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The present book of abstracts contains abstracts of talks given at 40th Enlarged Sessions (April 22-24, 2026) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

ABSTRACTS OF PLENARY TALKS

LIFE AND SCIENTIFIC ACTIVITIES OF ANDRO BITSADZE

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The present paper is devoted to life and scientific activities of the second director after the first founder director of VIAM Ilia Vekua.

Andro Bitsadze was born on May 22, 1916 in the village of Tskhrukveti, in the province of Kutaisi. He went first to the primary school in Tskhrukveti and then to the nine-year school in Chiatura. After finishing the school, in 1935-1940 he studied at the Faculty of Physics and Mathematics of Tbilisi State University. In 1940-43 A. Bitsadze was a post-graduate student of A. Razmadze Mathematical Institute of the Georgian Academy of Science. In 1944 he successfully defended his candidate's (Ph. D.) thesis under the guidance of Academician Ilia Vekua.

In 1944-48 he was a researcher at Tbilisi Mathematical Institute.

In 1948-51 A. Bitsadze was a doctoral student at V. Steklov Mathematical Institute of the USSR Academy of Science. In 1951 he defended his Doctor Science thesis.

In 1951-59 he was a senior researcher at V. Steklov Mathematical Institute.

In 1958 he was elected Corresponding Member of the USSR Academy of Science.

In 1959 A. Bitsadze with his family moved to Novosibirsk. There he greatly assisted the first Rector of Novosibirsk University, Academician I.Vekua in founding and creating a powerful scientific centre in Novosibirsk.

In 1959 his well-known monograph "Mixed type equations" was published by "Nauka". The monograph was later translated into different languages.

In 1966 A.Bitsadze's another notable monograph "Boundary value problems for second order elliptic equations" came out at "Nauka" Publishing House. Afterwards it was translated into English and published by "North Holland Publishing Company".

In 1969 he was elected Full member of the Georgian Academy of Science.

From 1971 he returned to his work at V. Steklov Mathematical Institute as a Head of Department and worked in this position until the end of his life. In 1972 he was invited by the Board of Directors of the Moscow Physical-Engineering Institute as Professor and Head of Chair of Higher Mathematics.

In 1978 A. Bitsadze was invited to Tbilisi as a director and scientific leader of I Vekua Institute of Applied Mathematics. During his stay in Tbilisi in 1981 his another monograph "Some classes of partial differential equations" was published by "Nauka" which was soon translated into English Unfortunately, at the end of 1983 A.Bitsadze had to leave the position of the director of I.Vekua Institute of Applied Mathematics.

After returning to Moscow A. Bitsadze went on with his scholarly and educational activities and from 1983 until the end of his life he was Professor at M. Lomonosov Moscow State University.

A. Bitsadze died on September 6, 1994, in Moscow.

UNIT CIRCLES AND TOTALLY OSCULATING SHAPES

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In the study of natural shapes and phenomena and in various fields of science and technology Gielis curves and surfaces have been widely used [1]. They are generalizations of conic sections, with Lamé curves as special cases. They are unit circles in their own right, and it has been proposed to to use the shape itself as totally osculating curve [1].

The definition of inner and vector products developed in [2], and the differential apparatus with the star derivative in [3], provide the methodological basis. These methods codify the central notion of stretchable radius [1], similar to the stretched Laplacian for the solution of boundary value problems [5].

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SEMI-MARKOV PROCESSES AND THEIR APPLICATIONS IN QUEUEING AND RELIABILITY NEW SOLUTIONS

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Traditionally, control decisions at various stages of the life cycle of complex systems are made using the methods of classical Queuing Theory (QT) and Mathematical Theory of Reliability (MTR) [1], [2], [5].

