : i \in I\}$ of subsets of E such that $card(I) > card(E)$ (see [1]). This classical result can be generalized in various directions. In particular, the next statement is valid.

Theorem. *Let E be an infinite set and let $\{Y_k : k \in K\}$ be a family of subsets of E satisfying the following two conditions:*

(1) $card(K) \leq card(E)$;

(2) $card(Y_k) = card(E)$ for each $k \in K$.

Then there exists an almost disjoint family $\{X_i : i \in I\}$ of subsets of E such that $card(I) > card(E)$ and $card(X_i \cap Y_k) = card(E)$ for all $i \in I$ and $k \in K$.

Some applications of this theorem are presented.

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ONE ALGEBRAIC CHARACTERIZATION OF UNCOUNTABLE CARDINALS LESS THAN THE CARDINALITY OF THE CONTINUUM

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In despite of the results of Gödel and Cohen (see [1], [2]) and numerous other publications, the problem of the continuum is one of the most interesting questions of set theory and each nontrivial result connected with the continuum problem is of certain interest.

We give one algebraic characterization of the set of cardinal numbers: $\{\lambda : \omega < \lambda < 2^{\omega}\}$, where ω is the least infinite cardinal. The similar characterization of the set $\{\lambda : \omega \leq \lambda \leq 2^{\omega}\}$ is done in [3].

Let's specify some terms: if A is a nonempty set and $f : A \to A$ is a function, then for any element $a \in A$ the set $f^{-1}(\{a\})$ is called the *level* of the function f; if $G_f \subset A^2$ is the graph of the function $f : A \to A$, then instead of the term "an automorphism of the structure (A, G_f) ", we shall use the term "an automorphism of the function f "; the function f is called *rigid* if it has only trivial automorphism.

For any cardinal number λ, let $P(\lambda)$ and $R(\lambda)$ denote the following propositions:

$P(\lambda)$ - "There exists a rigid function $f : \lambda \to \lambda$ whose one level has λ many elements and other levels have at most two elements",

$R(\lambda)$ - "There is no function $g : \lambda \to \lambda$ which has precisely λ many automorphisms".

Theorem. *Let $\lambda \neq 0$ be a cardinal. Then the following equivalence is true*:

$$\omega < \lambda < 2^\omega \;\Leftrightarrow\; P(\lambda)\&R(\lambda).$$

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SOME APPLICATIONS OF SET-THEORETICAL METHODS IN THE THEORY OF QUASI-INVARIANT MEASURES

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Let $(G, \cdot)$ be an arbitrary group and let $Y \subset G$. We say that Y is G-absolutely negligible in G if, for any σ-finite G-quasi-invariant measure μ on G, there exists a G-quasi-invariant measure $\mu^{'}$ on G extending μ and such that $\mu^{'}(Y) = 0$ (see, e.g., [1]-[3]).

Theorem. *Let $(G, \cdot)$ be an uncountable group and let card(G) be a regular cardinal. Then, for each $X \subset G$ with card(X) = card(G), there exists a G-absolutely negligible set $Y \subset G$ such that $X \cdot Y = G$.*

The proof of the above-mentioned theorem for $\alpha = \omega_1$ can be found in [1] and [2].

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ON FREE CYCLIC MV-ALGEBRAS

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We prove that the 1-generated free MV-algebra is isomorphic to a quotient of the disjoint union of the 1-generated free MV_n-algebras. We consider a partial reduct of MV-algebras in which all the binary operation are forgotten. An algebra $A = (A; \oplus, \odot, \neg, 0, 1)$ is said to be an MV-algebra if it satisfies the following equations: (i) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$,
(ii) $x \oplus y = y \oplus x$ (iii) $x \oplus 0 = x$, (iv) $x \oplus 1 = 1$, (v) $\neg 0 = 1$, (vi) $\neg 1 = 0$, (vii) $x \odot y = \neg(\neg x \oplus \neg y)$, (viii) $\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x$.
It is known that the variety $\mathbb{MV}$ of all MV-algebras is not locally finite and that, remarkably, it is generated by all simple finite MV-algebras. In addition, we have that the subvarieties of $\mathbb{MV}$, which are generated by finite families of simple finite MV-algebras, are locally finite.
Finite simple MV-algebras are finite MV-chains, i.e. finite linearly ordered MV-algebras, of any cardinality $n \geq 2$. We focus on the subvarieties of $\mathbb{MV}$, $\mathbb{V}_n$ generated by all finite MV-chains of cardinality $m \leq n + 1$ with $1 \leq n, n < \omega$.
We define special embeddings $\varepsilon_{ij} : F_{\mathbb{V}_i}(1) \to F_{\mathbb{V}_j}(1)$ between free algebras in corresponding locally finite subvarieties $\mathbb{V}_i$ and $\mathbb{V}_j$ generated, respectively, by all finite MV-chains of cardinality $m \leq i$ and $m \leq j$ preserving negation $\neg$ and constants 1 and 0. Then $\{(F_{\mathbb{V}_i}(1), \varepsilon_{ij}) | ji \in \omega \text{and} i \leq j\}$ is a direct system, let D be its direct limit. D can be seen as the quotient of the disjoint union $\biguplus\{F_{\mathbb{V}_k}(1) : k \in \omega\}$ over the equivalence relation E defined by xEy if, and only if, $x \in F_{\mathbb{V}_i}(1), y \in F_{\mathbb{V}_j}(1)$ for some $i \leq j$ and $\varepsilon_{ij}(x) = y$. We define the operations $\oplus$ and $\odot$ converting D into MV-algebra. Our assertion is expressed in following

Theorem 1. $(D, \oplus, \otimes, \neg, 0, 1)$ *is cyclic free MV-algebra.*

ON STRONGLY EFFECTIVELY LEVELABLE SETS

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Let $\omega = \{0, 1, 2, \ldots\}$, $\{\varphi_i\}_{i\in\omega}$ be an acceptable numbering of the partial computable functions and $\{\Phi_i\}_{i\in\omega}$ step countable functions which constitute a complexity measure in the sense of Blum [1]. Namely, assume that:

(1) $\varphi_i(x)$ converges if and only if $\Phi_i(x)$ converges, and

(2) the function

$$M(i, x, y) = \begin{cases} 1, & \text{if } \Phi_i(x) = y, \\ 0, & \text{otherwise}, \end{cases}$$

is (total) computable.

Definition (Blum and Marques [2]). A c.e. set A is effectively levelable if there exist computable functions f and r such that for all i, l, if $W_i = A$ and φ_l is computable, then

1) $W_{f(i,l)} = A$,

2) $(\exists_x^\infty)\ \big[\Phi_i(x) > \varphi_l(x)\ \text{ and }\ \Phi_{f(i,l)}(x) \le r(x)\big]$.

If, in addition, there exist a computable function g such that

3) $(\forall\, y)(\forall\, i)(\forall\, l)\ \big(y \in W_{f(i,l)} \setminus A \Longrightarrow y < g(\langle i, l\rangle)\big)$,

then we say that A is strongly effectively levelable.

Definition (Filotti [3]). A c.e. set A is undercreative if there exist computable functions δ and δ' such that

(1) $(\forall\, j)\ \big(W_{\delta(j)} = \overline{W}_{\delta'(j)}\big)$,

(2) $(\forall\, j)\ \Big(W_j \cap A\ \text{ finite }\ \Longrightarrow W_{\delta(j)} \subseteq \overline{W_j \setminus A}\ \ \&\ \ W_{\delta(j)} \cap \overline{W_j \cup A} \neq \varnothing\Big)$.

If, in addition, there exist a computable function g such that

(3) $(\forall\, j)(\forall\, y)\ \big(y \in W_{\delta(j)} \setminus A \Longrightarrow y < g(j)\big)$,

then we say that A is effectively undercreative.

Theorem. A c.e. set A is strongly effectively levelable if and only if it is effectively undercreative.

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A RELATION BETWEEN SOME TRANSLATION-INVARIANT BOREL MEASURES ON $\mathbb{R}^\infty$

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We study relations between translation-invariant Borel measures constructed in [1] and [2]. In particular, we show that Baker's measures [1] and [2] are not equivalent. Also, we show that Baker's measure [2] is not a final version of the infinite-dimensional Lebesgue measure on $\mathbb{R}^\infty$.

Let $\mathbb{R}$ be the real line and $\mathbb{R}^\infty$ stand for the space of all real-valued sequences, equipped with the Tychonoff topology (i.e., the product topology).

Let us denote by $\mathcal{B}(\mathbb{R}^\infty)$ the σ-algebra of all Borel subsets in $\mathbb{R}^\infty$. Further, let $\mathcal{R}_2$ be the class of all infinite dimensional rectangles $R \in \mathcal{B}(R^\infty)$ of the form $R = \prod_{i=1}^\infty R_i,\ R_i \in \mathcal{B}(\mathbb{R})$, such that $0 \le \prod_{i=1}^\infty m(R_i) := \lim_{n\to\infty} \prod_{i=1}^n m(R_i) < \infty$, where m denotes one-dimensional classical Borel measure on $\mathbb{R}$.

Let λ_1 and λ_2 be translation-invariant Borel measures constructed in [1] and [2], respectively.

The following theorem gives an answer on R.Baker's certain question [2]: "We do not know whether or not the measure λ_2 coincides with the original version (i.e., with the measure λ_1)."

Theorem 1 *The measures λ_1 and λ_2 are not equivalent and $\lambda_2 \ll \lambda_1$, where $\ll$ denotes the symbol of absolutely continuity.*

The next statement shows that the measure λ_2 is not a final version of infinite-dimensional Lebesgue measure in $\mathbb{R}^\infty$(cf. [2]).

Theorem 2 *Let S be a class of all Borel subsets in $\mathbb{R}^\infty$ which can be covered by the union of countable family of elements of $\mathcal{R}_2$. Then there exists such a translation-invariant Borel measure in $\mathbb{R}^\infty$ that $\mu(E) = \lambda_2(E)$ for $E \in S$, $\mu \ll \lambda_2$, and μ and λ_2 are not equivalent.*

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SECTION: *Applied Logics and Programming*

MACHINE TRANSLATION FROM GEORGIAN LANGUAGE

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In the article is considered the problem of machine translation from Georgian to other language, using special method for morphological and syntactic analysis. With this purpose there are prepared programming means, which simplify the whole process of translation ([1-3]).

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THE METHODS OF THE EFFECTIVE DATA SEARCH FOR THE LISTED STRUCTURES

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The development of the science and technics provoked the informational outburst that is expressed in the informational volume rise in the world, which submits to the exponential rise in the time unit. In such conditions the natural language automatized treatment and search by modern, effective methods is getting especially actual. In this article the search algorithm "wave" is proposed and its realization by parallel programming principles using broadening LISP on the multi-processor computer or LISP-machines is considered.

INTERNATIONAL MONEY TRANSFER SYSTEM "ALPHA EXPRESS"

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This system is for money transfer without opening an account. Each bank (agent) in the system sends addresses of his branches and service offices in the main database. In the main database all this information is written and it also divides service fee between sender, receiver and center. System contains client and server modules and both are installed in each service office. System works in following operating systems: Windows XP, Windows Vista and Windows 2003 Server. Server module works with one or more client module by the local network. Each service office changes information about money transfer with higher office by the Internet. This process is done by the synchronization operation. In that moment all information about service offices in the system is updated automatically (update means adding a new office, removing existing offices and so on). Money transfer system gives unique number (transfer code) to each transfer. Sender should tell this code to receiver. With this code it is known from where and at what time transfer was sent. Sender office has regular information about transfer (whether it was delivered or not at the address, whether receiver got it and so on) by the synchronization operation.

System contains five parts (programs):

1. AlphaClient.Exe - In the client module (for the service offices).
2. AlphaMain.Exe - In the client module (for the administration of the main database).
3. AlphaSocket.Exe - In the server module (Socket Server). It connects system users to the server module and also changes information between service offices.
4. Synchro.Exe - It is used for the synchronization operation. It is loaded when server of the service office changes information with higher office.
5. AlphaApp.Exe - In the server module (Application Server). It is used to connect client module and branches of the bank to the Socket Server. It is loaded for each connection. This program works with database, checks usernames and passwords, processes and sends information to the addressee.

A SYSTEM FOR RULE-BASED PROGRAMMING WITH SEQUENCE AND CONTEXT VARIABLES

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ρLog programs are collections of transformation rules described by strategies that provide a declarative semantics for answering queries. The expressive power of a rule-based programming language can be estimated by looking at: (1) the kind of queries that can be answered, (2) the strategic constructs recognized by the language, and (3) the programming constructs for specifying transformation rules. In our work we describe the current implementation and capabilities of system ρLog. Our implementation is based on the theoretical work proposed by Marin and Kutsia [MK06]. The computational core of the system is based on powerful pattern matching mechanism with individual, functional, context and sequence variables which can be instantiated, respectively, by terms, functional symbols and variables, contexts (i.e. terms with a single occurrence of the special constant hole), and term sequences. Regular hedge and tree constraints restrict the possible values of sequence and context variables, respectively. Such restrictions are defined by associating a variable with a regular expression that represents the set of its admissible values. Strategies provide a mechanism to control complex rule-based computations in a highly declarative way. Since the inference engine in ρLog is the SLDNF-resolution with leftmost literal selection, we chose Prolog as the implementation language. This system allows users to implement various solvers, tools for querying and translating XML, etc. in a declarative programming style. We illustrate these capabilities of the system on examples.

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ONE-DIMENSIONAL BIN PACKING CLASS: FAST BOUNDS OF OBJECTIVE FUNCTIONS

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We research a class of 14 combinatorial models [1], that are semantically near to the known One-Dimensional Bin Packing Problem (1DBPP). All models have a practical applications in the different areas: One-Dimensional Stock Cutting, placing of files on CDs, Scheduler Theory, a Container Loading and so on. A general description of class is following. Given a set of items $A = \{a_1, a_2, \ldots, a_n\}$, to each item a_k corresponds a positive integral number (weight) $s(a_k)$, $s(a_k) \geq s(a_{k+1})$. We need to divide the initial set A into M disjoint subsets $A_1, A_2, \ldots, A_M$, $\bigcup_{i=1}^{M} A_i = A$, $A_i \cap A_j = \emptyset$, $i \neq j$, $i, j \in [1, M]$ with the given properties. These properties use the features of each model. For a **Bin Packing** (**Model 1**) we wish to divide the A into a minimum number (M) of subsets: $C_i = \sum_{a_k \in A_i} s(a_k) \leq B$, $i \in [1, M]$, the B is a bin capacity, C_i is a content of i-th bin. For a **Bin Covering** (**Model 2**) we wish to divide the A into a maximum number (M) of subsets:

$C_i \geq B$, $i \in [1, M]$, the B is a bin quota. For a **Bin Packing & Bin Covering Hybrid** we wish to divide A into a minimum (**Model 3**) or maximum (**Model 4**) number (M) of subsets: $B_{\min} \leq C_i \leq B_{\max}$, $i \in [1, M]$, where the parameters $B_{\min}$ and $B_{\max}$ are the lower and upper thresholds respectively. All models 1-13 we lead to a **Model 0** in process of solving. Given a fixed list of bins $L = \{B_1, B_2, \ldots, B_M\}$, the B_i is a capacity of i-th bin. We need to pack the A to the L: $C_i \leq B_i$, $i \in [1, M]$. An answer is YES, if we can pack the A to the L and NO otherwise. All models are the **NP**-hard problems to find the optimal solutions for the arbitrary initial data and are solved in practice as rule using the approximate algorithms. But approximate solutions it is necessary to evaluate somehow. In this case we find the bounds of objective function: a lower bound $LB(A)$ for the tasks "to minimum" and an upper bound $UB(A)$ for the tasks "to maximum". One can write "$UB(A) =$ *approximate solution*" for the tasks "to minimum" and "$LB(A) =$ *approximate solution*" for the tasks "to maximum". Thus, we get $LB(A) \leq OPT(A) \leq UB(A)$ for the both cases. Since $OPT(A)$ is not known, we consider a value $p = ((UB(A) - LB(A))/LB(A)) \cdot 100\%$ as a measure of closeness to $OPT(A)$. In case $p = 0$ we claim "*approximate solution = optimal solution*". A finding of both fast and quality bounds has a practical importance especially for the tasks of large parameters m (a number of different weights). We offer an estimation technology to form the fast bounds of objective functions for our models. This technology may be used as base to make the bounds of objective functions for the other models that use an idea to divide the initial set A into the disjoint subsets with the given properties. The estimation technology is of two blocks: a reduction block and an estimation content block. The first block removes the dominant groups of weights from the initial data. The second block estimates the existence of reasonable solutions for a given number (M) of subsets. The reduction block lets to lead the initial problem A to a problem A^0 with a property $OPT(A) = OPT(A^0) + OPT(A')$, where $A = A^0 \cup A'$, $A^0 = \bigcup_{i=2}^{H} A^i$, $A^i = \bigcup_{j=1}^{M_i} A_j^i$, $A' = A \setminus A^0$. Each subset A_j^i is a dominate group of i weights. Hence the A^2 is a list of dominate pairs, the A^3 is a list of dominate triplets, the A^4 is a list of dominate quarters and so on. The estimation content block builds a corridor $[C_i^{\min}, C_i^{\max}]$, that any reasonable solution $\{C_i\}$ will pass within $[C_i^{\min}, C_i^{\max}]$, $C_i^{\min} \leq C_i \leq C_i^{\max}$, $i \in [1, M]$, $C_i \geq C_{i+1}$, the M is a fixed number of bins. Our approach to form the fast bounds of objective functions can be used in practice for the largest parameters m (50000 and more) as an alternative to the other approaches (e.g. a known **LP**-approach with use a **L**inear **P**rogramming technique) if a time factor is very important. A program is written in Microsoft Visual C++.

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ABOUT 1-STAGE VOICE MANAGED GEORGIAN INTELLECTUAL COMPUTER SYSTEM

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In the paper we will discuss the main technologic aim of one year subproject "Foundations of Logical Grammar of Georgian Language and its Methodological and Technological Applications" of TSU State Priority Program "Free and Complete Programming Inclusion of a Computer in the Georgian Natural Language System". The final aim of the TSU State Priority Program "Free and Complete Programming Inclusion of a Computer in the Georgian Natural Language System" is a complete mathematical foundation of Georgian Language and Thinking (GLT) and construction of Basic Georgian Intellectual Computer System, which, in turn, is just a computer equipped with the mathematical theory of GLT. The theoretic purpose of the one year subproject "Foundations of Logical Grammar of Georgian Language and its Methodological and Technological Applications" is to completely represent and fully systematize new sights and results elaborated on the basis of researches pursued with the purposes of creating Mathematical Theory of GLT (MTofGLT). The subproject's technological aim is to construct the first stage Voice Managed Georgian Intellectual Computer System $(VMGICS_1)$ through integration of the first stage MTofGLT and Georgian speech recognizer and synthesizer systems, which are already elaborated by our researches. The $VMGICS_1$ is a systemic unit of its basic sub-systems $GWmanager_1,\ GWreader_1,\ GOlistener_1,\ GWintel_1$ and non-basic, i.e. produced sub-system $GOintel_1$,which, in turn, is constructed through integration of the basic subsystems $GWreader_1,\ GOlistener_1,\ GWintel_1$. Computer, which will be software by $VMGICS_1$ system allows user to give user-defined standard and nonstandard commands via Georgian voice signal and will be equipped with:

1. Abilities to solve Georgian logical tasks and critically analyze Georgian logical judgments, given by Georgian written and oral languages;

2. Abilities to translate from Georgian written and oral languages into English and German written Languages by using mathematical language as a mediator Language.