In QT and MTR, Markov and semi-Markov(SM) processes have been historically prioritized over all others due to their practical applicability. At the same time, SM models provide much more adequate descriptors of real systems. That is why SM models constitute the major part of QT and MTR. On the other hand, both the construction and investigation of SM models are significantly more difficult than those of Markov ones. Transient investigation of the SM models is especially complicated. Over the course of the development of QT and MTR, a certain gap between classical theory and application has emerged. Our goal is to narrow this gap by rooting our models and methods in assumptions guided by the real world.

We have developed a completely novel, simple, and very effective, partially probabilistic method for investigating semi-Markov models: [3-4], [6-8]. This method represents a special variety of the supplementary variables technique (SVT) — a modified SVT avoiding partial differential equations (Kolmogorov equations).

That is a new contribution in studies of SMP on the whole, and significantly enhances its applicability in practice. At the same time, it provides a general form to solutions for some interesting class of PDEs and their finite and infinite systems, no matter where they were arisen. This is another interesting aspect of our approach.

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ABSTRACTS OF SECTIONAL TALKS

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

A NOTE ON THE INTERSECTIONS OF MAZURKIEWICZ TYPESETS WITH PLANE ALGEBRAIC CURVES

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It is known that for any natural number $n \geq 2$, there exists a subset of the Euclidean plane that intersects every straight line in the plane in exactly n points. Such a set is called a Mazurkiewicz n-type set with respect to the family of all straight lines in the plane (see [1], [2]); briefly - a Mazurkiewicz n-type set with respect to straight lines.

It is also known that for any natural number $k \geq 3$, there exists a subset of the Euclidean plane that intersects every non-degenerate circle in the plane in exactly k points. Such a set is called a Mazurkiewicz k-type with respect to the family of all circles in the plane (see [3], [4]); briefly - a Mazurkiewicz k-type set with respect to circles.

In the present talk it is shown that:

1. For any natural number $n \geq 2$ and for every plane algebraic curve L of degree 2 or higher, irreducible over the field of all real numbers, there exists a Mazurkiewicz n-type set with respect to straight lines such that its intersection with each non-degenerate simple arc of L has the cardinality of the continuum and, moreover, the above-mentioned intersection satisfies certain additional conditions in terms of the linear Lebesgue measure and Baire category.
2. For any natural number $k \geq 3$ and for every plane algebraic curve C of degree 3 or higher, irreducible over the field of all real numbers, there exists a Mazurkiewicz k-type set with respect to circles such that its intersection with each non-degenerate simple arc of C has the cardinality of the continuum and, moreover, the above-mentioned intersection satisfies certain additional conditions in terms of the linear Lebesgue measure and Baire category.

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ON CERTAIN PROPERTIES OF SIERPIŃSKI–ZYGMUND FUNCTIONS

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In the classical work "Sur une fonction qui est discontinue sur tout ensemble de puissance du continu (1923)", W. Sierpiński and A. Zygmund constructed a function $f: R \to R$ with the following property: for each subset Y of R with $card(Y) = c$, the restriction $f \mid Y$ is not continuous on Y. This classical result was fundamentally motivated by Blumberg's theorem, which states that for any real-valued function, there exists an everywhere dense subset on which the restriction is continuous.

The study of Sierpiński–Zygmund (SZ) functions demonstrates significant connections between real analysis, point-set topology, and set-theoretic principles.

More precisely, assuming Martin's Axiom (MA), it can be proven that there exists an injective additive SZ function that is simultaneously a solution to the Cauchy functional equation and absolutely non-measurable.

In this talk, we consider the properties of these functions in the context of specific point sets, such as Mazurkiewicz sets (which can contain the graph of an SZ function) and generalized Luzin sets.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [FR-25-19092 Komori type epistemic MV-algebras with constants].

ON COMBINATORIAL PROPERTIES OF POLYHEDRONS WITH TRIANGULAR FACES

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This talk explores the combinatorial properties of polyhedra with triangular faces. Polyhedra whose faces are exclusively triangles play a significant role in both geometry and combinatorics. Studying their structure allows us to establish relationships between the numbers of vertices, edges, and faces. One of the main tools used in this analysis is Euler's formula, which provides a fundamental relation among these quantities. In the case of triangular polyhedra, an additional constraint arises from the fact that each face consists of exactly three edges, leading to simplified combinatorial relations.

In the talk we discuss both regular and irregular polyhedra, as well as their graph representations. Special attention is given to the distribution of vertex degrees and symmetry properties. The obtained results are important for developments in geometric modeling, topology, and graph theory.

ON Q_1-DEGREES OF NONSPEEDABLE SETS

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Tennenbaum (see [1, p.159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is *Q-reducible* to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B.$$

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via f and for all x, y,

$$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset,$$

then we say that A is *Q_1-reducible* to B (in symbols: $A \leq_{Q_1} B$).

Suppose that $\{\varphi_i\}_{i \in \omega}$ is an effective numbering of all partial computable functions and let $\{\Phi_i\}_{i \in \omega}$ be a sequence of partial computable functions satisfying the following Blum's axioms [1]:

(1) the function $\varphi_i(x)$ is defined if and only if the function $\Phi_i(x)$ is defined.

(2) The function

$$M(i, n, m) = \begin{cases} 1, & \text{if } \Phi_i(n) = m, \\ 0, & \text{otherwise,} \end{cases}$$

is computable.

A c.e. set A is speedable if for all i such that $W_i = A$ and for all computable function h, there exists j such that $W_j = A$ and

$$(\exists^{\infty} x)(\Phi_i(x) > h(x, \Phi_j(x))),$$

and A is nonspeedable otherwise.

Our notations and terminology are standard and can be found in [2] and [3]. In this talk we will present the following result :

Theorem. Let A, B be noncomputable nonspeedable c.e. sets such that B is not a simple set, $A <_{Q_1} B$ and $A \oplus B \leq_{Q_1} B$, then there exist infinitely many pairwise Q_1-incomparable nonspeedable sets $\{C_i\}_{i \in \omega}$ such that for all $i \in \omega$,

$$A <_{Q_1} C_i <_{Q_1} B.$$

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ASSOCIATIVE R-FUNCTIONS AS DIRECTIONAL OPERATORS

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Constructing associative R-functions remains a significant challenge in functional modeling, as traditional nested binary compositions lead to asymmetric scalar fields and gradient distortions. This paper partly addresses the challenge of constructing associative N-ary R-functions and explores potential solutions. We present a strictly associative solution for the ternary case (N=3) based on spherical geometry and explore research avenues for extending these properties to N > 3 by treating R-functions as directional operators.

This evolution is grounded in a fundamental shift in perspective: reinterpreting R-functions not merely as algebraic combinations, but as directional operators that govern the progression of the potential field. While the ternary formulation ensures perfect symmetry and unit normal gradients in 3D, its extension to higher dimensions remains non-trivial. Inspired by the angular decomposition of spherical coordinates, we propose a "quasi-associative" N-ary approach for high-dimensional search spaces. This directional operator allows for a linear O(n) evaluation of the potential field, effectively guiding optimization solvers while highlighting the fundamental trade-off between computational scalability and C^k continuity at the zero-level set.

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STABILITY OF ISOMORPHISM OF FUNCTION DIGRAPHS UNDER COMPOSITION AND ARITHMETIC OPERATIONS

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The work is dedicated to the study of certain properties of directed graphs corresponding to mono-unary algebras. Based on the types of connected components of the graph of a mono-unary algebra (A, f), the types of connected components for the graph of the mono-unary algebra (A, f^m), $(m \in \mathbb{N})$ are determined for cases where f is a bijection, and the interrelationships between the connected components of these algebras are investigated. Furthermore, analogous problems are considered for certain non-bijective mono-unary algebras. It is shown that, in the general case, neither composition nor arithmetic operations preserve the isomorphism of mono-unary algebras.

SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz (Austria)
Co-chair: Besik Dundua, Mikheil Rukhaia

Dedicated to the 80th Birthday of Professor Khimur Rukhaia

SCALABILITY OF PROBABILISTIC INDUCTIVE LOGIC PROGRAMMING TO REAL AI SYSTEMS

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Probabilistic Inductive Logic Programming (PILP) offers a powerful framework for learning interpretable relational models under uncertainty, yet its adoption in real-world artificial intelligence systems remains limited due to scalability challenges [2, 4]. Most existing ILP-based approaches suffer from high computational cost and are unable to handle the scale and complexity of modern deep learning datasets [1].

This talk considers these limitations by exploring the design of efficient hybrid systems that integrate ILP with deep learning and probabilistic reasoning. The approach aims to overcome the traditional trade-off between expressiveness and efficiency.

In this talk, we outline a roadmap toward scalable neurosymbolic PILP systems and highlights their potential to support complex AI applications that require both high performance and transparent reasoning [3].

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UNRANKED PROBABILISTIC REASONING: ADDRESSING FUNDAMENTAL GAPS IN UNCERTAINTY MODELING FOR ARTIFICIAL INTELLIGENCE

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Existing uncertainty frameworks, interval-based probabilities, belief functions, and possibility theory, remain weakly integrated with modern Artificial Intelligence systems. In particular, they are not well aligned with machine learning [1] and deep learning paradigms, limiting their applicability in large-scale, data-driven environments. As a result, real-world adoption of these models [2] remains limited, with few practical systems deploying them effectively.

In this talk, we aim to bridge the gap between theoretical uncertainty models and real-world AI applications. We argue that improving compatibility with learning-based systems and addressing scalability challenges are essential for effective reasoning [3] under uncertainty.

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A LOGIC PROGRAMMING FRAMEWORK WITH HIGHER-ORDER PATTERN UNIFICATION MODULO SIMILARITY RELATIONS

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We present a logic programming framework that integrates higher-order pattern unification with similarity-based reasoning. Within this framework, we define a unification algorithm for higher-order patterns modulo similarity relations and prove its termination, soundness, and completeness. Moreover, as in the crisp setting, the unification problem is unitary, and whenever two terms are unifiable, the algorithm computes a most general unifier with the highest possible degree of approximation. We also study the operational and declarative semantics of the framework.

GEORGIAN DENOMINATIVE VERB MODELS AND THEIR ENGLISH EQUIVALENTS

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Lexical semantics enables a dialogue system to grasp the nuances of word meanings and interrelations in the Georgian language. The integration of the lexical network GeWordNet into the system supports the development of an intelligent Georgian-language dialogue interface, facilitating not only effective communication but also the generation of semantically appropriate responses in context. GeWordNet further contributes to the creation of a lexical-semantic resource aimed at enhancing language understanding.

For GeWordNet to function correctly, accurate Georgian translations of corresponding English lexemes are essential. This necessity, in turn, leads to the development of synthesis and analysis modules for both languages. Consequently, it becomes crucial to generate precise translations for deverbal verbs that are not included in the core lexicon.

The talk discusses models of Georgian denominative verbs and explores the possibility of establishing corresponding English structural models for them.

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DERIVATIVES OR REGULAR EXPRESSIONS WITH SIMILARITY RELATION

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In this talk, we extend Brzozowski's derivative by incorporating a similarity relation that allows fuzziness at the level of symbols, words, languages, and regular expressions. We define fuzzy derivatives, prove their correctness properties and show how they can be used for fuzzy automata construction.

ELECTROMAGNETIC FIELD LEVELS IN AVLABARI (TBILISI) AND FUZZY LOGIC MODEL FOR RISK ASSESMENT

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This research presents an initial analysis of electromagnetic field (EMF) exposure in the Avlabari district of Tbilisi. The research is based on collected data from mobile base stations, power lines, and Wi-Fi zones. The main goal is to understand how EMF is distributed in this urban area.

The results show that EMF levels are not the same in all locations. Higher levels are found near mobile towers and main roads, while lower levels are observed in open and less populated areas. The research also identifies important problems, such as limited access to public data and lack of continuous monitoring.