3. General thinking abilities in Georgian written and oral languages. It means that these systems equipped computer with abilities to make morphologic, syntactic, logical and common-semantic spell-checking of texts of Georgian written and oral languages. In the same time, it means that the systems equipped computer with abilities to reduce any sentence of core part of Georgian written and oral languages (CPofGWL and CPofGOL) to core part of Georgian thinking language (CPofGTL), and vice versa (i.e. to produce any well-formed expression of CPofGTL in CPofGWL and CPofGOL).

4. Fully content non-sensitive and partially content sensitive reading possibilities in Georgian written language and partially content non-sensitive and partially content sensitive listening possibilities in Georgian oral language.

In the paper we will present in detail 1-Stage Voice Managed Georgian Intellectual Computer System and, also, we will present 1-Stage Logical Grammar of Georgian Language and 1-Stage Voice Recognition and Synthesizer Systems for Georgian as a theoretic grounding of the 1-Stage Voice Managed Georgian Intellectual Computer System.

GEORGIAN LANGUAGES THESES

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The main purpose of one year subproject "Foundations of Logical Grammar of Georgian Language and its Methodological and Technological Applications" of the TSU State Priority Program is to completely represent and fully systematize new sights and results elaborated on the basis of researches pursued with the purposes of creating Mathematical Theory of GLT $(\mathrm{MTofGLT})$. In the paper we will discuss natural specifics of GLT, which in fact play the main role in our researches and our lingual ideology. It must be noticed that our general views on Language was formed as a result of our researches and of that general semantic approaches which was developed by Sh. Pkhakadze on the basis of his Notation Theory (NT). General view on a language of Montague, which is based on Frege's artificial mathematical language, differs from the Chomsky's view, who tried to study natural languages in the confines of natural languages. According to our view, like Chomsky's one, it is almost unfair using elements which are out of that natural language and thinking, which we are studying. This is as clear as unfairness of researching one's physical world using elements, which are out of this physical world. In spite of this clear fact, we do not exclude mathematical language from the lingual researches. Though, differently from Montague, we do not consider mathematical language as a separately standing artificial language from the natural languages. Moreover, we think that any natural language and thinking, in its basic part, is founded on Primary Mathematical Language (PML) and Primary Mathematical Concepts (PMC), which naturally exist in any human. Herewith, according to our lingual ideology primary semantic date, universally existing in the human, is the Primary Mathematical Theory (PMT), constructed on the basis of the PML and PMC. It is clear that this PMT subconsciously exists in a human. That is why we call the PML of this PMT as primary natural language of subconscious stage $(\mathrm{PNLofSS})$. We must mention, that one of the basic aim of our researches is full recovering of the Georgian Thinking Language (GTL) and, also, full recovering PML and PMT, which are standing at the grounding level of GTL. Now it should be clear, why we call Frege's artificial mathematical language as a natural language of subconscious level, and natural languages as natural languages of conscious stage. Our researches, which are based on prof. Sh. Pkhakadze's NT and his sufficiently General $\Im$ mathematical language $\Im_{SGML}$, lead us to declare the following very important statements from ideological as well as from technological point of view, which we call Georgian language Theses:

1. **Georgian Language is a result of formally extension of PML.**
2. **Georgian language is a language of $\Im_{SGML}$ type.**

In this way, according to our lingual ideology we are ready to make following three important conclusions: As it is known, Chomsky was interested not only in complete machine foundation of natural languages, but he also researched universal genetic linguistic program. Also, Montague researched a universal grammar. It is also well known about Wierzbicka's attempts to find universal linguistic concepts. The processes to find so-called linguistic universals always accompany lingual researches:**We think that while we are researching natural languages, it is more important to research various lingual specifics of other natural languages, than to research linguistic universals. Herewith, we say that as linguistic universals may consider only elements of PML, which by its different extensions gives different natural languages.**

MODIFIED $M\tau SR$ THEORY

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We defined contracted symbols and proved main theorems about properties of contracted forms and operations on it in [1]. This results, as well results stated in [2], concludes that I-IV, II' and IV' types of contracted symbols are rational by following meaning:

"On one side System is so general that we can define almost every contracted symbols which is used in classical mathematical theories. On second side System has so rich properties we have guarantee of freedom to do operating on contracted forms."[1]. Therefore in formal and "shinaarsuli" mathematical theories is desired to use I-IV, II and IV types of contracted symbols. Note that III, IV and IV' types of contracted symbols represent generalization of I, II and II' types of contracted symbols respectively. III, IV and IV' types of contracted symbols are quite complex and hard to use in automated reasoning. The priority of τSR [3] logic is that its alphabet gives possibilities by using only I, II and II' types of definitions of contracted symbols to introduce all operators which was introduced by rational system. In this paper we are building artificial modification of $M\tau SR$ logic which will support an implementation of mathematical problems into computer.

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SECTION: *Function theory*

ON THE SUMMABILITY OF FOURIER TRIGONOMETRIC SERIES BY THE GENERALIZED CESÀRO METHOD

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Let for a sequence (α_n) of real numbers ($\alpha_n > -1, n = 1, 2, ...$)

$$\sigma_n^{\alpha_n}(x,f) = \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n-1} S_\nu(x,f)/A_n^{\alpha_n}$$

where $A_\nu^k = (k+1)(k+2)...(k+\nu)/\nu!$ and $S_\nu(x,f)$ are partial sums of trigonometric Fourier series of f. If (α_n) is a constant sequence ($\alpha_n = \alpha,\ n = 1, 2, ...$) then $\sigma_n^{\alpha_n}$ coincides with the usual Cesáro σ_n^{α}-means ([1], Chapter III).

$C[0,2\pi]$ denotes the space of 2π-periodic continuous functions with the norm $\|f\|_C = \max_{x\in[0,2\pi]} |f(x)|$, and H^ω designates the well-known class of functions introduced by Nikol'skii [2].

Definition 1. (Nash [3]). Let Φ be a sequence of positive numbers. We say that 2π-periodic continuous function $f \in \Phi$ if $\|f\|_C \le 1$ and for every real numbers a, b ($|b-a| \le 2\pi$) uniformly in $x \in [0, 2\pi]$

$$\left|\int_a^b f(x+t)\cos ntdt\right| \le 1/\Phi(n),\ n = 1, 2, ...$$

Theorem 1. Let (α_n) be a sequence of numbers of the interval (0,1). If $f \in H^\omega \frown \Phi$ then

$$\left\|\sigma_n^{-\alpha_n}(\cdot,f) - f(\cdot)\right\|_C \le \frac{C(\omega,\Phi)}{\alpha_n(1-\alpha_n)}\omega\left(\frac{1}{n}\right)\left[\left(\frac{n}{\Phi(n)\omega(1/n)}\right)^{\alpha_n} - 1\right],\ n = 1, 2, ... ,$$

where $C(\omega,\Phi)$ is a positive constant depending only on the indicated parameters.

This theorem for constant sequence ($\alpha_n = \alpha,\ n = 1, 2, ...$) proved by Sat.

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ON NORDLANDER'S CONJECTURE IN THE THREE-DIMENSIONAL CASE

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Let X be a real normed space and let $S = \{x : x \in X, ||x|| = 1\}$. The modulus of convexity of X is the function $\delta_X : [0, 2] \to [0, 1]$, defined by

$$\delta_X(a) = \inf\{1 - \frac{1}{2}||x + y|| : x, y \in S, ||x - y|| = a\}.$$

If H is an inner product space then $\delta_H(a) = 1 - \frac{\sqrt{4-a^2}}{2}$. Nordlander discovered that for an arbitrary normed space X the following inequality is true

$$\delta_X(a) \le \delta_H(a), \forall a \in (0, 2).$$

Day proved that if

$$\delta_X(a) = \delta_H(a), \forall a \in (0, 2)$$

then X is an i.p.s.. Nordlander conjectured that the same conclusion should hold if the above equality takes place for some fixed $a \in (0, 2)$. Alonso and Benitez, making use of real two-dimensional arguments, proved the validity of Nordlander's conjecture for $a \in (0, 2)\backslash D$, where $D = \{2\cos(k\pi/(2n)) : k = 1, \ldots, n-1; n = 2, 3, \ldots\}$, and they gave counterexamples for $dimX = 2$ and $a \in D$. The validity of Nordlander's conjecture for any $a \in (0, 2)$ was open in the case $dimX \ge 3$ and in the present paper we give an affirmative answer on it.

ON THE MODULUS OF CONTINUITY OF K-TH ORDER OF CONJUGATE FUNCTIONS OF MANY VARIABLES

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The estimates of the partial moduli of continuity of k-th order of the conjugate functions of many variables are obtained in the space $C(T^n)$. The exactness of these estimates are established by proper examples.

THE RESTRICTED MAXIMAL OPERATORS OF FEJÉR MEANS OF DOUBLE WALSH-FOURIER SERIES

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Weisz **[Cesaro summability of two-dimensional Walsh-Fourier series, Trans. Amer. Math. Soc. 348(1996), 2169-2181**] proved that restricted maximal operators $\sigma^*, \widetilde{\sigma}^{*,(t)}$ of Fejér means of double Walsh-Fourier series and conjugate Walsh-Fourier series are bounded from Hardy space H_p to the space L_P when $P > 1/2$. In **[U. Goginava, Maximal operators of Fejér means of double Walsh-Fourier series. Acta Math. Hungar. 115 (2007), no. 4, 333–340]** it is proved that restricted maximal operators of Fejér means of double Walsh-Fourier series is not bounded from the Hardy space $H_{1/2}$ to the space weak-$L_{1/2}$.

We shall prove a stronger result than unboundedness of the maximal operators from the Hardy space $H_{1/2}$ to the space weak-$L_{1/2}$. In particular, we prove that there exists a martingale $f \in H_{1/2}$ such that

$$\|\sigma^* f\|_{weak-L_{1/2}} = +\infty \ \ , \ \left\|\widetilde{\sigma}^{*,(t)} f\right\|_{weak-L_{1/2}} = +\infty \, .$$

ON THE SUFFICIENT CONDITIONS OF ABSOLUTE CONVERGENCE OF DOUBLE TRIGONOMETRIC SERIES

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Let $f \in L(T^2),\ T = [-\pi; \pi]$ and let

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{mn}(f) e^{i(mx+ny)}$$

be is double Fourier series. Let

$$\rho_{mn}(f) = |C_{mn}(f)| + |C_{-mn}(f)| + |C_{m-n}(f)| + |C_{-m-n}(f)|, \ \ m \geq 1, n \geq 1,$$

$$\rho_{0n}(f) = |C_{0n}(f)| + |C_{0-n}(f)|, \ \ \rho_{m0}(f) = |C_{m0}(f)| + |C_{-m0}(f)|, \ \ m \geq 1, n \geq 1.$$

As usual, for $f \in L_p(T^2), 1 \leq p < \infty$ denote by

$$\omega\left(\frac{1}{m}, \frac{1}{n}, f\right)_{L_p}, \ \ \omega_1\left(\frac{1}{m}, f\right)_{L_p}, \ \ \omega_2\left(\frac{1}{n}, f\right)_{L_p},$$

respectively, mixed and partial moduli of continuity of the function f in the norm of the space $L_p(T^2)$. Denote by A_r the set of those functions for which

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \rho_{mn}^r(f) < \infty.$$

Theorem 1. Let $f \in L_p(T^2),\ p \in (1;2],\ r \in (0;q],\ q = \dfrac{p}{p-1}$. If

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\omega^r\left(\frac{1}{m},\frac{1}{n},f\right)_{L_p},\quad (mn)^{-\frac{r}{q}} < \infty,$$

$$\sum_{m=1}^{\infty}\omega_1^r\left(\frac{1}{m},f\right)_{L_p}\quad m^{-\frac{r}{q}} < \infty,$$

$$\sum_{n=1}^{\infty}\omega_2^r\left(\frac{1}{n},f\right)_{L_p},\quad (n)^{-\frac{r}{q}} < \infty,$$

then $f \in A_r$.

It is known that there exists the function of two variables with continuous partial derivatives whose Fourier trigonometric series is not absolutely convergent.

Theorem 2. Let $f_x{}', f_y{}' \in L_p(T^2), p \in (1;2], r \in \left(\frac{q}{q+1}; q\right]$. If

$$\sum_{m=1}^{\infty} m^{-r\frac{q+2}{q}+1}\left[\omega^r\left(\frac{1}{m},\frac{1}{m}f_x'\right)_{L_p} + \omega^r\left(\frac{1}{m},\frac{1}{m}f_y'\right)_{L_p}\right] < \infty.$$

then $f \in A_r$.

ON THE APPROXIMATION NUMBERS OF HARDY-TYPE OPERATORS IN BANACH FUNCTION SPACES

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We study the approximation numbers of the Hardy integral operator T given by

$$Tf(x) = v(x)\int_0^x u(t)f(t)dt$$

for $x \in \mathbb{R}^+ := [0,\infty)$, u, v are functions satisfying the local integrability condition and $f \in E$, where E is Banach function space on $\mathbb{R}^+$ with property: the pair of Banach function spaces (E, E) satisfy weak Minkowski's inequality. These operators appear naturally in the theory of differential equations and it is important to establish when operators of this kind have properties such as boundedness, compactness and to estimate their eigenvalues, or their approximation numbers. We prove that under appropriate conditions on u and v the approximation numbers $a_n(T)$ of T satisfy

$$c_1\int_0^{\infty}|u(t)v(t)|dt \le \liminf_{n\to\infty} na_n(T) \le \limsup_{n\to\infty} na_n(T) \le c_2\int_0^{\infty}|u(t)v(t)|dt.$$

ON UNCONDITIONAL CONVERGENCE IN BANACH SPACES WITH UNCONDITIONAL BASES

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Characterization of the Banach spaces isomorphic to the Banach space c_0 in terms of unconditionally converging series are obtained.

ON WEYL MULTIPLIERS FOR ALMOST EVERYWHERE SUMMABILITY OF DOUBLE BLOCK-ORTHOGONAL SERIES

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Block-orthonormal systems were introduced by Gaposhkin [1]. He proved that the Menshov-Rademacher‘s theorem and the strong law of large numbers are valid for such systems in certain conditions. In [3-4] there were obtained some results on convergence and summability of series with respect to block-orthonormal systems. In particular, Menshov-Rademacher’s and Gaposhkin’s theorems were generalized and the exact Weyl multipliers for the convergence and summability almost everywhere of series with respect to block-orthogonal systems were established in the case, when Menshov-Rademacher’s and Gaposhkin’s theorems are not true.

The double block-orthonormal systems were introduced in [4] and was considered the convergence almost everywhere of double series with respect to double block-orthonormal systems.

Definition. Let $\{M_p\}$ and $\{N_q\}$ be the increasing sequences of natural numbers and

$$\Delta_{p,q} = (M_p, M_{p+1}] \times (N_q, N_{q+1}], (p, q \geq 1).$$

Let $\{\phi_{mn}\}$ be a system of functions from $L^2((0,1)^2)$. The system $\{\phi_{mn}\}$ will be called a $\Delta_{p,q}$ - orthonormal system if:

1) $\| \phi_{mn} \| = 1, m = 1, 2, ..., n = 1, 2, ...,$;

2) $(\phi_{ij}, \phi_{kl}) = 0$, for $(i, j), (k, l) \in \Delta_{p,q}$, $(i, j) \neq (k, l)$, $(p, q \geq 1)$.

Moricz [2] established the analogue of Kacmarz‘s theorems for (C,1,1), (C,1,0) and (C,0,1) summability almost everywhere of double orthogonal series.

Now we generalized Moricz’s theorems [2] and established exact Weyl multipliers for (C,1,1), (C,1,0) and (C,0,1) summability almost everywhere of double block-orthonormal series.

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INTEGRAL REPRESENTATION OF ANALYTIC AND PLURIHARMONIC FUNCTIONS IN THE UNIT BALL

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We denote by B the unit ball in the space $\mathbb{C}^n$, and by S the sphere, i.e. $B = \left\{z \in \mathbb{C}^n : \ |z|^2 = \sum_{j=1}^{n} |z_j|^2 < 1\right\}$, $S = \partial B = \{z \in \mathbb{C}^n : \ |z| = 1\}$. A point $z \in \mathbb{C}^n$ is written in the from $z = rt$, where $r = |z|$, and $t = z : |z|$. The normalized Lebesgue measure $d\mu(z) = cdx_1dy_1 \cdots dx_ndy_n$ is written in terms of spherical coordinates (r,t) as $d\mu|z| = 2n \cdot r^{2n-1}drd\sigma(t)$, where $d\sigma(t)$ is the induced Lebesgue measure on S normalized so that $\int\limits_B d\mu|z| = \int\limits_0^1 \int\limits_S r^{2n-1}drd\sigma(t) = 1$.
Denote by $RP(B)$ the class of all functions in B that are the real parts of analytic functions.
Let D be an open set in $\mathbb{C}^n$. A function $u \in C^2(D)$ is called pluriharmonic if it satisfies the differential equations $D_j\overline{D}_ku = 0$, $j = \overline{1,n}$, $k = \overline{1,n}$.
Let $0 < p < 1$, $n = \dim \mathbb{C}^n$. An analytic (pluriharmonic) function f in B is said to belong to $A^p(B)$ $(h^p(B))$ if

$$\|f\| = \int\limits_0^1 \int\limits_S (1-r)^{n(p^{-1}-1)-1}|f(rt)|\, dsd\sigma(t) < +\infty.$$

Consider the functions $K(z,t) = c_n(1-r)^{n(p^{-1}-1)-1}/(1-\langle z,t\rangle)^{np-1}$, $H(z,t) = 2K(z,t)-1$, $P(z,t) = \operatorname{Re} H(z,t)$, $z \in B$, $t \in S$. $\langle z,t\rangle = \sum_{j=1}^{n} z_j\bar{t}_j$, $z = (z_1,\dots,z_n)$, $t = (t_1,\dots,t_n)$, $c_n = \Gamma(np^{-1}) : n!\gamma(np^{-1}-n)$. Let $f \in L^1(B)$, and introduce the operators:

$$K[f](z) = \int\limits_0^1 \int\limits_S K(z,t)f(rt)r^{n-1}\, drd\sigma(t),$$

$$H[f](z) = \int\limits_0^1 \int\limits_S H(z,t)f(rt)r^{n-1}\, drd\sigma(t),$$

$$P[f](z) = \int\limits_0^1 \int\limits_S P(z,t)f(rt)r^{n-1}\, drd\sigma(t).$$

The following theorems are valid.

Theorem 2. *Let $f \in A^p(B)$. Then for $\forall\, z \in B$, $f(z) = K[f](z)$.*

Theorem 3. *Let $f \in A^p(B)$. Then for $\forall\, z \in B$, $f(z) = H[\operatorname{Re} f](z) + i \operatorname{Im} f(0)$.*

Theorem 4. *Let $f \in RP(B)$. Then the following propositions are equivalent:* 1) $f \in h^p(B)$, 2) $\forall\, z \in B$, $f(z) = P[f](z)$.

Theorem 5. *Let $f \in H(B)$. Then the following propositions are equivalent:* 1) $f \in A^p(B)$, 2) $\forall\, z \in B$, $f(z) = K[f](z)$, 3) $\forall\, z \in B$, $f(z) = P[f](z)$.