In addition, this paper proposes a conceptual model for EMF risk assessment using fuzzy logic and probability methods. The model includes different factors, such as EMF strength, distance from sources, and exposure time. These factors are used to classify EMF levels into categories like low, moderate, high, and very high.

A simple example is presented to show how fuzzy logic can help in making decisions under uncertainty. The proposed approach can be useful for environmental monitoring and smart city planning.

Acknowledgements. This work was supported by the Shota Rustaveli National Science Foundation of Georgia under the project YS-25-4322.

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3. A Fuzzy Logic Technique for the Environmental Impact Assessment of Marine Renewable Energy Power Plants Pamela Flores and Edgar Mendoza (2025)

CONSTRAINT-SOLVING ALGORITHM WITH PROXIMITY RELATIONS IN COMMUTATIVE THEORIES

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We present a constraint-solving algorithm for commutative theories extended with proximity relations. The proposed method supports approximate symbolic matching, allowing terms to be compared even when they are not strictly identical. We prove that the algorithm terminates on all input constraints and is both sound and complete. In addition, we discuss its computational behavior and outline several potential applications of the solver in areas requiring flexible symbolic reasoning.

SCIENTIFIC-RESEARCH INSTITUTE FOR CULTURAL PROTECTION AND TECHNOLOGICAL DEVELOPMENT OF GEORGIAN STATE LANGUAGES AT THE GEORGIAN TECHNICAL UNIVERSITY: AIMS, TASKS, METHODS

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At Georgian Technical University, on May 15, 2025, „Scientific-Research Institute for Cultural Protection and Technological Development of Georgian State Languages“ was established. Through this initiative, the research processes concerning the complete mathematical study and technological support of Georgian state languages have been reinstated at the budgetary level within the Georgian university space after 18 years.

Among the Institute primary aims and tasks are to ensure the digital sovereignty of the Georgian State through complete technological support of the Georgian and Abkhazian languages, to unify Georgia through the improvement of Georgian-Abkhazian relations and, also, to develope basic and deepened texbook in logic and logical grammar of the Georgian language.

In addition to above, research methods of the institute are based on Shalva Pkhakadze's Notation Theory and, also, on K.Pkhakadze's foundations of logical grammar of the Georgian language and methods of formal-logical description of the Georgian language.

Acknowledgements. To the Ministry of Education, Science and Youth of Georgia.

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TOWARDS CUT-ELIMINATION IN PROBABILIZED UNRANKED SEQUENT CALCULUS

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All probabilistic logic formalisms studied so far are either propositional, or permit only individual variables, i.e., variables that can be instantiated by a single term. On the other hand, theories and systems that use not only individual variables but also sequence variables (these variables can be replaced by arbitrary finite, possibly empty, sequences of terms) have emerged. There are systems for programming with sequence variables. Probably the most prominent one is Mathematica [4], with a powerful rule-based programming language that uses (essentially first order, equational) unranked matching with sequence variables [2]. The unranked term is a first-order term, where the same function symbol can occur in different places with different number of arguments. Unranked function symbols and sequence variables bring a great deal of expressiveness in a language, permitting writing a short, concise, readable code.

We make one step forward in hybridizing logical and probabilistic methods, and develop probabilized first-order sequent calculus with sequence variables and unranked function symbols. In such formalism, sequence variables, unranked terms and probabilistic primitives are available together. We probabilize the Gentzen-style inference system G, given in [3], in a similar way, as Marija Boričić probabilized classical propositional sequent calculus in [1]. We show that the new system keeps properties like soundness and completeness.

We are interested to study cut-elimination in the probabilized unranked sequent calculus. The original system G has the cut-elimination property and it is our ongoing work to prove that this priperty is preserved in the new system as well. For this purpose, we follow Gentzen's original proof procedure and develop cut-elimination rewrite rules for different cases.