ON THE GENERALIZED SPLINE LINEAR ALGORITHMS AND CONDITION OF ITS CENTRALITY

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Central algorithms φ^s are the best because their local error is equal to the Chebyshev radius of information I and is minimal. Observe that a central algorithm is an optimal error algorithm, but not every optimal error algorithm is central. By us is established a condition for Ritz method in energetic Hilbert spaces to be a central spline algorithm. Central algorithms have deviation equal to one, but they are often difficult to obtain or might even not exist. On the other hand there are easy to implement so called linear algorithms φ^L. It is proved that to the linear and central algorithms have direct connection spline algorithms φ^s. Existing spline algorithm depends only on the proximality of the subspace $KerI$ and does not depend on the solution operator, if this one does not depend on the set of problem elements.
We introduce the definition of generalized spline as strongly proximality $KerI$ in a Frechet space E. It is proved that for any nonadaptive information I cardinality 1 generalized interpolating spline exists if and only if the space E is reflexive and quojection. This result can be considered as generalization of James well known theorems for a Frechet space. In the same place are given definitions of generalized central and generalized spline algorithms.
The extended Ritz method is used for obtaining an approximate solution of the equation $A^\infty u = f$ in the Frechet space $D(A^\infty)$. The restriction of a self-adjoint operator A defined on a dense set $D(A)$ of the Hilbert space H to the Frechet space $D(A^\infty)$ is considered. This restriction is denoted by A^∞ and coincides with the restriction of A^N from the Frechet space H^N to $D(A^\infty)$ (Due to this notation, the space $D(A^\infty)$ acquires a new meaning that differs from classical case where $D(A^\infty)$ was whole symbol, where A^∞, if taken separately, meant nothing). It is proved that approximate solutions of this equation constructed by the extended Ritz method do not depend on the number of norms that generate the topology of the space $D(A^\infty)$. Hence this approximate method is both a generalized central and generalized spline algorithm. Examples of self-adjoint and positive-definite strong degenerate elliptic differential operators satisfying the above conditions are given. The validity of theoretical results in case of the harmonic oscillator operator is confirmed by numerical calculations.

SECTION: *Computational Mechanics*

THE THERMO-ELASTICITY PROBLEM OF DEFORMATION OF FLEXIBLE MULTILAYERED SHELLS OF REVOLUTION WITH LAYERS OF VARIABLE THICKNESS IN A REFINED SETTING

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A version of a refined theory of deformation of flexible multilayered shells of revolution with layers of variable thickness which counts non-homogenity of deformation of lateral shear strains is considered. Using the approach for shells of revolution we get a non-linear boundary problem for the system

of ordinary differential equations. The solution of this problem is obtained using the methods of linearization and discrete orthogonalization.
Based on the given approach we investigate on the concrete examples the stress-deformed state of shells under the action of temperature-field. Some numerical results will be also discussed.

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DIFFERENCE METHOD OF SOLVING THE DARBOUX PROBLEM FOR NONLINEAR KLEIN-GORDON EQUATION

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The first Darboux problem for cubic nonlinear Klein-Gordon equation is considered, with nonhomogeneous condition on characteristic line. Solvability and convergence of the corresponding difference scheme is investigated in Sobolev spaces.

ON APPLICATION OF ALTERNATING TO PERTURBATION TECHNIQUES METHOD TO SINGULAR INTEGRAL EQUATIONS CONTAINING AN IMMOVABLE SINGULARITY

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Problems of approximate solution of some linear nonhomogeneous operator equation is studied with an approach alternative to asymptotic method. Our alternative method is based on representation of unknown vector over the small parameter with orthogonal series instead of asymptotic one. In such a case system of three-point operator equations of special structure in received. For system solving a certain regular method is used. On the basis of the suggested method the programming production is created and realized by means of computer.

A NUMERICAL ALGORITHM FOR A NONLINEAR INTEGRO-DIFFERENTIAL OSCILLATION EQUATION

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The following initial boundary value problem is considered

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} - \left(\alpha + \beta \int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right) \frac{\partial^2 u}{\partial x^2} = 0, \tag{1}$$

$$0 < x < L, \quad 0 < t \le T,$$

$$\begin{gathered} u(x,0) = u^{(0)}(x), \quad \frac{\partial u}{\partial t}(x,0) = u^{(1)}(x), \\ u(0,t) = u(L,t) = 0, \quad \frac{\partial^2 u}{\partial x^2}(0,t) = \frac{\partial^2 u}{\partial x^2}(1,t) = 0, \end{gathered} \tag{2}$$

$$0 \le x \le L, \quad 0 \le t \le T,$$

where α and β are positive constants, $u^{(0)}(x)$ and $u^{(1)}(x)$ are given functions. Equation (1) describes a dynamic state of the beam [1].
To construct an approximate algorithm for the solution of problem (1), (2) we use Galerkin's method and a symmetrical difference scheme. The stability and accuracy of the proposed algorithm is studied.

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AN APPROXIMATE SCHEME FOR CAUCHY TYPE SINGULAR INTEGRALS ON THE INFINITE INTERVAL

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An approximate scheme for Cauchy type singular integrals on the infinite interval is elaborated. Application to numerical solution of some singular integral equations is indicated.

VON KARMAN TYPE SYSTEMS OF EQUATION FOR POROUS, PIEZO AND VISCOUS ELASTIC PLATES

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One of the most principal objects in development of mechanics and mathematics is a system of non-linear differential equations for elastic isotropic plate constructed by von Karman. This system with corresponding boundary conditions represents the most essential part of the main manuals in elasticity theory. In spite of this in 1978 Truesdell expressed an idea about neediness of "Physical Soundness" of von Karman system. This circumstance generated the problem of justification of von Karman system. Afterwards this problem is studied by many authors, but with most attention it was investigated by Ciarlet [1]. In particular, he wrote: "the von Karman equations may be given a full justification by means of the leading term of a formal asymptotic expansion" [1,p.368]. This result obviously is not sufficient for the justification of "Physical Soundness" of von Karman system as representations by asymptotic expansions is dissimilar, leading terms are only coefficients of power series without any "Physical Soundness". Based on the [2], the method of constructing such anisotropic inhomogeneous 2D nonlinear models of von Karman–Mindlin-Reissner(KMR) type for binary mixture of poro, piezo and viscous elastic thin-walled structures with variable thickness is given, by means of which terms take quite determined "Physical Soundness". The corresponding variables are quantities with certain physical meaning: averaged components of the displacement vector, bending and twisting moments, shearing forces, rotation of normals, surface efforts. In addition, the corresponding equations are constructed, taking into account the conditions of equality of the main vector and moment to zero. By choosing parameters in the isotropic case from KMR type system (having a continuum power) the von Karman system as one of the possible models is obtained. The given method differs from the classical one by the fact, that according to the classical method, one of the equations of von Karman system represents one of Saint-Venant's compatibility conditions, i.e. it's obtained on the basis of geometry and not taking into account the equilibrium equations. This remark is essential for dynamical problems. In this case, along the quantities describing the vertical directions and surface wave processes, it is necessary to take into account the quantity $\Delta\partial_n\Phi$, corresponding to wave processes in the horizontal direction. Further for isotropic and generalized transversal elastic plates in linear case from KMR the unified representation for all 2D BVP (considered in terms of planar expansions and rotations) is obtained. So this report is devoted to problems of constructing the KMR type 2D BVP with respect to spatial variables for binary mixture of visco-poro-elastic and piezo-electric and electrically conductive elastic thin-walled structures. At first will be introduced the nonlinear dynamic 3D (with respect to spatial variables) mathematical model for poro, piezo and viscous elastic media. At last we shall report the new iterative methods and numerical schemes for solving the corresponding BVP for 2D nonlinear systems of differential equations of KMR type.

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GROUPS OF REDUCED G-IDENTITIES FOR NILPOTENT GROUPS OF STEP 2

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In paper [1], G. Baumslag, A. Myasnikov and V. Remeleslennikov expounded the fundamental principles of algebraic geometry over a fixed group G. In particular, this paper introduces the category of G-groups, the notion of a G-free group and the category of algebraic sets over the group G. The morphisms of the latter category are verbal mappings.
Denote by $F(X)$ a free group with a basis $X = \{x_1, \dots, x_n\}$, $n \geq 1$. In [1] it is shown that a free product $G[X] = G * F(X)$ is **a G-free group**.
Let $v(g_1, \dots, g_m, x_1, \dots, x_n)$ be an element of the G-free group $G[X]$ and H be some G-group. We call this element **a G-identity** for H if for any set of elements $h_1, \dots, h_n$ from H the value $v(g_1, \dots, g_m, h_1, \dots, h_n)$ is unit in H. The notions of a G-manifold and a G-verbal subgroup are introduced in the usual manner.
We denote by V the set of all identities without coefficients, which are true on the group G, and by $V(G)$ its corresponding verbal subgroup from $G[X]$. We call it **a subgroup of proper identities**. Along with V, we consider the set V_c of all G-identities which are true on the group G. We denote by $V_c(G)$ the verbal subgroup from $G[X]$ corresponding to the set V_c. It is obvious that $V_c(G) \supseteq V(G)$. The factor-group $V_{n,red}(G) = V_C(G)/V(G)$ is called the **group of reduced G-identities of rank** n.
We are interested in the following questions:
1. What is a structure of the group $V_{n,red}(G)$?
2. We want to know how the G-manifold generated by the group $V - var(G)$ is finitely based.
The following results are obtained in [2] and [3]. Let G be a 2-stepped nilpotent group. We introduce the following notation. Let G' be the commutant of the group G, $Z(G)$ be its center, $\overline{G} = G/G'$. Using this notation, the following theorem is valid.

Theorem 6. *Let G be a nilpotent group satisfying the condition $var(G) = \mathfrak{n}_2$. Then $V_{n,red}(G) \cong (\overline{Z})^n$, where $(\overline{Z})^n = \underbrace{\overline{Z} \times \cdots \times \overline{Z}}_{n\text{-times}}$.*

This theorem is used to prove the following

Theorem 7. *Let G be a nilpotent group of step 2 and $var(G) = \mathfrak{N}_2$. Then the G-manifold is finitely based if and only if $G - var(G)$ is finitely generated.*

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SUBGROUP LATTICE OF HALL'S W–POWER GROUPS

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Ph. Hall introduced one class of groups which he called W–power groups or simply W–groups (for the definition and some properties of W–groups see [1,2,3]). This class is a generalization of the notion of W–module to the case of an arbitrary locally nilpotent group. The importance of W–power nilpotent groups in the general theory of abstract groups is due to the fact that any torsion-free, finitely generated nilpotent groups are embedded into some W–power group. Hall generalized some results from the theory of nilpotent groups [1].

All W–subgroup of W–group G generate lattice.

Theorem 1. *A W–power group G over an arbitrary binomial ring W has a modular lattice of subgroups if and only if G is Abelian.*

Theorem 2. *If G is a locally cyclic W–power group over an arbitrary binomial ring W, then a lattice $L(G)$ is distributive.*

Theorem 3. *A W–power group G over a principal ideal domain has a distributive lattice of W–subgroups if and only if it is locally cyclic.*

Similar results for Lie algebras are stated by A.Lashkhi [4].

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ON GENERAL ESTIMATE OF THE SINGULAR SERIES CORRESPONDING TO POSITIVE QUATERNARY QUADRATIC FORMS AND ITS IMPROVEMENT IN SOME SPECIAL CASES

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Let $r(f,m)$ denote the number of representations of $m \in \mathbb{N}$ by a positive definite, integral, primitive, quaternary quadratic form f of determinant d and let $\rho(f,m)$ be the corresponding singular series. The sum of the series is the main term of asymptotic formulas for $r(f,m)$.

In papers [1] and [2] are investigated problems concerned to the best estimates for $\rho(f,m)$ with respect to d and m.

The paper [3] proves a more precise estimate

$$\rho(f,m) = O(d_0^{-\frac{1}{3}} d_1^{-\frac{1}{2}} m \ln b(d_1) \ln\ln b(m)) \;\text{ if }\; n = 4,$$

where $d = d_0 d_1$, $d = \prod\limits_{p|2^5 d} p^{h(p)}$, $d_0 = \prod\limits_{p|2^5 dp|m} p^{h(p)}$, $d_1 = \prod\limits_{p|2^4 dp\nmid m, p>2} p^{h(p)}$,
$h(p) \geqslant 0$ if $p > 2$; $h(2) \geqslant -4$.
The last estimate as a general result for the quaternary quadratic forms of above mentioned type is unimprovable in a certain sense.
We construct some special type quaternary forms for which the mentioned estimate may be sharpened.

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ON THE BASIS OF THE SPACE OF CUSP FORMS OF THE LEVEL 8

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The basis of the space of cusp forms $S_{k/2}(\Gamma_0(8), \chi)$ is constructed for any integer k and character $\bmod 8$.

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A LATTICE OF FULLY INVARIANT SUBGROUPS OF A COTORSION GROUP

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The report considers a lattice of fully invariant subgroups of a cotorsion hull of a direct sum of torsion-complete groups. It is shown that this lattice is isomorphic to a lattice of filters of a semilattice made up of infinite matrices.

PERIODIC GROUPS WITH PRESCRIBED ELEMENT ORDERS

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For a periodic group G, denote by $\omega(G)$ the *spectrum*, i.e. the set of element orders, of G. It is obvious that $\omega(G)$ is finite if and only if G is of finite exponent. Thus, a group with finite spectrum is not necessarily a locally finite group.
The talk contains a survey of known spectra which ensure the local finiteness of corresponding groups. The following recent results are typical.

Theorem 1. *Let* $\omega(G) = \{1, 2, 3, 5, 6\}$. *Then* G *is locally finite*.

Theorem 2. *Let* $\omega(G) = \{1, 2, 3, 4, 8\}$. *Then* G *is locally finite*.
Theorem 1 is obtained in collaboration with A.S. Mamontov.

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ON DISTRIBUTION OF PRIME NUMBERS

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Relatively to the distribution of prime numbers were stated many interesting questions. It should be justified that to answer those questions were needed centuries, but for some ones the answers are unknown at this time too.
I present in my talk new results about distribution of the prime numbers.
Theorem 1. For every natural number n, it takes place the inequality $\pi(n) \geq \sqrt{n} - 1$.
Theorem 2. If $ngt;33$, $n \in \mathbb{N}$ we have $\pi(n)lt;\frac{n}{3}$.
Theorem 3. For every natural number $n \in \mathbb{N}$ there exists a pair of prime numbers $(p;q)$ such that $p = n - t, q = n + t$ and $t \in \mathbb{N}$.

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ON THE DIMENSION OF SOME SPACES OF GENERALIZED THETA-SERIES

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Let

$$Q(x) = Q(x_1, \cdots, x_f) = \sum_{0 \le r \le s \le f} b_{rs} x_r x_s$$

be an integral positive definite quadratic form in an even number f of variables.
Let $R(\nu, Q)$ denote the space of the spherical polynomials $P(x)$ of even order ν with respect to $Q(x)$ and let $T(\nu, Q) = \{\vartheta(\tau, P_, Q) \;:\; P \in R(\nu, Q)\}$ is the space of generalized theta-series, where

$$\vartheta(\tau, P_, Q) = \sum_{x \in \mathbb{Z}^f} P_(x) z^{Q(x)}, \qquad z = e^{2\pi i \tau}, \;\; \mathrm{Im}\tau > 0, \;\; \tau \in \mathbb{C}.$$

In [1], [2] is obtained the upper bound for the dimension of the space $T(\nu, Q)$ for some quaternary quadratic forms. Here is calculated the dimension of the space $T(4, Q)$ and $T(8, Q)$.
We calculate the dimension of the space $T(6, Q)$.

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ON CUSP FORMS OF WEIGHT 9/2

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It is well known that $r(n; f)$ - the number of representations of a positive integer n by the positive definite quadratic form f, can be expressed as the sum

$$r(n; f) = p(n; f) + v(n; f),$$

where $p(n; f)$ is singular series. This series has been exhaustively studied and the formulae for computing it are known. The second summand $v(n; f)$ is a Fourier coefficient of a cusp form. It can be expressed in terms of modular forms as follows:

$$\theta(\tau; f) = E(\tau; f) + X(\tau),$$

$$\theta(\tau; f) = 1 + \sum_{n=1}^{\infty} r(n; f) Q^n$$

where $\tau \in H = \{\tau : Im\tau > 0\}$, $Q = exp^{2\pi i \tau}$, $X(\tau)$ is a cusp form, and

$$E(\tau; f) = 1 + \sum_{n=1}^{\infty} \rho(n; f) Q^n$$

is an Eisenstein series corresponding to f; If the genus of the quadratic form f contains one class, then according to Siegel's theorem $\theta(\tau; f) = E(\tau; f)$ and therefore the problem of obtaining "exact" formulas for $r(n; f)$ is solved completely. If the genus contains more then one class, then it is necessary to find a cusp form $X(\tau)$. In this paper, using modular properties of the so-called generalized theta-functions with characteristics, cusp forms of weight $\frac{9}{2}$ are built. With the help of these results, the exact formulas for the number of representations of positive integers by diagonal quadratic forms in nine variables can be obtained.

SECTION: *Complex Analysis and Application*

CAUCHY-LEBESGUE CLASSES FOR Q-HOLOMORPHIC VECTORS

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B.Bojarski showed that the methods of the theory of generalized analytic functions admit far-going generalization in case of the first order elliptic system, which has the following complex form

$$\partial_{\bar{z}} w - Q(z)\partial_z w + A(z)w + B(z)\overline{w} = 0 \tag{1}$$

where $Q(z), A(z), B(z)$ are given square matrices of order n, $Q(z)$ is a matrix of special quasidiagonal form.

Later G.Hile noted that the property which appears to be essential for generalization of the theory of analytic functions to general elliptic systems of the form (1) is the self-commuting property of the matrix $Q(z)$, i.e.

$$Q(z_1)Q(z_2) = Q(z_2)Q(z_1). \tag{2}$$

We investigate the generalized Cauchy-Lebesgue classes, which are natural classes for the study of discontinuous boundary value problems posed for the system (1) satisfying the condition (2).

COMPLEX POINTS OF TWO-DIMENSIONAL SURFACES

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As is known, in many problems of complex analysis and differential geometry an important role is played by the complex points of submanifolds of complex vector space. Similar situations arise in algebraic geometry and singularity theory. In this talk we will present several general results on complex points of real two-dimensional surfaces it two-dimensional complex vector space.

Namely, we consider a smooth compact surface X given by two real polynomial equations in $\mathbb{R}^4 \cong \mathbb{C}^2$ and describe an effective method of finding the number of its complex points. To this end we construct a system of real polynomial equations in auxiliary variables such that the number of real solutions to this system is equal to the number of complex points of X. Next, we apply the Bruce formula for the Euler characteristic and obtain an explicit formula for the number of complex points of X in terms of the local topological degree of an explicitly given polynomial mapping. Some corollaries of our results and several concrete examples will be also presented.

THE MIXED BOUNDARY VALUE PROBLEM FOR THE SPHERICAL ZONE, WHEN THE DISPLACEMENT VECTOR IS INDEPENDENT FROM THE THICKNESS COORDINATE

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In the present paper the non-shallow spherical bodies of shell type are discussed. Their internal geometry is alterable towards the thickness (the non-shallow spherical shells). Here we consider the case, when the displacement vector is independent from the thickness coordinate x_3.
We introduce the isometric coordinate system at the sphere. The equilibrium equations and formulas of components of the displacement vector are expressed using three holomorphic functions in the system of isometric coordinates. The mixed boundary value problem for the spherical shell, the stereographic production of which in the equatorial space gives us the circular zone, has been solved. We have to find the elasticity balance, when some components of stresses and displacements are marked on the boundary points.
The circular zone is bounded with two concentric circles.
The formulas of the displacement vector are expressed with the help of three holomorphic functions. The holomorphic functions are introduced by series and on this way the elasticity balance has been found.

ON SOME PROBLEMS IN THE THEORY OF GENERALIZED ANALYTIC FUNCTIONS

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Generalized analytic functions which we will consider are the solutions of equations of the form $\partial_{\overline{z}} = aw + b\overline{w}$ (CBV). As it is well known, such functions share many properties with analytic functions [1]. Nevertheless there is also essential difference in properties between analytic and generalized analytic functions (see e. g. [2]). Some important classes (e. g. that of multivalued analytic functions) of generalized analytic functions are not investigated from the point of view of pure mathematics.
In the talk is considered one of the problems not sufficiently studied in the theory of generalized analytic functions, notwithstanding the fact that by the end of the XX century the theory of analytic functions acquired its conclusive form (see [5]).
I have in mind relationship between the Riemann-Hilbert boundary value problem [5], [11] and the monodromic Riemann-Hilbert problem [5]. An obvious problem, studied only on the plane [3],[4] and partially on compact Riemann surfaces [9] is that of singular equations CBV. With the aid of methods from algebraic topology it is proved that similarly to the case of ordinary differential equations, a CBV singular equation induces a vector bundle with an L_p-connection on a Riemann surface [6].
In this context, an analog of the 21st Hilbert problem consists in the problem of constructing on a Riemann surface a holomorphic bundle with a connection whose holonomy representation coincides with a given representation.
Investigation of moduli of complex structures is performed using the Beltrami equation. To investigate complex structures on a vector bundle over a Riemann surface one can use the matrix Beltrami equation and properties of the space of Q-holomorphic vectors over Riemann surfaces (see [8],[10]). In terms of Q-holomorphic vectors one can also investigate deformations of complex structures on vector bundles.
The way of using systems of CBV type in contemporary mathematical physics does in our opinion follow the pattern similar to the ideology developed in [7].

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ON THE PROBLEM OF LINEAR CONJUGATION FOR SOME BOUNDARY LINES

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We consider the boundary value problem of linear conjugation

$$\Phi^+(t) = G(t)\Phi^-(t) + f(t), \quad t \in \Gamma$$

in case where Γ is an unclosed Carleson arc, $f(t) \in L_p(\Gamma)$, or $f(t) \in L^{p(\cdot)}(\Gamma)$. An unknown function $\Phi(z)$ is represented by the Cauchy type integral with density from $L_p(\Gamma)$, or respectively, from $f^{p(\cdot)}(\Gamma)$. The solution is written explicitly. The dependence of the index on the characteristics of lines Γ is shown.

ABOUT CONSTRUCTION OF ANALYTIC AND NUMERICAL SOLUTIONS OF SOME PLANE BOUNDARY VALUE PROBLEM FOR MIXTURE THEORY

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In this paper one variant of the mixture theory of two elastic materials is considered. In case of plane deformation are solved the following statical problems: Flaman problem (in the boundary of middle-plane is acting a point force), Kelvin problem (point force is applied to a point of plane) and the problem, when the displacements have constant discontinuity of the segment of infinite plane and they are continuous outside of this segment. The obtained singular solutions are used for applying on the boundary element methods called the fictitious load method and displacement discontinuous method to the numerical solution of various boundary value problems for mixtures.

ON ONE BOUNDARY VALUE PROBLEM

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Let Γ be some smooth curve which is the boundary of the finite domain D^+ and infinite domain D^- of the complex plain E. Let coefficients be as follows:

$$a \in L^{loc}p^{(E)}, \quad b \in L^{loc}p, 2^{(E)}, \quad p > 2,$$

see[1]. The following boundary value problem

$$\partial_{\bar{z}}\omega + a\omega + b\bar{\omega} = 0 \quad \text{in} \;\; D^+ \cup D^-,$$

$$\omega^+(t) = G(t)\omega^-(t) + g(t), \quad t \in \Gamma$$

will be studied.

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HOLOMORPHIC CURVES IN LOOP SPACES OF 3-FOLDS

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As was shown by M.Atiyah in 1984, holomorphic curves in the loop space of 3-sphere can be used to describe instantons in four-dimensional space. We will present two further applications of holomorphic curves in loop spaces of 3-dimensional manifolds.

We will begin with a precise definition of loop spaces and holomorphic curves to be considered. To illustrate these concepts, two geometric constructions and several concrete examples of such curves will be explicitly described. As the first application, we will show that solutions to certain generalized Riemann-Hilbert problems in loop spaces can be constructed from the aforementioned examples. Further connections between the two topics will be outlined.

The second application is related to the concept of the contact boundary of an isolated plane curve singularity. Namely, we will show that such a singularity yields a well-defined isomorphism class of holomorphic discs in the Brylinski loop space of 3-sphere and this isomorphism class determines the contact boundary of the singularity in question. Possible applications of the latter result to the classification of plane curve singularities will also be mentioned.

ON VEKUA'S INTEGRAL REPRESENTATION OF HOLOMORPHIC FUNCTIONS AND THEIR DERIVATIVES

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Many boundary value problems of the function theory involve not only the values of an unknown function, but also the values of its derivatives. Therefore it is very useful to have formulas providing us with integral representations of a number of sequential derivatives of the given holomorphic function. One of such representations, highly convenient for applications, was given by I. N. Vekua. He proved the following

Theorem (I. Vekua). *Let D^+ be a finite domain, bounded by a simple closed Lyapunov curve Γ, and $\Phi(z)$ be a holomorphic in D^+ function which has the derivatives of order m, which is continuous in $\overline{D^+}$ and takes on Γ the boundary value from the Hölder class H. Then assuming that the origin of coordinates lies in D^+, the function Φ is representable as follows:*

$$\Phi(z) = \int_\Gamma \frac{\mu(t)ds}{1-\frac{z}{t}} + id, \tag{1}$$

for $m = 0$;

$$\Phi(z) = \int_\Gamma \mu(t)\Big(1-\frac{z}{t}\Big)^{m-1} \ln\Big(1-\frac{z}{t}\Big)ds + \int_\Gamma \mu(t)ds + id, \tag{2}$$

for $m \geq 1$; here $\mu(t)$ is the real function of the class H, and d is the real constant; $\mu(t)$ and d are defined uniquely with respect to $\Phi(z)$.

Later on, B. Khvedelidze generalized the above theorem to the case, in which the derivative of order m of the function $\Phi(z)$ is representable in D^+ by the Cauchy type integral with density from the Lebesgue space $L^p(\Gamma;\omega)$, where $p > 1$ and

$$\omega(t) = \prod_{k=1}^{\nu} |t-t_k|^{\alpha_k}, \quad t_k \in \Gamma, \quad \alpha_k \in \Big(-\frac{1}{p}, \frac{1}{p'}\Big), \quad p' = \frac{p}{p-1}. \tag{3}$$

Recently, particular attention of researches has been directed to the Lebesgue spaces with a variable exponent $p(t)$ and also to the problems in which the boundary values of unknown functions belong to those spaces.

In this connection, it is desirable to have representations of type (1)–(2) for holomorphic functions possessing the boundary values with the above-mentioned property for possibly wide classes of functions $p(t)$, curves Γ, and weighted functions ω. We have managed to generalize I. N. Vekua's formula in two directions, i.e., instead of the Lyapunov boundary curve we consider the piecewise smooth boundary, and assume that density of the Cauchy type integral is from the Lebesgue space with a variable exponent and power weight.

ELASTIC EQUILIBRIUM OF THE HALF-PLANE WITH A STRAIGHT LINE CUT

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A problem of plane elasticity theory related with the elastic equilibration of semi-plane with the cut to the semi-plane boundary was considered by many authors. In their works were examined different ways of approximate solutions of the corresponding singular integral equation.
We obtained an effective solution of this problem by means of Cauchy-type integrals.

NON-ISOLATENESS AND INFINITENESS OF THE ORDERS OF ZEROES OF THE GENERALIZED ANALYTIC FUNCTIONS

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The analysis of elliptic system of Cauchy-Riemann differential equations is one among the classical concepts of construction of the theory of analytic functions. The essential generalization of this system in terms of the monograph [3]-the generalized Cauchy-Riemann system (its solutions are called the generalized analytic functions) is the canonical form of the general elliptic system on the plane, which was first investigated by Hilbert [2]. In the fundamental works of Vekua I., Bers L. and their disciples and followers the complete theory of the generalized analytic functions is constructed and the most important applications of this theory in various problems of analysis, geometry, mechanics and physics are indicated, for the generalized Cauchy-Riemann system with regular coefficients is established, that all main features of the classical theory of the analytic functions are preserved; in particular, the main principle of isolateness and finiteness of the orders of zeroes of the solutions remain valid (this fundamental result was first obtained by Carleman[1]).
In the present work we state the results of our research in the theory of the generalized Cauchy-Reimann system with non-regular, more precisely, with quasi-regular coefficients[3]. Further results on the existence of non-analytic structure of the indicated classes of the generalized analytic functions are obtained. In particular, we established the precise limits of sufficiently wide classes of the generalized Cauchy-Reimann system with quasi-regular coefficients for which the principle of isolateness and finiteness of the orders of zeroes of the solutions is not preserved.

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ON THE INTEGRATION OF THE DIFFERENTIAL EQUATIONS SYSTEM FOR NONLINEAR AND NON-SHALLOW SHELLS

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In this article the geometrically nonlinear and non-shallow shells are considered. Here under non-shallow shells will be meant three-dimensional elastic bodies satisfying the conditions $|hb_\beta^\alpha| \leq q < 1$ $(\alpha, \beta = 1, 2)$, in contrast to shallow shells, for which the assumption $hb_\beta^\alpha \cong 0$ is accepted, where h is the semi-thickness, b_β^α are mixed components of the curvature tensor of the midsurface S of the shell Ω.

Making use of vector and tensor notations, the equilibrium equations of the 3-D elastic bodies and stress-strain relations can be written as follows:

$$\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}\boldsymbol{\sigma}^i}{\partial x_i} + \boldsymbol{\Phi} = 0 \Leftrightarrow \quad \nabla_i\boldsymbol{\sigma}^i + \boldsymbol{\Phi} = 0, \;\; \boldsymbol{\sigma}^i = \tau^{ij}(\boldsymbol{R}_j + \partial_j\boldsymbol{U}),$$
$$\tau^{ij} = [\lambda g^{ij}g^{pq} + \mu(g^{ip}g^{jq} + g^{iq}g^{jp}]\ell_{pq} \quad (g^{mn} = R^m R^n),$$
$$2e_{pq} = \boldsymbol{R}_p\partial_q\boldsymbol{U} + \boldsymbol{R}_q\partial_p\boldsymbol{U} + \partial_p\boldsymbol{U}\partial_q\boldsymbol{U} \quad (i, j, p, q = 1, 2, 3),$$

where g is the discriminant of metric quadratic form of the 3-D domain Ω, ∇_i are covariant derivatives with respect to the space coordinates $x^i, \boldsymbol{\Phi}$ is vector of volume forces, τ^{ij} and e_{ij} are contravariant components of the stress and covariant components of the strain tensor, $\boldsymbol{U}$ is the displacement vector, $\boldsymbol{R}_i$ and $\boldsymbol{R}^i$ are the covariant and contravariant basis vectors of space.

Basis vectors $\boldsymbol{R}_i(\boldsymbol{R}^i)$ are expressed by the formulas

$$\boldsymbol{R}_\alpha = (a_\alpha^\beta - x_3 b_\alpha^\beta)\boldsymbol{r}_\beta, \;\; \boldsymbol{R}_3 = \boldsymbol{n}, \quad \boldsymbol{R}^\alpha = \frac{(1 - 2Hx_3)a_\beta^\alpha + x_3 b_\beta^\alpha}{1 - 2Hx_3 + Kx_3^2}\boldsymbol{r}^\beta, \quad \boldsymbol{R}^3 = \boldsymbol{n},$$

where $\boldsymbol{r}_\alpha(\boldsymbol{r}^\alpha)$ are the basis vectors of the midsurface $S, 2H = b_\alpha^\alpha, K = b_1^1 b_2^2 - b_1^2 b_2^1$.

Using the method reduction of I. Vekua and the method of a small parameter two-dimensional system of equations for the geometrically nonlinear and non-shallow shells is obtained. For any approximation of order N the complex representation of general solution and boundary conditions are obtained.

SOME PROPERTIES OF THE GENERALIZED POWER FUNCTIONS

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The generalized power functions of Carleman-Vekua equation

$$\partial_{\bar{z}}U + AU + B\bar{U} = 0,];]; A, b \in L_{p,2}, \quad p > 2 \tag{1}$$

are representable in the form [1] :

$$U_{2k}(z,t) = (z-t)^k u_{2k}(z,t),$$
$$U_{2k+1}(z,t) = i(z-t)^k u_{2k+a}(z,t), \quad k = 0, \pm 1, k,$$

where the functions $u_k(z,t)$ satisfy the following conditions:

$$\begin{aligned} &1)\ \ u_k(z,t) \neq 0; \\ &2)\ \ u_k(t,t) = 1; \\ &3)\ \ \text{for every fixed } t \text{ they are Holder} - \text{continuous in the whole plane.} \end{aligned}$$

Having established some relations between the generalized power functions of the equation (1) and its conjugate equation, we prove that for every fixed z the functions $u_k(z,t)$ are Holder-continuous in the whole plane.

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SECTION: *Differential Equations and Optimal Control*

ON AN IMPLEMENTATION OF THE PERTURBATION TO THE SOLUTION OF THE NON-SMOOTH VARIATIONAL PROBLEMS

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At first we consider perturbed functional

$$\Phi(u,\vartheta) = \frac{\alpha}{2}\int\limits_{\Omega} |grad(u(x)+\vartheta(x))|^2 dx + \beta\int\limits_{\Omega} |grad(u(x)+\vartheta(x))| dx - \int\limits_{\Omega} f(x)u(x)dx,$$

here $u_x(x) = (u_{x_1}(x),...,u_{x_n}(x)) = gradu(x)$, α and β are positive constants, $f \in L_2(\Omega)$ is the given function, $\Omega \in C^1$ open and bounded set, $\Phi : H_0^1(\Omega) \times H_0^1(\Omega) \to \overline{R}$.
Then, the necessary condition for the minimization of the functional is obtained as a special case.
$\Phi(u,0) = J(u) = \frac{\alpha}{2}\int\limits_{\Omega} |gradu(x)|^2 dx + \beta\int\limits_{\Omega} |gradu(x)|\, dx - \int\limits_{\Omega} f(x)u(x)dx.$

Then it is proved that the problem $\inf\limits_{u \in H_0'(\Omega)} \Phi(u)$ is stable.

OPTIMIZATION OF THE SYSTEM WITH STATIC FEEDBACK BY OUTPUT VARIABLE

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The optimization problem for the continuous system with static feedback by output variable is formulated. The considered functional involves cross terms. To solve synthesis problem for the feedback chain, it is necessary to define the constant matrix describing controlling influences by the admissible part of the phase vector. Calculation algorithm is offered for the solution of this problem.

FORMULAS OF VARIATION OF SOLUTION FOR DELAY CONTROLLED DIFFERENTIAL EQUATION WITH MIXED INITIAL CONDITION AND THEIR APPLICATIONS IN OPTIMAL PROBLEMS WITH INCOMMENSURABLE DELAYS IN CONTROLS

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For the nonlinear controlled differential equation with variable delays, local formulas for variation of solution are proved. For the optimal control problems with incommensurable delays in controls and with mixed initial condition, necessary conditions of optimality are obtained: for the optimal initial function and control - in the form of linearized maximum principles; for the initial and final moments - in the form of equalities and inequalities. The general results are concretized for optimal problem with integral functional, fixed endpoint and linear optimal problem.

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FORMULAS OF VARIATION OF SOLUTION FOR VARIABLE STRUCTURE DELAY DIFFERENTIAL EQUATION WITH MIXED INTERMEDIATE CONDITION AND THEIR APPLICATIONS IN OPTIMAL PROBLEMS

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For the nonlinear variable structure differential equation with constant delays in phase coordinates, formulas of variation of solution are proved. For the variable structure optimal problems with mixed intermediate condition, the necessary conditions of optimality are obtained: for the initial function and optimal control - in the form of maximum principles; for the structure optimal changing (switching) and final moments - in the form of equalities and inequalities. One of them, the essential novelty, is necessary condition of optimality for the switching moment, which contains the effect of mixed intermediate condition. The general results are concretized for optimal problem with integral functional and fixed endpoint and for linear optimal problem. As an application of the obtained results the economical problem of optimal distribution of the invested capital and determination of optimal investment periods for various branches of the economy are considered.

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ON OSCILLATORY PROPERTIES OF THIRD ORDER SYSTEM OF DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

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A certain sufficient condition for the oscillation of proper solutions of the third order system of differential equations with deviating arguments is established.

Let us consider the following system of differential equations

$$x_i'(t) = f_i(t, |x_{i+1}(\tau_{i+1}(t))|)\text{sign}x_{i+1}(\tau_{i+1}(t)) \ \ (i = 1, 2, 3), \ \ (1)$$

where

$$(-1)^{\nu_1} f_i(t, x_1, x_2, x_3)\text{sign}x_{i+1} \geq p_i(t)|x_{i+1}|^{\lambda_i},$$

whenever

1) $t \in R+,\ x_i \in R\ (i = 1, 2, 3),\ x_4 = x_1;$
2) $\nu_i \in \{0; 1\}, \sum_{i=1}^{3} \nu_i = \nu;$
3) $\lambda_i > 0, \prod_{i=1}^{3} \lambda_i = \lambda;$
4) $p_i \in L_{\text{loc}}(R_+; R_+)\ (i = 1, 2, 3), \int_0^\infty p_i(t)dt = +\infty (i = 1, 2).$

Theorem 1. *Let ν be an odd natural number. If the following conditions*
a) $\int_0^\infty p_3(t)dt < +\infty,$
b) $\int^\infty p_2(t)(\int_{\tau_3(t)}^{+\infty} p_3(s)ds)^{\lambda_2} dt = +\infty.$
c) $\lambda < 1, \tau_3(\tau_2(\tau_1(t))) \leq t$, *whenever* $t \geq t_1$
d) $\int^{+\infty} p_3(t)\big(\int_{t_1}^{\tau_1(t)} p_1(s_1)\big(\int_{\tau_2(t_1)}^{\tau_2(s_1)} p_2(s_2)ds_2\big)^{\lambda_1} ds_1\big)^{\lambda_3} dt = +\infty;$
hold, then the system (1) *has the property* A.

FORMULAS OF VARIATION OF SOLUTION FOR QUASI-LINEAR NEUTRAL DIFFERENTIAL EQUATION WITH MIXED INITIAL CONDITION AND THEIR APPLICATIONS IN OPTIMAL PROBLEMS

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For the quasi-linear neutral differential equation with variable delays, local formulas of variation of solution are proved. For the optimal control problems with commensurable and incommensurable delays in controls and with mixed initial condition, necessary conditions of optimality are obtained: for the optimal control - in the form of maximum and linearized maximum principles; for the optimal initial function - in the form of linearized maximum principle; for the initial and final moments - in the form of equalities and inequalities. The general results are concretized for optimal problem with integral functional, for fixed endpoint and for linear optimal problem.

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THE ANALYTICAL SOLUTION OF THE EQUATION OF KLEIN-FOCK-GORDON FOR TWO-DIMENSIONAL PIONIC ATOM IN THE PRESENCE OF A CONSTANT HOMOGENEOUS MAGNETIC FIELD

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In this work we solve Klein-Fock-Gordon equation for the two-dimensional pionic atom presence in a constant homogeneous magnetic field. The relativistic spectrum of energy has been calculated at some value of the angular moment and a magnetic field $\overrightarrow{H}$. The analysis of the obtained calculations allows us to draw a conclusion, that Klein - Fock-Gordon equation as against of Shrodinger equation, in case when a particle is in the magnetic field, does not present s - states. We have been found also that the correction of a constant magnetic field to a relativistic level of energy is appreciable when a magnetic field $H > 100$.

GENERAL APPROACH TO THE RESEARCH OF STABILITY OF SOLUTIONS IN PROBLEMS OF BOOLEAN OPTIMIZATION

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The general method for research of stability of solutions in problems of Boolean optimization is offered. In particular, the notions of set of stability for optimal, local-optimal and approximate solutions are introduced. The formulae for calculation of the radius of stability are obtained.