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RESOLUTION RULE TO τ-LOGIC FORMULAS

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Computer programs have been developed to automate the task of reasoning and proving theorems. Many of these programs make use of a rule of inference known as resolution. τ-Logic is constructed on the basis of $\tau$the main principles of the notation theory and would be interesting to apply this rule to τ-logic formulas.

SECTION OF ALGEBRA, GEOMETRY AND NUMBER THEORY

Chairs: Mikhail Amaghlobeli, George Khimshiashvili, Teimuraz Vepkhvadze, Malkhaz Bakuradze
Co-chair: Ketevan Shavgulidze

THE STRUCTURE OF THE TWO-GENERATED FREE 2-NILPOTENT R-GROUP, WHERE $R = \mathbb{Q}(t)$

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Let R be an arbitrary associative ring with unity. Using this ring, we define a new category of R-groups in three different ways. For this purpose, we extend the group language $L_{gr}\left\{\cdot, {}^{-1}, e\right\}$ as follows: $L_{gr}^{R} = L_{gr} \cup \left\{ f_{\alpha}(g) \middle| \ \alpha \in \mathbb{R} \right\}$, where $f_{\alpha}(g)$ is a unary operation defined by $f_{\alpha}(g) = g^{\alpha}$ for all $g \in G$.

A set G is called a Lyndon exponential R-group [1] if it is equipped with operations $\cdot$, ${}^{-1}$, e, f and satisfies the following axioms:

1) the group axioms;

2) $g^{\alpha+\beta} = g^{\alpha} g^{\beta}$, $g^{\alpha\beta} = \left(g^{\alpha}\right)^{\beta}$, $\left(h^{-1}gh\right)^{\alpha} = h^{-1} g^{\alpha} h$.

Unfortunately, Lyndon exponential R-groups are not always R-modules, which complicates the use of this concept in the case of non-free groups.

In[2]A. Myasnikov and V. Remeslennikov introduced a new category $\mathbb{M}_{R}$ (MR-groups) , in which Lyndon's axioms are supplemented by the following quasi-identity:

(MR-axiom) $\forall g, h \in G \ [g,h] = e \to (gh)^{\alpha} = g^{\alpha} h^{\alpha}$, $\alpha \in \mathbb{R}$, $[g,h] = g^{-1} h^{-1} gh$.

Clearly, every R-module satisfies this property.

For nilpotent groups and binomial rings, Ph. Hall introduced in [3] a category of R-groups that differs from $\mathbb{M}_{R}$

In[4], the structure of free two-generated 2-nilpotent exponential R-groups over the ring $R = \mathbb{Q}[t]$ is studied.

This report is devoted to the study of 2-nilpotent exponential R-groups (MR-groups) over the field of rational functions $R = \mathbb{Q}(t)$[5]. The obtained results make it possible to investigate fundamental algorithmic problems for this class of groups [6].

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THE CAUCHY-BUNIAKOVSKY-SCHWARZ INEQUALITY IN PSEUDO-EUCLIDEAN SPACE

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It is known that any real numbers $a_1,...,a_n,b_1,...,b_n$ satisfy the Cauchy-Buniakovsky-Schwarz inequality:

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right) \tag{1}$$

and the equality is achieved if $a_i = kb_i, \forall k \in \mathsf{R},\ i = \overline{1,n}$. The well-known geometric interpretation of inequality (1) considers these numbers $a_k, b_k,\ k = \overline{1,n},$ as Cartesian components of vectors $\boldsymbol{a},\boldsymbol{b}$ in a linear vector Euclidean (real) E^n space. Moreover, (1) provides the possibility for defining the angle between vectors $\boldsymbol{a},\boldsymbol{b} \in \mathsf{E}^n$:

$$\cos\varphi = \left(\sum_{i=1}^{n} a_i b_i\right)\left(\sum_{i=1}^{n} a_i^2\right)^{-1/2}\left(\sum_{i=1}^{n} b_i^2\right)^{-1/2}. \tag{2}$$