NECESSARY CONDITIONS FOR EXISTENCE OF POSITIVE SOLUTIONS OF SECOND ORDER LINEAR DIFFERENCE EQUATIONS

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The difference equation

$$\Delta^2 u(k) + \sum_{j=1}^{m} p_j(k)\, u(\tau_j(k)) = 0 \tag{1}$$

is considered, where $m \in N$ and the functions $p_j : N \to R_+$, $\tau_j : N \to N$ $(j = 1, \ldots, m)$ are defined on the set of natural numbers N,

$$\lim_{k\to+\infty}\tau_j(k)=+\infty (j=1,\dots,m), \Delta u(k)=u(k+1)-u(k) \text{ and } \Delta^2=\Delta\circ\Delta.$$

Necessary conditions are obtained for the equation (1) to have a positive solution. The obtained results are then used for deriving new oscillation criteria generalizing some earlier known ones.

ON A SINGULAR BOUNDARY VALUE PROBLEM FOR THE INTEGRO-DIFFERENTIAL EQUATIONS

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Consider the following boundary value problem

$$x'(t)=p(t)\int_0^{\sigma(t)} q(s)\,x(\tau(s))\,ds, \tag{1}$$

$$x(t)=\varphi(t) \quad \text{for} \quad t\in[\tau_0,0), \quad \liminf_{t\to+\infty}|x(t)|<+\infty, \tag{2}$$

where $p\in L_{\text{loc}}(R_+;(0,+\infty))$, $q\in L_{\text{loc}}(R_+;R_+)$, $0\le\sigma(t)\le t$, $\tau(t)<t$ for $t\in R_+$ and $\lim\limits_{t\to+\infty}\sigma(t)=\lim\limits_{t\to+\infty}\tau(t)=+\infty$,

$$\tau_0=\inf\big\{\tau(t):t\in R_+\big\}, \quad \sup\big\{|\varphi(t)|:t\in[\tau_0,0)\big\}<+\infty.$$

We shall establish the sufficient conditions for the unique solvability and oscillation of the boundary value problem (1),(2).

SOLUTION OF THE CONTROLLABILITY PROBLEM FOR THE TWO-PARAMETRIC BILINEAR SEQUENTIAL MACHINES

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The sequential machines under finite fields or, so-called, finite sequential machines present very important class of finite-automate systems. For the present time the questions on controllability of linear finite sequential machines and also linear discrete systems are very well investigated. The investigations of questions on theory of bilinear sequential machines (BSM) and also bilinear systems represent the highly developed division of discrete mathematics. But such questions of the theory of BSM as diagnostics, controllability and convertibility are still of the insufficient research. The present paper is attempt to implement such desideratum. Here the two-parametric bilinear sequential machine (BSM) is considered, the definition of its controllability, the determination of rang with respect to the one of the parameters are given. Sufficient conditions of controllability of two-parametric BSM are established, as a remarks more simple sufficient conditions on controllability are given.

Two-parametric bilinear sequential machine (BSM) under the field $GF(p)$, where p is simple number is described by the following equation of the state

$$\begin{cases} s(t+1,\nu+1)=[A+u(t,\nu)G]s(t,\nu)+Bs(t,\nu+1)+Ds(t+1,\nu), \\ s(t,0)=s(0,\nu)=w_0\neq 0, \end{cases}$$

where A,G,B,D are $(n\times n)$ matrices with components from $GF(p)$, s is $(n\times 1)$ vector with components from $GF(p)$, u is scalar.

OPTIMIZATION OF DELAY CONTROLLED SYSTEMS WITH NON-FIXED INITIAL MOMENT AND MIXED INITIAL CONDITION

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For the controlled differential equation with variable delays in phase coordinates and variable commensurable delays in controls, optimal control problem with general boundary conditions and functional is considered. Necessary optimality conditions are obtained: for the optimal initial function and control - in the form of maximum principles; for the initial and final moments - in the form of equalities and inequalities. One of them, the essential novelty, is necessary condition of optimality for the initial moment, which contains the effect of mixed initial condition. The general results are concretized for optimal problem with integral functional and fixed endpoint, for linear optimal problem and time-optimal problem.

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THE EXTREMAL PRINCIPLE IN QUASILINEAR CONTROL SYSTEMS OF NEUTRAL TYPE

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The extremal principle for a specific class of smooth-convex minimization problems is formulated. Using this principle, the necessary conditions of optimality for quasilinear control systems with a deviating argument of a neutral type in the presence of mixed restrictions are given. As against before executed works, the necessary conditions of optimality for a problem with intermediate conditions are proved.

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LARGE TIME BEHAVIOR AND NUMERICAL SOLUTION OF NONLINEAR INTEGRO-DIFFERENTIAL SYSTEM ASSOCIATED WITH THE PENETRATION OF A MAGNETIC FIELD INTO A SUBSTANCE

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We investigate initial-boundary value problem for the following nonlinear system of integro-differential equations:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial U}{\partial x}\right], \quad \frac{\partial V}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial V}{\partial x}\right], \quad (x,t) \in (0,1)\times(0,\infty), \tag{1}$$

$$U(0,t) = U(1,t) = V(0,t) = V(1,t) = 0, \quad t \geq 0, \tag{2}$$

$$U(x,0) = U_0(x), \quad V(x,0) = V_0(x), \quad x \in [0,1], \tag{3}$$

where

$$S(t) = \int_0^t \int_0^1 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] dx d\tau, \tag{4}$$

$a(S) = (1+S)^p$, $p > 0$; $U_0 = U_0(x)$ and $V_0 = V_0(x)$ are given functions.

Theorem. *If* U_0, $V_0 \in H_3(0,1) \cap H_0^1(0,1)$, *then for the unique solution of the problem (1)-(4) the following asymptotic relations hold as* $t \to \infty$*:*

$$\frac{\partial U(x,t)}{\partial x} = O\left(\exp\left(-\frac{t}{2}\right)\right), \quad \frac{\partial V(x,t)}{\partial x} = O\left(\exp\left(-\frac{t}{2}\right)\right),$$

$$\frac{\partial U(x,t)}{\partial t} = O\left(\exp\left(-\frac{t}{2}\right)\right), \quad \frac{\partial V(x,t)}{\partial t} = O\left(\exp\left(-\frac{t}{2}\right)\right),$$

uniformly in x *on* $[0,1]$.

The problem with the following boundary conditions is also studied:

$$U(0,t) = V(0,t) = 0,\ U(1,t) = \psi_1,\ V(1,t) = \psi_2, \quad t \geq 0, \tag{5}$$

where $\psi_1 = const \geq 0$, $\psi_2 = const \geq 0$, $\psi_1^2 + \psi_2^2 \neq 0$.

Note that in one our previous work the stabilization of the solutions of problem (1),(3)-(5) has been proven. In this work we have achieved only power-like stabilization. Recently we have proved that if $a(S) = 1 + S$ then the exponential stabilization of solutions of problem (1),(3)-(5) takes place.

The algorithms of numerical solution are constructed and investigated. Numerous numerical experiments were carried out that agree with the theoretical researches.

ON A NON-LINEAR VERSION OF GOURSAT PROBLEM WITH PARTIALLY FREE CHARACTERISTIC SUPPORT

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The following second order hyperbolic equation is given

$$y\big[(u^2-u_y)u_{xx}-(2u_xu_y+u_y-u_x-1)u_{xy}+(u_x^2-u_x)u_{yy}\big]==f(u_x,u_y). \tag{1}$$

This equation is hyperbolic along all those functions which fulfill the following condition

$$u_x-u_y+1\neq 0.$$

The parabolic degeneracy of the equation takes place where this condition is not fulfilled.

For the given equation we consider the Goursat problem with the partially free characteristic support: let us assume that the characteristic line y of the (1) equation is given, which is defined in explicit form by relations: $y=\varphi(x)$, $\varphi\in C^2[x_0,x_1]$. This line belongs to the family of the λ_1 characteristics. From the point x_0, another unknown characteristic line derived, which belongs to another λ_2 family characteristics.

Let us find the solution of the (1) equation if in the point $(x_0,\varphi(x_0))$ it equals to u_0 and if on an unknown characteristic the following condition is fulfilled

$$\alpha(x)\,u_x+\beta(x)\,u_y=\theta(x),\quad x\in[x_0,x_2],\quad \alpha,\beta,\theta\in C^1[x_0,x_2].$$

Theorem. *If the following conditions are fulfilled*

$$\begin{aligned}
&\beta(x_0)\neq\alpha(x_0)\,\varphi'(x_0),\quad \Big(\frac{\beta y}{\alpha+\beta}\Big)\neq 0,\quad \Lambda(\beta-\theta)(\theta+\alpha)<0,\\
&\Lambda\big[\alpha'(\beta-\theta)-\alpha(\beta'-\theta')\big]+\beta(\beta-\theta-\alpha)=0,\\
&\psi(x_0)-\frac{\beta(x_0)-\theta(x_0)}{\beta(x_0)-\theta(x_0)-\alpha(x_0)}\,\Lambda>0,
\end{aligned}$$

where $\Lambda=\dfrac{\varphi'(x_0)\,\varphi(x_0)}{\varphi'(x_0)+1}$,

then the unknown $y=\psi(x)$ *characteristic line is defined uniquely:*

$$\begin{aligned}
y=&\bigg[-\int_{x_0}^{x}2\Lambda\,\frac{(\beta-\theta)(\theta+\alpha)}{(\beta-\theta-\alpha)^2}\,dx+\Big(\varphi(x_0)-\frac{\beta(x_0)-\theta(x_0)}{\beta(x_0)-\theta(x_0)-\alpha(x_0)}\Big)^2\bigg]^{1/2}-\\
&-\varphi(x_0)-\frac{\beta(x_0)-\theta(x_0)}{\beta(x_0)-\theta(x_0)-\alpha(x_0)}\,\Lambda.
\end{aligned}$$

ON A CYLINDRICAL VIBRATION OF A DOUBLE-LAYER PRISMATIC BODY

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We consider double-layer prismatic body consisting of an elastic shell and viscous fluid occupying prismatic, in general, cusped domains. We use I.Vekuas dimension reduction method [1]. In the $N=0$ approximation of hierarchcal models of multi-layer mixtures [2] we study an initial boundary value problem of cylindrical deformation of of the above-metioned double-layer prismatic body.

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PARTIAL DIFFERENTIAL EQUATIONS IN THIN DOMAINS

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The present paper deals with initial and boundary value problems for first and second order partial differential equations and systems in thin domains. Using I.Vekua's [1] dimension reduction method, investigation of initial and boundary value problems for first and second order partial differential equations and systems in n-dimensional domain is reduced to the corresponding problems for equations and systems in $(n-k)$ -dimensional domain, where k is a number of coordinate axes of the rectangular Cartesian coordinate system along which the measures of the n-dimensional domain are essentially less than its measures along the other coordinate axes. I.Vekua's dimension reduction method was successfully applied and developed for prismatic shells [1-5] and beams [6,7], cusped ones included [6,8] (see also references therein).

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LARGE TIME BEHAVIOR OF SOLUTIONS AND FINITE DIFFERENCE SCHEME TO A NONLINEAR INTEGRO-DIFFERENTIAL SYSTEM

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We consider following integro-differential system

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial U}{\partial x}\right], \quad \frac{\partial V}{\partial t} = \frac{\partial}{\partial x}\left[a(S)\frac{\partial V}{\partial x}\right], \quad (x,t) \in (0,1)\times(0,\infty), \tag{1}$$

where

$$S(x,t) = \int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] d\tau. \tag{2}$$

To complete the problem we include the initial and boundary conditions:

$$U(0,t) = U(1,t) = V(0,t) = V(1,t) = 0, \quad t \geq 0, \tag{3}$$

$$U(x,0) = U_0(x), \quad V(x,0) = V_0(x), \quad x \in [0,1]. \tag{4}$$

Theorem 1. *Suppose that* $a(S) = 1 + S$, $U_0, V_0 \in H^3(0,1)$, $U_0(0) = U_0(1) = V_0(0) = V_0(1) = 0$, *then for the unique solution of problem (1)-(4) the following relations hold:*

$$\left|\frac{\partial U(x,t)}{\partial x}\right| + \left|\frac{\partial V(x,t)}{\partial x}\right| + \left|\frac{\partial U(x,t)}{\partial t}\right| + \left|\frac{\partial V(x,t)}{\partial t}\right| \leq C\exp\left(-\frac{t}{2}\right), \quad t \geq 0$$

uniformly in x *on* $[0,1]$.
The problem with following boundary conditions is also studied:

$$U(0,t) = V(0,t) = 0,\ U(1,t) = \psi_1,\ V(1,t) = \psi_2, \quad t \geq 0, \tag{5}$$

where $\psi_1 = const \geq 0$, $\psi_2 = const \geq 0$.
Theorem 2. *Suppose that* $a(S) = 1 + S$, $U_0, V_0 \in H^3(0,1)$, $U_0(0) = V_0(0) = 0$, $U_0(1) = \psi_1 = Const \geq 0$, $V_0(1) = \psi_2 = Const \geq 0$, $\psi_1^2 + \psi_2^2 \neq 0$, *then for the unique solution of problem (1),(2),(4),(5) the following estimates are true:*

$$\left|\frac{\partial U(x,t)}{\partial x} - \psi_1\right| + \left|\frac{\partial V(x,t)}{\partial x} - \psi_2\right| \leq C(1+t)^{-2}, \quad t \geq 0,$$

$$\left|\frac{\partial U(x,t)}{\partial t}\right| + \left|\frac{\partial V(x,t)}{\partial t}\right| \leq C(1+t)^{-1}, \quad t \geq 0$$

uniformly in x *on* $[0,1]$.
The algorithms of numerical solution are constructed and investigated. Numerous of numerical experiments were carried out that agree with the theoretical researches.

ON CONSTRUCTION AND INVESTIGATION OF VARIABLE DIRECTIONS SCHEME FOR A TWO-DIMENSIONAL NONLINEAR DIFFUSION MODEL

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The considered model is connected with process of vein formation in meristematic tissues of young leaves. The mentioned so-called Mitchison's model has the following form:

$$\begin{gathered}\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(W\frac{\partial U}{\partial y}\right),\\ \frac{\partial V}{\partial t}=-V+f\left(V\frac{\partial U}{\partial x}\right),\quad \frac{\partial W}{\partial t}=-W+g\left(W\frac{\partial U}{\partial y}\right).\end{gathered}\tag{1}$$

Here f and g are given sufficiently smooth functions of their arguments, which satisfy the following conditions: $0<d\le f(r)\le D$, $0<d\le g(s)\le D$, $|f'(r)|<D,\ |g'(s)|<D$, where d and D are constants.

The essential difficulties arise in the process of constructing, investigating and realizing the numerical algorithms for model (1). Besides nonlinearity, the complexity of studying such problems are conditioned also by its two-dimensionality. Therefore, naturally the question of reduction of this problem to easier ones arises. In particular, it is very important to reduce the two-dimensional problem to the set of one-dimensional problems.

At present there are some effective algorithms for solving the multi-dimensional problems. These algorithms mainly belong to the methods of variable directions, splitting-up or sum-approximation according to their approximative properties.

Construction and investigation of one kind of variable directions scheme for the system (1) are discussed.

ON ONE NONLOCAL BOUNDARY VALUE PROBLEM FOR FOURTH ORDER ORDINARY DIFFERENTIAL EQUATION AND ITS VARIATIONAL FORMULATION

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The following nonlocal boundary value problem is investigated:

$$\begin{gathered}(k_1(x)\,u''(x))''-(k_2(x)\,u'(x))'+k_3(x)\,u(x)=f(x),\quad x\in\,]-a,\ 0[\,,\\ u(-a)=0,\quad u'(-a)=0,\quad u'(0)=0,\\ \int\limits_{-\xi}^{0}k_2(x)\,u'(x)\,dx-k_1(0)\,u''(0)+k_1(-\xi)\,u''(-\xi)=0,\end{gathered}\tag{1}$$

where $f(x),\ k_3(x)\in C[-a,\ 0]$, $k_1(x)\in C^{(2)}[-a,\ 0]$, $k_2(x)\in C^{(1)}[-a,\ 0]$; $k_1(x)\ge K_1=Const>0$, $k_2(x)\,,\ k_3(x)\ge 0$ and ξ is arbitrary fixed point from $]0,\ a[$.

Variational formulation of problem (1) is given. Necessary and sufficient conditions for equivalence of variational formulation and problem (1) is established.

ON THE CHARACTERISTIC BOUNDARY VALUE PROBLEMS FOR NONLINEAR EQUATIONS WITH ITERATED WAVE OPERATOR IN THE PRINCIPAL PART

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The characteristic boundary value problems for a hyperbolic equation with power nonlinearity and iterated wave operator in the principal part are considered in the conical domains. Depending on the exponent of nonlinearity and spatial dimensionality of the equation, the existence and uniqueness of the solutions of the boundary value problems are established. The non-solvability of these problems is also considered here.

ON ERROR ESTIMATION OF SYMMETRIC DECOMPOSITION FORMULA FOR THE SEMIGROUP

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In case of applying a decomposition method, the solving operator of the corresponding decomposed problem generates Trotter's formula or Chernoff's formula. Therefore, the error estimation of decomposition method is equivalent to the problem of approximating of continuous semigroup using Trotter's type formulas. Works [1], [2] are dedicated to error estimations of Trotter's type formulas in uniform topology. In these works there were considered two-dimensional case. In the present work there is obtained the error estimation of symmetric decomposition formula for the semigroup in uniform topology for multidimensional case.

Theorem. *Let the following conditions be fulfilled: (a)* $A_1, A_2, ..., A_m$ $(m \geq 2)$ *are self-adjoint positively defined operators in the Hilbert space* H*; (b)* $D(A_1) \subset D(A_j)$, $j = 2, ..., m$ *and operator* $A = A_1 + A_2 + ... + A_m$ *is self-adjoint on* $D(A_1)$.
Then for any $n \geq 2$*, the following estimation is true:*

$$\left\| \exp(-tA) - \left[V\left(\frac{t}{n}\right) \right]^n \right\| \leq c \frac{\ln(n)}{\sqrt{n}}, \quad t \geq 0,$$

where $c = const > 0$, $\|\cdot\|$ *is a norm in* H*,*

$$V(t) = \exp\left(-\frac{1}{2}tA_1\right) ... \exp\left(-\frac{1}{2}tA_{m-1}\right) \exp(-tA_m) \exp\left(-\frac{1}{2}tA_{m-1}\right) ... \exp\left(-\frac{1}{2}tA_1\right).$$

Let us note that the above-stated estimation is true if $\exp(-tA_j)$ $(j = 1, ..., m)$ is replaced by operator $(I + tA_j)^{-1}$.

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ON THE SOLUTION OF BOUNDARY VALUE PROBLEMS FOR EQUATION OF MULTIPLE SCATTERING OF THE LIGHT IN GASEOUS MEDIA

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The theory of the solution of boundary value problems for equation describing the multiple scattering of the light in gaseous media is constructed. A theorem on the expansion of the general solution in terms of eigenfunctions of discrete and continuous spectra of the characteristic equation is proved. The proof reduces the solving of the Riemann-Hilbert boundary value problem.

ABOUT NONHYPERBOLISITY OF SOME EQUATIONS OF MOTION OF VISCOELASTICITY

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Is shown nonhyprbolisit of some equations of motion of viscoelasticity, when the constitutive relationship contain fractional derivatives.
For the proof of the received results Paley-Wiener well-known theorem essentially is used.
Mathematical Subject Classification: 26A33,74D99

AN ANALOGUE OF THE BITSADZE–SAMARSKI PROBLEM FOR A MIXED TYPE EQUATION

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Let us consider the following equation

$$\operatorname{sgn} y u_{xx} + u_{yy} + a u_x + b u_y + c u = 0, \tag{1}$$

where a, b and c are the given analytic functions of their arguments, real for the real (x, y), and $u(x, y)$ is an unknown real function.

Let Ω be a simply connected domain of a plane of complex variables $z = x + iy$, bounded by a curve σ of the class C^2 with ends $C_1(0,0)$ and $C_2(1,0)$ lying in the upper half-plane $y > 0$ and by the characteristics $CC_1 : y = -x$, $CC_2 : y = x - 1$, $C = (\frac{1}{2}, -\frac{1}{2})$, of equation (1). The use is made of the following notation: Ω^+ and Ω^- are, respectively, the elliptic and hyperbolic parts of the mixed domain Ω, $J = \{x : 0 < x < 1\}$ is the interval of the straight line $y = 0$, $\theta = \frac{x}{2} - i\,\frac{x}{2}$.
Assume that $\partial\Omega^+ \in C^{2,h}$, $\alpha h < 1$, $a = b = c$, when $(x, y) \in \Omega$.
Our problem is to find a regular in the domain Ω solution $u(x, y)$ of equation (1), satisfying the conditions

$$(pu_x + qu_y + \lambda u)|_\sigma = \varphi \;\; \forall (x, y) \in \sigma, \tag{2}$$

$$\frac{d}{dx}\, u[\theta(x)] = a(x)u_y(x, 0) + b(x) \;\; \forall x \in J, \tag{3}$$

where $\varphi, p, q, \lambda \in C^{1,h}$, $\alpha h = const < 1$, $a \in C(\overline{J}) \cap C^1(J)$, $b \in C(J)$, and $p^2 + q^2 \neq 0$.

Theorem. *Let the conditions*

$$H(t) = (p + iq)(t) = 0, \quad t \in \sigma,$$
$$(1 + i)p(0) = 1 + 2a(0) + i, \quad p(0) + q(0) = 0,$$
$$\varphi(0) = b(0) = 0$$

be fulfilled. Then this problem is Noetherian one.

INITIAL APPROXIMATIONS OF NONLINEAR HIERARCHICAL MODELS FOR MEDIUM CONSISTING OF POROUS-SOLID AND FLUID PARTS

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Using uniform presentation of basic systems of differential equations of continuum mechanics[1] by Vekua-Kantorovich projective methods, $N = 0, 1, 2$ approximations of hierarchical nonlinear models are constructed, when on the face surfaces stress vector components or on one part of the face surfaces displacement vector components and on the other part stress vector components are given. The conditions of continuity of displacement vector and stress tensor are given on the interface. Then the relations of $N = 0, 1$ approximations and models constructed by physical hypotheses are investigated. Further there are constructed and investigated cases $N = 0, 1$ when the solid part of consisting medium is a porous-elastic one. In particular, there is proved that the principle part of operator corresponding to operators of spatial part of both 3D and for all N corresponding systems of partial differential equations is strongly elliptic.

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SOME NON-CLASSICAL PROBLEMS FOR QUASI-LINEAR EQUATION

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For the quasi-linear mixed second order equation

$$u_{xx} + (1 + u_x + u_y)u_{xy} + (u_x + u_y)u_{yy} = 0 \tag{1}$$

the following two problems are studied:
Problem 1. Find the regular solution of (1) with its domain of definition under the conditions

$$u_x(x,x) + u_y(x,x) = \varphi(x), \quad x \in [0,a],$$

$$\alpha(x)u(x,x) + \beta(x)u(x,\varphi(0)x) = \psi(x), \quad x \in [0,a],$$

where $\alpha, \beta, \varphi \in C^1[0,a], \psi \in C^2[0,a], \varphi(x) > 1, x \in [0,a], a > 0$, are the given functions.
Problem 2. Find the regular solution of (1) with its domain of definition under the conditions

$$u_x(x,x) + u_y(x,x) = \varphi(x), \quad x \in [0,a],$$

$$\alpha(x)u(x,x) + \beta(x)u(x, x + (\varphi(0) - 1)(\exp(x) - 1)) = \psi(x), \quad x \in [0,a],$$

where $\alpha, \beta, \varphi \in C^1[0,a], \psi \in C^2[0,a], \varphi(x) > 1, x \in [0,a], a > 0$, are the given functions.
The existence and uniqueness theorems of the solutions of the considered problems are given.

SECTION: *ProbabilPity and Mathematical Statistics*

REDUCTION IN OPTIMAL STOPPING PROBLEM WITH INCOMPLETE DATA

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The problem of optimal stopping of a stochastic process with incomplete data is reduced to the problem of optimal stopping of a stochastic process with complete data.
Let us consider a partially observable stochastic process (θ_t, ξ_t), $0 \le t \le T < \infty$, of Kalman–BucyŠs model, where θ_t is the nonobservable process and ξ_t is the observable process (R. Lipcer, A. Shiryaev, Statistics of random processes. *Moscow*, 1974).
We consider a continuous gain function $g(t,x)$ and define payoffs by the equalities

$$s^\circ = \sup_{\tau \in \mathfrak{M}^\theta} Eg(\tau, \theta_\tau), \quad s = \sup_{\tau \in \mathfrak{M}^\xi} Eg(\tau, \theta_\tau),$$

where $\mathfrak{M}^\theta$, $\mathfrak{M}^\xi$ are the classes of stopping times with respect to the families of σ-algebras $\mathcal{F}_t^\theta$ and $\mathcal{F}_t^\xi$.

Theorem. Let η be a standard normal random variable. Then the relation

$$Eg(\tau, \theta_\tau) = Eg(\tau, m_\tau + \eta\sqrt{\gamma_t})$$

holds for any stopping time $\tau \in \mathfrak{M}^{\xi}$, where

$$m_t = E(\theta_t|\mathcal{F}_t^{\xi}), \quad \gamma_t = E\big[(\theta_t - m_t)^2\big].$$

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THE OPTIMAL FORECASTING FOR THE STOCK PRICE EVOLUTION MODEL DESCRIBED BY GEOMETRICAL GAUSSIAN SEMIMARTINGALE WITH "DISORDER"

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On the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_n)_{0\le n\le N}, P)$, consider the following process with discrete time

$$S_n = S_0 \exp\{\sum_{k=1}^{n}[I(\theta \le k)\Delta M_k^1 + I(\theta > k)\Delta M_k^2]\}$$

or

$$S_n = S_{n-1} \exp\{I(\theta \le k)\Delta M_k^1 + I(\theta > k)\Delta M_k^2\}.$$

Here S_0 is deterministic; $M^1 = (M_n^1, \mathcal{F}_n), M^1 = (M_n^2, \mathcal{F}_n)$ are independent Gaussian martingales with quadratic characteristics $\langle M^1\rangle$ and $\langle M^2\rangle$; θ is a random variable which takes the values $1, 2, ...N$ with known probability distribution $\pi_n = P(\theta = n); n = 1, 2, ..., N, \theta$ is independent from M^1 and M^2.

This process with disorder in the random moment θ we consider as the stock price evolution model. We investigate the problem of finding the optimal in mean square sense forecasting estimation of S_n. For example, the optimal forecasting on one step has the following form

$$\tilde{S}_n(1) = E(S_n/\mathcal{F}_{n-1}^S) = S_{n-1}[\exp\{\tfrac{1}{2}\Delta\langle M^1\rangle_n\}(1 - P(\theta > n/\mathcal{F}_{n-1}^S))+$$
$$\exp\{\tfrac{1}{2}\Delta\langle M^2\rangle_n\}P(\theta > n/\mathcal{F}_{n-1}^S)],$$

where $\mathcal{F}_n^S = \sigma\{S_0, S_1, ..., S_n\}$. For $P(\theta > n/\mathcal{F}_{n-1}^S)$ we have obtained the explicit representation. Also, we find the coefficient of Curtosis (excess) for

$$h_n = I(\theta \le n)\Delta M_n^1 + I(\theta > n)\Delta M_n^2,$$

$n = 1, 2, ..., N$.

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POLYAK'S AVERAGING FOR ROBBINS–MONRO TYPE SDE

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The Polyak approach to stochastic approximation problems is the use of averaging outputs of primary schemes. In several cases the averaging can provide asymptotically efficient strongly consistent estimate with the rate of convergence higher than the rate of the primary process.

We study the asymptotic behaviour of the Polyak averaging procedure for the Robbins–Monro type (RM type) SDE introduced in [1-2]

$$z_t = z_0 + \int_0^t H(s, z_{s-})\, dK_s + \int_0^t M(ds, z_{s-}), \tag{1}$$

where $K = \{K_t,\ t \geq 0\}$ is an increasing predictable process, $H(t,u)$ and $M(t,u)$, $t \geq 0$, $u \in R$, are random fields given on some stochastic basis. We assume that for each $t \geq 0$ $H(t,0) = 0$, $H(t,u)u < 0$ for $u \neq 0$ P-a.s. for each $u \in R$, $M(u) = \{M(t,u),\ t \geq 0\} \in \mathcal{M}^2_{loc}$. Equation (1) naturally includes both generalized RM stochastic approximation algorithm with martingale noises [2] and recursive estimation procedures for parametric semimartingale statistical models, and enables one to study them by a common approach.

The presented work is the final part of series of papers [1-2] concerning the asymptotic behavior of solution $z = (z_t)_{t\geq 0}$ of equation (1).

We define the Polyak averaging procedure for the process $z = (z_t)_{t\geq 0}$ by the formula

$$\overline{z}_t = \frac{1}{\mathcal{E}_t^{-1}(-g \circ K)} \int_0^t z_s d\mathcal{E}_s^{-1}(-g \circ K), \tag{2}$$

where $g \circ K_t < \infty$ for all $t \geq 0$, $g \circ K_\infty = \infty$ P-a.s. Denote $\mathcal{E}_t^{-1} = \mathcal{E}_t^{-1}(-g \circ K)$.

We show that under sufficiently mild conditions the normed process $\overline{z} = (\overline{z}_t)_{t\geq 0}$ admits an asymptotic representation which allows to derive its asymptotic distribution. As a special cases we obtain the results of [2] concerning the asymptotics of averaging procedure for "standard" RM stochastic approximation algorithm, as well as for RM algorithms with slowly varying gains.

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NATURALNESS OF CONCEPT OF THE GENERALIZED STOCHASTIC INTEGRAL IN A SEPARABLE BANACH SPACE, THE CASE OF $C[0,1]$; WEAK SECOND ORDER CONTINUOUS STOCHASTIC PROCESSES

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In this paper we define the generalized stochastic integral for a wide class of Banach space valued random processes (generalized random processes) with respect to real Wiener processes, which is the generalized random element. If it is decomposable by the random element, then we say that this random element is the stochastic integral. Therefore, the problem of existence of the stochastic integral is reduced to the problem of decomposability of the generalized random element. To show the naturalness of this definition, we consider the case, when the Banach space is $C[0,1]$. To this end we introduce the weakly mean square continuous stochastic processes and develop some of its behaviors.

EXPONENTIAL HEDGING AND SUFFICIENT FILTRATIONS

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We assume that the dynamics of the price process of the asset traded on the market is described by a continuous semimartingale $S=(S_t, t\in[0,T])$ defined on a filtered probability space $(\Omega,\mathcal{F},F=(F_t,t\in[0,T]),P)$, satisfying the usual conditions. Suppose the price process admits the decomposition $S_t=S_0+M_t+\int_0^t\lambda_u d\langle M\rangle_u$, where M is a continuous F-local martingale and λ is a F-predictable process. Let G be a filtration smaller than F, i.e., $G_t\subseteq F_t$ for every $t\in[0,T]$.
We consider the exponential hedging problem (EHP)

$$\text{to minimize}\quad E[e^{-\alpha(\int_0^T\pi_u dS_u-H)}]\quad\text{over all}\quad \pi\in\Pi(G), \tag{1}$$

where H is a bounded F_T-measurable random variable, $\Pi(G)$ is a certain class of admissible strategies and $(\int_0^t\pi_u dS_u, t\in[0,T])$ represents the wealth process related to the self-financing strategy π.
Let $V_t(F)$ and $V_t(G)$ be the value processes of the problem corresponding to the cases of of the full and partial information

$$V_t(F)=\operatorname*{ess\,inf}_{\pi\in\Pi(F)}E[e^{-\alpha(\int_t^T\pi_u dS_u-H})|F_t],\quad V_t(G)=\operatorname*{ess\,inf}_{\pi\in\Pi(G)}E[e^{-\alpha(\int_t^T\pi_u dS_u-H)}|G_t].$$

It is convenient to express the optimization problem as a set EHP $=\{(\Omega,F,P),S,H\}$ of the probability space, asset price process S and the terminal reward H.
Definition. The filtration G is said to be sufficient for the problem EHP if $V_0(F)=V_0(G)$. Filtration G is sufficient in the strong sense if $V_t(F)=V_t(G)$ for all $t\in[0,T]$.
Let us consider the following conditions: **A)** $\langle M\rangle$ is G-predictable, **B)** any G-martingale is an F-martingale, **C)** filtration G is continuous, **D)** $\langle\lambda\cdot M\rangle_T\le C$.

We shall use the notation $\widehat{Y}_t$ for the G-predictable projection of Y_t and κ_t^2 for the Radon-Nicodym derivative $d\langle\widehat{M}\rangle_t/d\langle M\rangle_t$.

Theorem 1. Let conditions A)-D) be satisfied. Then $V_t(F) = V_t(G)$ if and only if H is G_T-measurable and the process $V_t(F)$ satisfies the BSDE

$$Y_t = Y_0 + \frac{1}{2}\int_0^t \frac{(\psi_u\kappa_u^2 + \widehat{\lambda}_u Y_u)^2}{Y_u} d\langle M\rangle_u + \int_0^t \psi_u d\widehat{M}_u + L_t, \quad Y_T = e^{\alpha H}. \tag{2}$$

Corollary 1. Let conditions $C)$ and $D)$ be satisfied and let $F^S \subseteq G$. Then:
a) $V_t(F) = V_t(G)$ if and only if $H \in G_T$ and λ is G-predictable, b) If in addition B) is satisfied and $H \in G_T$, then $V_0(F) = V_0(G)$ if and only if λ is G-predictable.

Theorem 2. Let in addition to A)-D) the following conditions be satisfied: E) λ is G-predictable, F) for any G-local martingale m^G, $\langle M, m^G\rangle$ is G-predictable and G) H is G_T-measurable. Then $V_t(F) = V_t(G)$ for all $t \in [0, T]$.

ON REPRESENTATIONS OF INTEGRAL FUNCTIONAL FOR THE ESTIMATION OF THE DISTRIBUTION DENSITY

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Let $X_1, X_2, \ldots, X_n$ be a sequence of independent, identically distributed random variables in R, which have the distribution density $f(x)$; $K(x)$ is a given kernel and $h - h_n$ is a sequence of positive constants converging to zero.
Let

$$\widehat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{h_n} K\left(\frac{x - X_i}{h_n}\right)$$

is the kernel density estimator; $f_h(x) = E\,\widehat{f}_h(x)$. We consider integral functional of the form

$$T(f) = \int_R \varphi\big(x, f(x), f'(x), \ldots, f^{(k)}(x)\big)dx.$$

In some conditions we prove that $T(\widehat{f}_h) - T(f_h)$ has the form

$$T(\widehat{f}_h) - T(f_h) = \int_R A\big(f_h(x)\big)\big(\widehat{f}_h(x) - f_h(x)\big)dx + O_p\left(\frac{1}{nh_n}\right)^a, \quad 0 < a \le 1.$$

This difference representation implies the central limit theorem and the law of iterated logarithm.

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ON THE POWER OF THE GOODNESS-OF-FIT TEST BASED ON WOLVERTON–WAGNER DISTRIBUTION DENSITY ESTIMATES

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Let $X_1, X_2, \ldots, X_n$ be a sequence of independent, equally distributed random variables with values in the Euclidean p-dimensional space R_p, $p \geq 1$, which have the distribution density $f(x)$, $x = (x_1, \ldots, x_p)$. Using the sampling $X_1, X_2, \ldots, X_n$, it is required to test the hypothesis

$$H_0 : f = f_0.$$

We consider the checking test of the hypothesis H_0 based on the statistics

$$U_n = n a_n^{-p} \int \big(f_n(x) - f_0(x)\big)^2 r(x)\, dx,$$

where $f_n(x)$ is a kernel estimate of the Wolverton–Wagner probability density,

$$f_n(x) = \frac{1}{n} \sum_{i=1}^{n} a_i^p K\big(a_i(x - X_i)\big),$$

where $K(x)$ is a given kernel, (a_n) is a sequence of positive numbers monotonically converging to infinity.

We study the question of consistency of the goodness-of-fit test based on U_n and also consider the power asymptotics of the goodness-of-fit test for certain close alternatives.

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STOCHASTIC DERIVATIVE OF POISSON FUNCTIONALS

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Ma, Protter and Martin have proposed in [1] the stochastic derivative for normal martingales (the martingale is said to be normal, if $\langle M, M\rangle_t = t$): the derivative operator is analogous to what is often called the Malliavin derivative in the Brownian case, and it is defined as a linear operator D from

$$\mathbf{D}_{2,1} := \{F = \sum_{n=0}^{\infty} I_n(f_n) : \sum_{n=0}^{\infty} n n! \|f_n\|^2_{L^2([0,T]^n)} < \infty\} \subset L^2(\Omega)$$

into $L^2([0,T] \times \Omega)$ by

$$D_t F := \sum_{n=1}^{\infty} n I_{n-1}(f_n(\cdot, t)), t \in [0,T].$$

There are two ways to describe the Malliavin derivative, and they are equivalent in the Brownian case but not in the martingale case. In [1] an example is given, which shows the two definitions (Sobolev space and chaos expansion) are compatible if and only if $[M,M]$ is deterministic. In work of Purtukhia [2] the space $\mathbf{D}^M_{p,1}$ $(1<p<2)$ is proposed for a class of normal martingales and Ocone-Haussmann-Clark formula is established for functionals from this space.
Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ be a filtered probability space satisfying the usual conditions. Assume that the standard Poisson process N_t is given on it $(P(N_t=n)=t^ne^{-t}/n!, n=0,1,2,...)$ and that $\mathcal{F}_t$ is generated by N $(\mathcal{F}_t=\mathcal{F}^N_t)$, $\mathcal{F}=\mathcal{F}_T$. Denote by M_t-the compensated Poisson process $M_t:=N_t-t$ (note that M_t is a normal martingale $\langle M,M\rangle_t=t$), but $[M,M]_t=N_t$ is not deterministic). Let us denote $\bigtriangledown_x f(x):=f(x+1)-f(x)$ and $\bigtriangledown_x f(M_T):=\bigtriangledown_x f(x)|_{x=M_T}$.
Theorem. For any polynomial function $P_n(x)$ of order $n(n\geq)$ with respect to x the Malliavin derivative can be computed as follows:

$$D_tP_n(M_T)=\bigtriangledown_x P_n(M_T)\cdot I_{[0,T]}(t)$$

(note that in the Brownian case: $D_tP_n(M_T)=[P_n(x)]'_x|_{x=M_T}\cdot I_{[0,T]}(t)$).