1. We have studied relations analogous of inequality (1) for vectors $\boldsymbol{a},\boldsymbol{b} \in \mathsf{P}^n_{1,n-1}$, where $\mathsf{P}^n_{1,n-1}$ denotes a pseudo-Euclidean (real) linear vector space with a Cartesian metric

$$g_{ij} = g^{ij} = \mathrm{diag}(1,-1,-1,...,-1),\ \ i,j = \overline{0,n-1}.$$

Specifically, we considered Minkowski space $\mathsf{P}^4_{1,3}$ which is important for physical applications. For vectors $\boldsymbol{a},\boldsymbol{b} \in \mathsf{P}^n_{1,n-1}$ $(\boldsymbol{a} \neq k\boldsymbol{b})$, we proved the following:

Theorem:

- ***a***) *The inequality* $\langle \boldsymbol{a}|\boldsymbol{b}\rangle^2 > \langle \boldsymbol{a}|\boldsymbol{a}\rangle^2\langle \boldsymbol{b}|\boldsymbol{b}\rangle^2$ *holds iff the vectors* $\boldsymbol{a},\boldsymbol{b}$ $(\boldsymbol{a} \neq k\boldsymbol{b})$ *belong to a pseudo-Euclidean (real) 2-plane* $\mathsf{P}^2_{1,1} \subset \mathsf{P}^4_{1,3}$;
- ***b***) *The inequality* $\langle \boldsymbol{a}|\boldsymbol{b}\rangle^2 < \langle \boldsymbol{a}|\boldsymbol{a}\rangle^2\langle \boldsymbol{b}|\boldsymbol{b}\rangle^2$ *holds iff the vectors* $\boldsymbol{a},\boldsymbol{b}$ $(\boldsymbol{a} \neq k\boldsymbol{b})$ *belong to a Euclidean (real) 2-plane* $\mathsf{P}^2_{0,2} = \mathsf{E}^2 \subset \mathsf{P}^4_{1,3}$;
- ***c***) *The equality* $\langle \boldsymbol{a}|\boldsymbol{b}\rangle^2 = \langle \boldsymbol{a}|\boldsymbol{a}\rangle^2\langle \boldsymbol{b}|\boldsymbol{b}\rangle^2$ *holds iff the vectors* $\boldsymbol{a},\boldsymbol{b}$ $(\boldsymbol{a} \neq k\boldsymbol{b})$ *belong to a degenerate* (see, e.g., [1]) *(real) 2-plane.*

Here and bellow $\langle \boldsymbol{a}|\boldsymbol{b}\rangle$ denotes the scalar product of vectors $\boldsymbol{a},\boldsymbol{b} \in \mathsf{P}^n_{1,n-1}$:

$$\langle \boldsymbol{a}|\boldsymbol{b}\rangle = \sum_{i,j=1}^{n} g^{ij}a_i b_j = \sum_{j=1}^{n} a^j b_j = \sum_{i=1}^{n} a_i b^i, \quad \sum_{i=1}^{n} g^{ij}a_i = a^j, \quad \sum_{j=1}^{n} g^{ij}b_j = b^i.$$

2. We generalized the Lagrange identity for vectors $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d} \in \mathsf{P}^n_{1,n-1}$ of a pseudo-Euclidean (real) space $\mathsf{P}^n_{1,n-1}$ (summation is implied over repeated indices):

$$\langle (\boldsymbol{a}\wedge\boldsymbol{b})|(\boldsymbol{c}\wedge\boldsymbol{d})\rangle = (2!)^{-1} g^{ij} g^{kl} (\boldsymbol{a}\wedge\boldsymbol{b})_{ik} (\boldsymbol{c}\wedge\boldsymbol{d})_{jl} = (2!)^{-1} g^{ij} g^{kl} (a_i b_k - b_i a_k)(c_j d_l - d_j c_l) =$$
$$= \langle \boldsymbol{a}|\boldsymbol{c}\rangle\langle \boldsymbol{b}|\boldsymbol{d}\rangle - \langle \boldsymbol{a}|\boldsymbol{d}\rangle\langle \boldsymbol{b}|\boldsymbol{c}\rangle,$$
$$\langle (\boldsymbol{a}\wedge\boldsymbol{b})|(\boldsymbol{a}\wedge\boldsymbol{b})\rangle = \langle \boldsymbol{a}|\boldsymbol{a}\rangle\langle \boldsymbol{b}|\boldsymbol{b}\rangle - \langle \boldsymbol{a}|\boldsymbol{b}\rangle^2.$$