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LIMIT THEOREMS FOR OPERATOR ABEL SUMMATION

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Let H be a real separable Hilbert space with $1\leq \dim(H)$, $L(H)$ be the set of all continuous linear operators $A:H\to H$, I will stand for the identity operator. We write: $\mathbf{U}=\{A\in L(H):\|A\|<1\}$. Moreover, for a constant c, $1\leq c<\infty$, and for a fixed operator $R\in L(H)$, let

$$\mathbf{U}_c(R)=\{A\in\mathbf{U}:\|I-A\|\leq c(1-\|A\|)\ A=A^*, \text{and } AR=RA\}.$$

For an operator $A\in L(H)$ and an integer $n\geq 0$ we set $A^0:=I,\ A^{n+1}:=A^nA$.
Fix a sequence ξ_k, $k=0,1,\dots$ of independent identically distributed random elements in H such that $\mathbb{E}\|\xi_0\|<\infty,\ \mathbb{E}\xi_0=0$. We observe that then $\sum_{k=0}^\infty\|A^k\xi_k\|<\infty\ \ a.s.\ \ \forall A\in\mathbf{U}$ and the equality $\eta_A=\sum_{k=0}^\infty A^k\xi_k,\ \ A\in\mathbf{U}$, defines an H-valued random field $(\eta_A)_{A\in\mathbf{U}}$.
We shall discuss the following statements.
Theorem 1. $\|(I-A)\eta_A\|\longrightarrow 0$ *a.s. when for some constant* c, $1\leq c<\infty$, *we have* $A\in\mathbf{U}_c(I)$ *and* $A\to I$.
Theorem 2([3]). *If* $\mathbb{E}\|\xi_0\|^2<\infty$ *and* R *is the covariance operator of* ξ_0, *then* $(I-A^2)^{\frac{1}{2}}\eta_A\longrightarrow\gamma_R$ *in distribution when for some constant* c, $1\leq c<\infty$, *we have* $A\in\mathbf{U}_c(R)$ *and* $A\to I$; *here* γ_R *stands for the symmetric Gaussian measure on* H *with the covariance operator* R.

For $\dim(H) = 1$ Theorems 1 and 2 see in [2] and [1], respectively; the latter being one of a huge amount of papers devoted to the distributional properties of η_A called often *a discounted sum* which models the *present value* of consecutive independent payments $\xi_0, \xi_1, ...$ with a *discount factor* $0 < A < 1$.

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SECTION: *Mechanics of Solids*

THE PROBLEM OF VIBRATION OF THE ELASTIC HALF-PLANE WITH A CRACK PERPENDICULAR TO ITS BOUNDARY

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In the paper an elastic isotropic semi-plane with the crack perpendicular to its boundary is considered. The crack is affected by transversal effort. The displacement takes place in the direction perpendicular to the possible propagation of the crack. Components of displacement satisfy differential equation

$$\Delta W + k^2 W = 0.$$

Boundary conditions will be

$$\begin{aligned} \mu \frac{\partial W}{\partial y} &= \tau\delta(x_0), \quad && x < l, \quad y = 0, \\ W &= 0, && x > l, \quad y = 0, \\ \frac{\partial W}{\partial x} &= 0, && x = 0, \quad 0 < y < \infty. \end{aligned}$$

Let us write the differential equation and the boundary conditions in polar coordinates. The problem is solved through integral transformation of Kontorovich-Lebedev

$$\Phi(\lambda) = \int_0^\infty w(r)\, \frac{H_\lambda^{(2)}(kr)}{r}\, dr, \quad w = -\frac{1}{2}\int_{-i\infty}^{i\infty} \lambda\, \Phi(\lambda)\, I_\lambda(kr)\, d\lambda.$$

In the transformed area the problem is reduced to the functional equation of Wiener-Hopf and is solved by the method of factorization. Then we use reverse transformation of Kontorovich-Lebedev. The obtained integrals are calculated using the theory of residues.

THE SECOND BOUNDARY VALUE PROBLEM OF STATICS FOR TRANSVERSALLY ISOTROPIC BINARY MIXTURES FOR AN INFINITE STRIP

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In the present paper the basic two-dimensional second BVP of statics of elastic transversally isotropic binary mixtures is investigated for an infinite strip. We show that the potential can be successfully used for the effective solution of BVP for an infinite strip. Using the potential method and the Fourier method, the second BVP is solved. The solution is constructed in terms of elementary functions and Fourier transform, that has not been solved before.

SAINT-VENANT'S PROBLEM FOR MULTI-LAYER ELLIPTIC TUBS

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The Saint-Venant's problem for three layers concentrated circular cylinder is considered. The tube is composed by three different isotropic materials glued together along the boundary of the tube.
the three auxiliary problems of the plane deformations stated by N. Muskhelishvili are considered. The solutions of these problems are obtained in terms of the displacements and stresses.

THE DYNAMICAL BENDING PROBLEM OF BEAM LYING ON THE ELASTIC BASIS

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It is considered dynamic contact problem for half-plate, strengthened with elastic infinite or half-infinite beam, induced by normal harmonic forces. We assume that under cover plate (beam) along the line of contact there may occur normal contact stresses.
Considering problem with respect to corresponding contact forces is reduced to the solution of integral differential equation for certain boundary condition. These equation can by solved using the methods of analytical functions and integral transformations.

THE MIXED PROBLEM OF THE THEORY OF ELASTIC MIXTURE FOR A RECTANGLE WEAKENED BY UNKNOWN EQUIV-STRONG HOLES

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In the present paper we investigate the mixed problem of statics in the linear theory of elasticity mixtures for a rectangle weakened by equiv-strong holes. Using the methods of the theory of analytic functions, there are defined a stressed state of the plate, a form and mutual location of the hole contours.

PLANE WAVES AND VIBRATIONS IN THE BINARY MIXTURES OF THERMOELASTIC SOLIDS

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In this paper some basic properties of wave numbers of the longitudinal and transverse plane waves are treated. The existence theorems of eigenfrequencies of the interior homogeneous boundary value problems of steady oscillations for binary mixtures of thermoelastic solids are proved. The connection between plane waves and eigenfrequencies is established.

BOUNDARY-CONTACT VALUE PROBLEMS OF THERMOELASTICITY OF BINARY MIXTURES FOR A CIRCLE AND CIRCULAR RING

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The effective solutions of boundary-contact value problems of the theory of thermoelasticity of binary mixtures for a piecewise-homogeneous circle and for circular ring are obtained in the form of absolutely and uniformly convergent series.

SECTION: *Theory of Shells and Plates*

ELASTIC POLYGON WITH POLYGONAL HOLES WHICH HAS UNIFORM SOLID BOUNDARY HOLES IN THE CORNERS

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Elastic, isotropic, convex polygonal uniform plate, which is weakened by the convex polygonal holes and cuttings and in the corners of holes by cuttings with unknown forms, is considered. Let the plate with the outer borders be tangent to the perfectly solid body (is inserted in the absolutely solid body), on the inner bounder the linear parts press down also linearbased stamps, and the cutting bounds

are free or they have an effect of normal force (safe pressure). The friction along the bounders is neglected. The problem is: to find the forms of unknown cuttings taking in account that there were not a voltage concentration. These cuttings are called the cuttings with uniformly stable bounders. Two cases were considered:
a) the aggregate vectors acting on the stamps are given;
b) the displacement of the stamps are given.
In both cases the given problem is reduce to the same mathematical problem. In the given work the case b) is considered. In case of some symmetry, especially when the polygons are regular (rectilinear) and the plate is symmetric to there radiuses and lines including apothems (slant height), the problem is reduced to the some types of boundary problem of analytic functional theory for the simple conjugate area, the part of bounders of which is unknown. The solution of the problem in quadratics is obtained. The graph (diagram) of desired outline is constructed.

DEFINITION OF MODE OF DEFORMATION OF SPHERICAL BUILD-UP DOME WITH FILTERING PARAMETERS BY USING TAYLOR'S GENERALIZED FORMULA

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The spherical ring dome which is free on the internal contour and undergoes the distributed round ring loading is considered. The dome consists from the three ring parts which are connected each by others with hinges. The external contour of the dome is supported on the rigid diaphragm. The simultaneous differential equations by S. Mikeladze due to generalized TaylorŠs series are reduced to the first kind Voltaire type simultaneous integral equations solution of which is carried out by the obtained by S. Mikeladze recurrence formulas. The jump values which occur in the dome parts interconnected hinges are defined by continuity conditions which are characterized by the different selected factors. The formulas defining all load-bearing factors are set up, corresponding diagrams are constructed and on their basis is analyzed mode of deformation of the dome and are defined the strength and rigidity conditions.

SECTION: *Mathematical Problems of Hydromechanics*

THERMODYNAMICS OF THREE-DIMENSIONAL BAROCHRONIC FLOWS

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In the series of articles [1-4] there has been constructed a complete set of polynomial invariants - a polynomial basis of invariants of linear operators and of their matrices for orthogonal transformations in n-dimensional Euclidean space E^n. The classification problem of the operators by their orthogonal invariants has been solved (corresponding theorems for the unitary group U^n in n-dimensional unitary vector space U^n have been proved too). The method developed can be successfully used in various physics problems. The results obtained in this way usually are more correct and detailed then ones obtained by other authors with methods differ from ours. Our method allows to find some new solutions [5] which were not found by methods used previously [6]-[9]. In recent study [10], the complete set of orthogonal invariants of three-dimensional matrix is used to solve three-dimensional nonlinear equation in partial derivatives, describing some hydro- and aero-dynamical problems. The similar method has been used in [6-9] where the differential equations (DE) system was obtained for algebraic invariants of Jacoby matrix of hydrodynamical velocity field. In [6-9] authors investigate types of symmetries of general solutions of the DE system for some hydrodynamical models. But the set of algebraic invariants used in [6-9] is not complete. In [5], [10] we have shown that the application of complete polynomial basis of orthogonal invariants gets all (smooth) solutions of corresponding physics problems in covariant form. In common case the number of arbitrary parameters in obtained solutions can not be reduced. In the present study the results obtained in [5] and [10] are used to find thermodynamical (TD) parameters of barochronic flow which is defined in [8]. We have found a space-time dependence of a temperature, entropy and some thermodynamical potentials (e.g. entalpy) in all possible (two and only two, see [5], [10]) regimes of such flow, by exactly solving corresponding 3-dimensional nonlinear differential equations in partial derivatives. The results obtained seem to be interesting taking into consideration that barochronic flow naturally describes long-scale evolution of the Universe [5].

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EXACT SOLUTION OF THE PROBLEM OF DAMPED TWO-DIMENSIONAL BENARD FLOW

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On the basis of two-dimensional statement of the problem on the flow of viscous incompressible fluid the Navier-Stokes equations are built up which are dedimensionalized in a natural way. Numerical similarity factors: Strouhal number, Euler number, Froude number and Reynolds number are obtained. Considering the two-dimensional statement of problem, an analytical algorithm of finding the whole class of exact solutions of Navier-Stokes equations is set up. In particular case, exact solutions of damping cellular Benard flow with rectangular cells are constructed.

THE NUMERICAL INVESTIGATION OF INSTABILITY AND TRANSITION IN CURVED CHANNEL FLOWS WITH A TRANSVERSE PRESSURE GRADIENT

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The regimes arising after the loss of stability of viscous flows between two rotating cylinders for the case in which the basic velocity distribution is the sum of a velocity distribution due the rotation of the cylinders and a pumping velocity distribution due to a pressure gradient acting round the cylinders are considered. It is assumed that there occurs inward or outward flows through a surface.
These regimes are studied using the bifurcation theory of hydrodynamic flows with cylindrical symmetry and thus the object of numerical analysis of stability and transition is a dynamical system of amplitude equations, representing the generalization of the well-known amplitude equation due to Landau.
Transition schemes connected with sequential bifurcations of the dynamical system equilibria are given. It is established that for certain values of the problem parameters the dynamical system equilibria have bifurcations in the form of limiting cycles to everyone of which there corresponds a three-frequency quasiperiodic regime of the fluid flow.
A number of calculations of limiting cycles are carried out and their stability and bifurcation are studied.

MAGNETOHYDRODYNAMIC UNSTEADY FREE CONVECTION ACROSS AN INFINITE VERTICAL POROUS PLATE

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The present paper is devoted to a study of unsteady free convection across an infinite vertical porous plate under the action of an uniform transverse magnetic field. Initially the plate and the fluid are at the same temperature and there is no flow. At $t = 0$ the plate is given a thermal transient which causes a step-change in its temperature.
Consider the motion of a viscous electrically conducting fluid occupying the half space $y \geq 0$ under the action of an uniform external magnetic field B0 in the y-direction. Initially the plate and the fluid are at the same temperature T_0. At $t = 0$ the porous plate temperature suddenly increased to $T_0 + T_w$. Since the plate is infinite, all physical quantities will be functions of only y and t. If, at any time t, the velocity, the magnetic field and the temperature are given by $(U, V_o, 0)$, $(B, B_o, 0)$ and T respectively, the system of equations governing the flow and heat transfer are:

$$\begin{aligned}
&\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g\beta(T - T_0) + v\frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\rho} B_0 \frac{\partial B}{\partial y},\\
&0 = -\frac{\partial P_1}{\partial y} - \mu B \frac{\partial B}{\partial y},\\
&\frac{\partial B}{\partial t} - v_0 \frac{\partial B}{\partial y} = B_0 \frac{\partial u}{\partial y} + \eta \frac{\partial^2 B}{\partial y^2},\\
&\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = K \frac{\partial^2 T}{\partial y^2}.
\end{aligned}$$

Herein we have neglected the viscous and Joulean dissipation.
The initial and boundary conditions are:

$$t = 0: \quad u = 0, \quad B = 0; \quad T = T_0 \ \text{ for } \ y \geq 0;$$
$$\left. \begin{array}{l} y = 0: \quad u = 0, \quad T = T_w + T_0, \quad B_{y=0^+} = B_{y=0}, \\ \dfrac{1}{\sigma} \left(\dfrac{\partial B}{\partial y} \right) \Big|_{y=0^+} = \dfrac{1}{\sigma_w} \left(\dfrac{\partial B}{\partial y} \right) \Big|_{y=0^-}, \\ y \to \infty: \quad u \to 0, \quad B \to 0, \quad T \to T_0, \end{array} \right\} t \geq 0$$

The solution is obtained by Laplace transform technique. All physical properties of this problem are determined.

ON THE CONSTRUCTION OF SOLUTIONS OF SPATIAL AXI-SYMMETRIC STATIONARY PROBLEMS OF THE THEORY OF JET FLOWS WITH PARTIALLY UNKNOWN BOUNDARIES

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In the present work we present a general mathematical method of constructing stationary solutions of spatial axi-symmetric, with partially unknown boundaries, problems of jet theory, in particular, we consider a liquid flow of finite width round a spatial circular wedge. Unknown functions (velocity potential, flow function) and their arguments on each interval of the boundary satisfy two inhomogeneous boundary conditions. The system of differential equations with respect to the velocity potential and flow function are reduced to the normal form. Unknown functions are represented by a sum of holomorphic and generalized analytic functions. One problem from the theory of spatial jets is solved.

INFLUENCE OF ELECTRIC AND MAGNETIC FIELD ON HEAT TRANSFER UNDER LAMINAR FLOW OF LIQUID IN A FLAT CHANNEL

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Magnetic hydrodynamic of laminar flow in a flat channel is studied enough. Hurtman who was the first to investigate the problem, showed that the existence of outer magnetic field leads to developing parabolic speed profile and building up the coefficient of resistance. Simu-ltaneous adjustment on the flow of outer electricand magnetic field causes appearance of addi-tional pressurre gradient, the sign of which depends on the electric field.
The influence of magnetic and electric fields on heat transfer is not studied thoroughly.
In the given work the problem of heat transfer under laminar flow of electrical conduc-ting incompressible fluid in a flat channel with outer electric and magnetic fields is studied.Basic influence of magnetic field on heat transfer is conditioned by changing the speed profile.Under this condition the factor of heat transfer may be increased on account of magnetic field by 50%.

MIXED BOUNDARY VALUE PROBLEMS OF PIEZOELECTRICITY FOR SOLIDS WITH CRACKS

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We investigate the solvability and asymptotics of solutions to mixed boundary value problems of piezoelectricity for homogeneous anisotropic solids with cracks. Using the potential methods and theory of pseudodifferential equations on manifolds with boundary, we prove the existence and uniqueness of solutions. The full asymptotics of solutions are obtained near the crack edge and near the line where the boundary conditions are changed. The properties of singularity exponents of solutions are established. We show that these exponents can be calculated by means of the eigenvalues of the matrix constructed with help of the symbol matrices of the corresponding pseudodifferential operators. We present particular examples which show that the singularity exponents depend on the material constants.

CONTACT PROBLEMS WITH FRICTION FOR HEMITROPIC SOLIDS

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Contact problems for hemitropic elastic solids are studied with the help of the theory of variational inequalities. We treat the cases when the friction effects, described by the Coulomb's law, are taken into consideration either on some part of the boundary of the body or on the whole boundary. We prove the corresponding existence and uniqueness theorems and show the continuous dependence of solutions on the data of the problem.

GENERAL REPRESENTATIONS OF SOLUTIONS TO THE OSCILLATION EQUATIONS OF THERMO-HEMITROPIC ELASTICITY

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We derive the general representations of solutions to the oscillation equations of thermo-hemitropic elasticity by means of metaharmonic functions. By these representations we construct explicitly solutions of the Dirichlet and Neumann boundary value problems for domains with specific geometry.

EXTERIOR WEDGE DIFFRACTION PROBLEMS WITH DIRICHLET, NEUMANN AND IMPEDANCE BOUNDARY CONDITIONS

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Classes of problems of wave diffraction by a plane angular screen occupying an infinite 270 degrees wedge sector are studied in a Bessel potential spaces framework. The problems are subjected to different possible combinations of boundary conditions on the faces of the wedge. Namely, under consideration there will be boundary conditions of Dirichlet-Dirichlet, Neumann-Neumann, Neumann-Dirichlet, impedance-Dirichlet, and impedance-Neumann types. Existence and uniqueness results are proved for all these cases in the weak formulation. In addition, the solutions are provided within the spaces in consideration, and higher regularity of solutions is also obtained in a scale of Bessel potential spaces.

MIXED INTERFACE CRACK PROBLEMS FOR COMPOSED SOLID STRUCTURES

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We investigate three–dimensional interface crack problems (ICP) for metallic–piezoelectric composite bodies. We give a mathematical formulation of the physical problem when the metallic and piezoelectric bodies are bonded along some proper part of their boundaries where an interface crack occurs. In our analysis we employ Voigt's linear model for the piezoelectric part and the usual classical model of elasticity for the metallic part, to write the corresponding coupled systems of governing partial differential equations. As a result, in the piezoceramic part the unknown field is represented by a 4–component vector (three components of the displacement vector and the electric potential function), while in the metallic part the unknown field is described by a 3–component vector (three components of the displacement vector). Therefore, the mathematical modeling becomes complicated since we have to find reasonable efficient boundary, transmission and crack conditions for the physical fields possessing different dimensions in adjacent domains.

By potential methods the ICP is reduced to an equivalent strongly elliptic system of pseudodifferential equations on manifolds with boundary. We study the solvability of this system in Sobolev-Slobodetski (W_p^s), Bessel potential (H_p^s), and Besov ($B_{p,t}^s$) spaces and prove uniqueness and existence theorems for the original ICP. We analyse the regularity properties of the corresponding electric and mechanical fields near the crack edges and near the curves where the boundary conditions change. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators. We present some numerical calculations which demonstrate that the stress singularity exponents essentially depend on the material parameters.

THE BASIC BOUNDARY VALUE PROBLEMS OF STATICS OF THE THEORY OF INCOMPRESSIBLE VISCOUS FLUID FOR SPHEROID

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We use the Papkovich-Neiber type general representation formulas for the equations of statics of the theory of incompressible viscous fluid and construct explicit solutions to the Dirichlet and Neumann boundary value problems for a spheroid. The solutions are represented in the form of uniformly and absolutely convergent Fourier-Laplace series.

SECTION: *Mathematical Modeling and Numerical Analysis*

ON MATHEMATICAL MODELING OF MULTILAYER PRISMATIC SHELLS CONSISTING OF ELASTIC AND FLUID PARTS

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The paper is devoted to the construction of dynamical two-dimensional hierarchical models of multilayer prismatic shells consisting of elastic solid and fluid parts and the issues of approximation of the three-dimensional problem by constructed problems are investigated. The dynamical three-dimensional mathematical model of multilayer prismatic shell consisting of elastic and fluid parts is considered, when the solid part is occupied by nonhomogeneous anisotropic elastic body, and fluid part is filled by viscous barotropic fluid. Applying variational approach the hierarchies of dynamical two-dimensional models of multilayer prismatic shell are constructed, when surface forces or displacements are given on the upper and the lower faces of the multilayer prismatic shell or on one face of the prismatic shell the surface force and on the another face the displacement are given, the conditions of continuity of displacement vector function and stress are given. The hierarchical models are studied for multilayer prismatic shells, which occupy Lipschitz domain, but the thickness may vanish on a part of the boundary. Under suitable conditions on the parameters characterizing mechanical properties of the solid and fluid parts the existence and uniqueness of solutions of the reduced two-dimensional problems in the spaces of vector-valued distributions with respect to the time variable is proved, with values in corresponding weighted Sobolev spaces. The relation of the constructed hierarchies of two-dimensional models and original three-dimensional problems is investigated, the convergence in suitable spaces of the sequence of vector-functions of three space variables restored from the solutions of the reduced problems to the solutions of the three-dimensional problems is proved, and under additional regularity conditions the rate of convergence is estimated.

FINITE DIFFERENCE SCHEMES FOR ONE NONLOCAL BIHARMONIC PROBLEM

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A mixed problem with integral restrictions and Dirichlet conditions on the part of the boundary for biharmonic equation is considered. Unique solvability of the corresponding difference scheme is proved. An estimate of convergence rate in weighted Sobolev spaces is obtained.

THE INTEGRO-DIFFERENTIAL INEQUALITIES METHOD FOR SOLVING THE MODELING PROBLEMS OF GRAVITATING GAS THEORY

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The mathematical modeling of astrophysics processes is one of the most actual problems of modern applied mathematics. Gas dynamics processes, connected with the explosive phenomena and propagation of detonating waves, represents both theoretical and practical interest. These problems are rather actual, including modeling and calculation of possible catastrophic consequences of explosions and propagation of detonating burning of gas for mines, gas pipelines, gasholders, etc. Many problems of astrophysics demand for solution the research of dynamics of gas bodies which interact with a gravitational field. In modern astrophysics special interest is expressed to rough catastrophic processes of explosion of stars and the neutron stars received at it and collapsing bodies - black holes. It is obvious, that for research of the heavenly phenomena it is necessary to put in a basis of concepts the statements and decisions of some dynamic problems about motion of gravitating gas. They can be considered as mathematical models which cover essential features of motion and evolution of stars. As is known, mathematical models of many real gas dynamics processes are described by classical initial-boundary problems for systems of nonlinear differential equations in partial derivatives. It is clear that reception of exact solutions of such most complicated problems generally is impossible. In this connection, in the modern gas dynamics connected with explosive processes, rather actual development of the approached analytical and numerical methods for solution of such problems is represented. The basic essential and practically important parameter in these problems is the law of motion of a detonating wave (a spherical surface where the solution undergoes the first kind of discontinuity) which arises due to explosion or dynamic instability of balance. However, the classical formulation of a problem in terms of the differential equations usually assumes preliminary full local definition of properties of gas flow. On the other hand, the description of the phenomenon of explosion by ideal mathematical model naturally demands certain accuracy of calculations. In this connection, it is clear that the direct approached definition of the required integrated characteristic of a problem by an establishment of system of integro-differential inequalities is important, which allows to receive bilateral estimations. In many cases (for example, in automodel) these estimations are also sufficient for its solution. This work proposes an integrodifferential inequalities method of solution for a system of nonlinear nonhomogeneous equations of one class of initial-boundary problems with an unknown external boundary in the domain; for spherical and symmetrical non-stationary adiabatic flows of the perfect gravitating gas, on the basis of the equations of motion of medium and the equation of energy, deducing of system of integrodifferential inequalities for the law of motion of detonating wave and the moment of inertia of area of perturbating motion of gas. The detonating wave arises as the result of the nonhomogeneous gravitational collapse (parabolic or elliptical) of the gas at zero pressure or during the breakdown of the equilibrium position. A system of integrodifferential inequalities is constructed, determining the law of detonating wave and the moment of inertia of area of perturbating motion of gas with respect to a known initial state of the gas. A number of automodel problems are considered as examples.

MATHEMATICAL MODELING OF SOME PECULIAR PROPERTIES OF REGIONAL CLIMATE CHANGE

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The geographical situation of Georgia, its complex orographic and circulation conditions cause the climatic diversity. There are forming almost all the types of climate existing on the Earth from the high-mountainous zone of the main ridge of Caucasus (with constant snows and glaciers) to the humid subtropical climate of the Black Sea and continental climate of the East Georgia. The result of such a diversity of Georgian climate is that the statistical processing of the data of mean climatic temperature of 1906-1999 years exposed the regularity of the climate cooling in the West Georgia and warming in the East Georgia. There were also elicited those micro-regions where mean climatic temperature does not change in time. In the present work there is considered a non-stationary nonlinear equation of the atmosphere thermal conductivity, which describes the dissemination of the middle climatic temperature in time, in space and on meridian. A nonlinear mathematical model describes two processes of opposite directions, namely the influx of warm in atmosphere from nonlinear thermal sources, warm loss and their interaction. We have studied the nonlinear problem of atmospheric thermal conductivity, when in the atmosphere are functioning nonlinear thermal sources, actions of which generate "Thermal effect". We also studied the effect of thermal and advective-dynamic factors of atmosphere on the changes of the West Georgian climate. We have shown that non-proportional warming of the Black Sea and Kolkhi lowland provokes the intensive strengthening of circulation. In addition, by means of the diffusion equation we studied the transfer of thermal gases and aerosols by the monsoon circulation to the West Georgia. We have shown that the effect of climate cooling must exist in other regions of the Earth, especially there, where monsoon circulation and advective-orographic factors are sharply expressed.

ON NUMERICAL MODELING OF EMERGENCY SPILLING OIL PENETRATION INSHORE WATERS OF THE BLACK SEA

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In the present paper mathematical modelling of oil outflow and spreading in the Black Sea water is presented. The mathematical model takes into consideration oil transformation (evaporation, emulsification, dispersion and sedimentation). Oil distribution on the Bleak Sea water surface has three scenarios: the first - oil spill from the pipeline with the length of 2,5 km at the approach to the oil bay of Batumi Port; in the second case the accident may occur in the region of Poti seaside, when the military accident take place; the third case reveals the second variant of the scenario of accidental situation: railway accident at the bridges crossing, for instance on river Supsa, when the oil reaches the mouth of the river, transferred by water flow.

INVESTIGATION OF THE CIRCUMSTANCE OF THE LIGHT-SIGNALS AT THE STREETS' CROSSING POINT INFLUENCE UPON THE HARMFUL SUBSTANCES' CONCENTRATIONS DISTRIBUTION BY NUMERICAL MODELING

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At present Georgian industrial potential is vary low, a lot of plants and mills are not functioning. That is why the main pollutants of the air are exhaust gases in Georgia. So air pollution from exhaust gases is considered to be one of the most serious environment problem in the capital of Georgia - Tbilisi with population of 1.5 million. Tbilisi air pollution problem is not different from other urban areas in the world. Aerosols are considered to be one of the most serious air pollution problems in Tbilisi. According to Georgian Government statistics data, in areas with heavy traffic the air pollution quality is exceeded than in industrial areas. It is expected that the continuous economic growth in Tbilisi is inseparable with intensity of traffic and it will more degrade the air quality. Unfortunately for last 15 years, owing to hard economical situation, the net of meteorological stations and observation laboratories almost was destroyed in Georgia. At present there are functioning only six meteorological observation laboratories in Tbilisi from up to 34 in 1992. Also Tbilisi has rather compound orography. Therefore, investigation of the exhaust gases dispersion in Tbilisi street canyons by mathematical modelling is very important for the health of population, for management of environment and future economic planning. To learn the above-mentioned problem in the given work we have done the following activities: we have learnt background picture of air pollution in Tbilisi and the tendency of changing of the general level of pollution according to the years and seasons; we have learnt the meteorological conditions established in Tbilisi and the role of auto transport as a reason of air pollution; we have drowned a numerical model of exhaust gas spreading in Tbilisi streets canyons based on the integration of hydro-thermo-dynamic equations with accounting of compound relief of Tbilisi. With the help of the mathematical modeling NOx harmful substances' concentration distribution at Agmashenebeli and King Tamar streets' crossing and for the whole complex of streets adjoined to this territory has been studied. Also the influence of the light-signals at the streets' crossing point upon the harmful substances' concentration growth has been investigated.

THE NUMERICAL MODEL OF A METEO NONHOMOGENEOUS AIR MASSES INTERACTION IN A MESO-BOUNDARY LAYER OF ATMOSPHERE

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In the paper there was stated and solved the two-dimensional (x-z plane) nonstationary problem about a meso-boundary layer of atmosphere with taking into account an interaction of meteo nonhomogeneous air masses. An advection invasion at a breeze circulation was considered. In consequence, an advection fog was simulated; An artificial influence on one by heat source temperature and air downward stream velocity was realized. Elements of a frontogenesis (frontal surface, its inclination, cool and warm fronts) were simulated. An interaction between two air masses with different temperature and aerosol concentration was considered. Also there was simulated an interaction between two clouds. First numerical experiments suggest that our model simulates main moments of considered meteoprocesses satisfactorily.

ON THE ACCURACY OF AN ITERATION METHOD WHEN SOLVING A SYSTEM OF TIMOSHENKO EQUATIONS

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A numerical algorithm is constructed for the solution of the nonlinear initial boundary value problem [1, 2]

$$\frac{\partial^2 w}{\partial t^2} = \left(cd - a + b\int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx \right) \frac{\partial^2 w}{\partial x^2} - cd\,\frac{\partial \psi}{\partial x}\,,$$
$$\frac{\partial^2 \psi}{\partial t^2} = c\,\frac{\partial^2 \psi}{\partial x^2} - c^2 d\left(\psi - \frac{\partial w}{\partial x}\right),$$
$$0 < x < 1, \quad 0 < t \le T, \quad a,b,c,d > 0, \quad cd - a > 0,$$
$$\frac{\partial^l w}{\partial t^l}(x,0) = w^{(l)}(x), \quad \frac{\partial^l \psi}{\partial t^l}(x,0) = \psi^{(l)}(x),$$
$$w(0,t) = w(1,t) = 0, \quad \frac{\partial \psi}{\partial x}(0,t) = \frac{\partial \psi}{\partial x}(1,t) = 0,$$
$$0 \le x \le 1, \quad 0 \le t \le T, \quad l = 0,1.$$

As a result of the discretization of the problem with respect to spatial and time variables we obtain a nonlinear system of algebraic equations. As different from [3], where the discrete system is solved by Picard's iteration method, here we used Jacobi's iteration method. The error of the method is estimated.

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THE FOURTH ORDER OF ACCURACY SEQUENTIAL TYPE RATIONAL SPLITTING FOR THE SEMIGROUP APPROXIMATION

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Let A be a self-adjoint positively defined (generally unbounded) operator in the H Hilbert space. Besides, $A = A_1 + A_2$, where A_1 and A_2 are also self-adjoint positively defined operators on $D(A_1) \bigcap D(A_2)$. Then the following estimation takes place:

$$\left\| \left(\exp(-tA) - \left[V\left(\frac{t}{n}\right) \right]^n \right) \varphi \right\| = O\left(\frac{1}{n^4}\right), \quad \varphi \in D\left(A^5\right),$$

where

$$\begin{aligned} V(t) &= W\left(t, \frac{\overline{\alpha}}{4} A_1\right) W\left(t, \frac{\overline{\alpha}}{2} A_2\right) W\left(t, \frac{1}{4} A_1\right) W\left(t, \frac{\alpha}{2} A_2\right) W\left(t, \frac{\alpha}{2} A_1\right) \\ &\quad \times W\left(t, \frac{\alpha}{2} A_2\right) W\left(t, \frac{1}{4} A_1\right) W\left(t, \frac{\overline{\alpha}}{2} A_2\right) W\left(t, \frac{\overline{\alpha}}{4} A_1\right), \\ W(t, A) &= \left(I - t\frac{\alpha}{2} A\right) \left(I + t\frac{\overline{\alpha}}{2} A\right)^{-1} \left(I - t\frac{\overline{\alpha}}{2} A\right) \left(I + t\frac{\alpha}{2} A\right)^{-1}, \end{aligned}$$

and where $\alpha = \frac{1}{2} \pm i\frac{1}{2\sqrt{3}}$.
It is easy to show that $\|V(t)\| \leq 1$. From here follows stability of the corresponding decomposition scheme.
Let us note that the scalar function corresponding to the operator $W(t, A)$

$$Q(x) = \frac{1 - \frac{1}{2}x + \frac{1}{12}x^2}{1 + \frac{1}{2}x + \frac{1}{12}x^2},$$

represents Pade approximation for e^{-x}.
In [1] on the basis of rational splitting of the semigroup we have constructed the third order of accuracy decomposition scheme for evolution problem.

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ON CONSTRUCTION OF A HIGH ORDER OF ACCURACY DECOMPOSITION SCHEME FOR NON-HOMOGENEOUS ABSTRACT HYPERBOLIC EQUATION ON THE BASIS OF APPROXIMATION OF SINE AND COSINE OPERATOR FUNCTIONS

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As it is known, the solution of Cauchy problem for an abstract hyperbolic equation can be given by means of sine and cosine operator functions, where square root from the main operator is included in the argument. Using this formula, for the equally distanced values of the time variable, the precise three-layer semi-discrete scheme can be constructed, whose transition operator is a cosine operator function. Main purpose of the work is to construct decomposition scheme for abstract hyperbolic equation by means of the above-mentioned scheme basing on splitting of cosine-operator function.
In the present work, on the basis of rational splitting of cosine operator-function, there is constructed the following fourth order accuracy decomposition scheme for nonhomogeneous hyperbolic equation, when the main operator $(A = A_1 + A_2)$ is self-adjoint positively defined and is represented as a sum of two addends:

$$\begin{aligned} u_{k+1} &= V(\tau)\,u_k - u_{k-1} + \psi_k, \quad k = 1, ..., n-1, \\ u_0 &= \varphi_0, \quad u_1 = \frac{1}{2}\left(V(\tau)\,\varphi_0 + \tau V\left(\frac{\tau}{\sqrt{3}}\right)\varphi_1\right) + \tau^2\varphi_2, \end{aligned}$$

where

$$\begin{aligned} V(\tau) &= V_0(\tau; A_1, A_2) + V_0(\tau; A_2, A_1), \\ V_0(\tau; A_1, A_2) &= \left(I + \alpha\tau^2 A_1\right)^{-1}\left(I + \lambda\tau^2 A_2\right)^{-1}\left(I + \overline{\alpha}\tau^2 A_1\right)^{-1}, \\ \psi_k &= \tau^2\left(I + \frac{1}{12}\tau^2 A_1\right)^{-1}\left(I + \frac{1}{12}\tau^2 A_2\right)^{-1} f(t_k) + \frac{1}{12}\tau^4 f''(t_k), \\ \varphi_2 &= \frac{1}{2} f(0) + \frac{\tau}{6} f'(0) + \frac{\tau^2}{24} f''(0) - \frac{2\tau^2}{3} A f'(0), \end{aligned}$$

and where $\lambda = \frac{1}{2} \pm \frac{1}{\sqrt{6}}$, $\alpha = \frac{1-\lambda}{2} \pm i\frac{\sqrt{3-(1-\lambda)^2}}{2}$, $\overline{\alpha}$ is a conjugate of α, $f(t)$ is the right-hand side of the equation, φ_0 and φ_1 are initial values, τ is a time step, $n > 1$ is a number of division of time variable. Stability of the constructed scheme is shown and the error of approximate solution is estimated.

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THE THIRD ORDER OF ACCURACY SEQUENTIAL TYPE RATIONAL SPLITTING OF EVOLUTION PROBLEM

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In the present work, on the basis of rational splitting of semigroup, there is constructed the following third order of accuracy sequential type decomposition scheme for nonhomogeneous evolution problem, when the main operator $(A = A_1 + A_2)$ generates strongly continuous semigroup in the Banach space and is represented as a sum of two addends:

$$\begin{aligned} u_k &= V(\tau)u_{k-1} + \frac{\tau}{4}\left(3S\left(\frac{1}{3}\tau\right) f\left(t_{k-1/3}\right) + S\left(\tau\right) f\left(t_{k-1}\right)\right), \\ u_0 &= \varphi, \quad k = 1, 2, ..., \end{aligned}$$

where

$$\begin{aligned} V(\tau) &= W\left(\tau, \frac{\alpha}{2}A_1\right) W\left(\tau, \alpha A_2\right) W\left(\tau, \frac{1}{2}A_1\right) W\left(\tau, \overline{\alpha} A_2\right) W\left(\tau, \frac{\overline{\alpha}}{2}A_1\right), \\ W\left(\tau, A\right) &= \left(I - \frac{1}{3}\tau A\right)\left(I + \lambda\tau A\right)^{-1}\left(I + \overline{\lambda}\tau A\right)^{-1}, \\ S(\tau) &= K\left(\tau, \frac{1}{2}A_1\right) K\left(\tau, A_2\right) K\left(\tau, \frac{1}{2}A_1\right), \\ K(\tau, A) &= \left(I - \frac{1}{2}\tau A\right)\left(I + \frac{1}{2}\tau A\right)^{-1}, \end{aligned}$$

and where $\alpha = \frac{1}{2} \pm i\frac{1}{2\sqrt{3}}$, $\lambda = \frac{1}{3} \pm i\frac{1}{3\sqrt{2}}$, $f\left(t\right)$ is the right-hand side of the equation, φ_0 is initial value, τ is a time step. Let us note that the operator $K(\tau, A)$ is the transition operator of Krank-Nickolson scheme. Stability of the constructed scheme is shown and the error of approximate solution is estimated. In [1] we have constructed the third order of accuracy sequential type exponential splitting for homogeneous evolution problem.

References

[1] J. Rogava, M. Tsiklauri, Third Order of Accuracy Sequential Type Decomposition Schemes for Two and Multidimensional Evolution Problems, Tbilisi, AMIM, Vol. 10, No. 1, pp. 72-87, 2005.

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TO METHODS OF APPROXIMATE SOLUTION OF BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

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There are considered two methods of approximate solution and arising according to motion problems of numerical realization for ordinary differential equations:

$$Ly(x) = (k(x)y'(x))' + p(x)y(x) = f(x, y(x), y'(x)), \quad (-1 < x < 1), \tag{1}$$

with boundary conditions:

$$\alpha_1 y(-1) + \beta_1 y'(-1) = \gamma_1, \quad \alpha_2 y(1) + \beta_2 y'(1) = \gamma_2. \tag{2}$$

First of them extends the optimal schemes of [1],ch.3, parts 13.1-13.3, 15.1, when $p(x)$ is nontrivial function.
The second one represents a projective-iterated scheme for approximate solution of (1)-(2), when $p(x)$ is polynomial, $k(x) = (1-x)^\alpha(1+x)^\beta$ $(\alpha, \beta > -1)$. According of this scheme, for each iterated step we inert operator defining on the class of functions satisfying the homogeneous boundary conditions (2). As coordinate functions are used the linear combinations of classical orthogonal respect to weight $k(x)$ polynomial systems.
At last we reduce results according to numerical realization for some examples by methodology of [1], ch.3, parts 12.1-123 and 17.1.

References

[1] Tamaz Vashakmadze. The Theory of Anisotropic Elastic Plates. Kluwer Acad.Publ.-Springer- Verlag, Dordrecht/Boston/London,1999.

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