3. Based on above results we have defined the angles between vectors in Minkowski space $\mathsf{P}^4_{1,3}$

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GENERALIZED THETA-SERIES FOR SOME SIX VARIABLE QUADRATIC FORMS

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In [1, 2, 3], the upper bounds for the dimensions of the spaces T(ν; Q) for certain quadratic forms in three, four, five and r variables were established. In several cases, the dimensions were calculated and the bases for these spaces were constructed.

In this paper, some six variable positive definite diagonal and non-diagonal quadratic forms are considered, the spaces of spherical functions with respect to these forms are constructed, and the corresponding generalized theta-series are established.

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MÖBIUS PHENOMENON IN GML SURFACES AND BODIES WITH DIFFERENT RADIAL CROSS-SECTION

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The report analytically records the generalized Möbius-Listing's surfaces and bodies - (GML_m^n) with the Möbius phenomenon - That is, for certain values of the parameters (m,n) and in the case of a certain shape of the diametrical cross section, the results of a complete cut results in either multiple parts or exhibits the Möbius phenomenon when the results is a single surface or body. Among the radial cross-sections of the considered GML_m^n (plane and linear figures) are such objects as: the Euler spiral - "clothoid", Lamé curves, "Grandy roses", Gielis curves, epicycloids, hypocycloids, etc. For this purpose, the report discusses two large groups of the GML_m^n geometric figures (surfaces and three-dimensional bodies) and three types of "cutting" of these figures, namely a "B-cut" - i.e. cutting along the base line, and chordal and radial cuts.

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ON THE INTEGRABILITY OF TENSOR STRUCTURES OF THE TANGENT BUNDLE $T(V^n)$

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Consider a tangent bundle $T\big(T(Vn)\big)$ with the local coordinates $\big(x^i, y^i\big)$ where x^i are the coordinates of the basis Vn, and y^i are those of the layer $T_z, z \in T(Vn)$, local coordinates of the point of the space $T(Vn)$ are transformed as follows:

$$\bar{x}^i = \bar{x}^i(x^k), \bar{y}^i = x^i_k y^k, \;\; i,j,k = \overline{1,n}.$$

The following theorems are proved:

Theorem 1. If double tensor structures and almost product structures are defined on the base manifold Vn of the tangent boundle $T(Vn)$, then these structures are integrable if and only

if the linear connection $\Gamma_k^i$is smooth with vanishing curvature $R_{ij}^k = 0$, and the vector field $\xi^p$and the co-vector field η_k are covariantly constant.

Theorem 2. If double tensor structures and almost product structures are defined on the space $T(Vn)$, then these structures are integrable if and only if the linear connection $\Gamma_k^i$is smooth with vanishing curvature $R_{ij}^k = 0$, and the vector field $\xi^p$and the co-vector field η_k are covariantly constant.

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GENERALIZED THETA-FUNCTIONS WITH CHARACTERISTICS AND CUSP FORM CORRESPONDING TO QUADRATIC FORMS IN TEN VARIABLES

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The modular properties of generalized theta-functions with characteristics are used to build the cusp form corresponding to quadratic forms in ten variables. This will allow us to obtain formulas for the number of representations of positive integers by quadratic forms in ten variables.

SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
Co-chair: Aleksandre Aplakovi, Ana Danelia

ON THE UNIFORM CONVERGENCE OF TRIGONOMETRIC FOURIER SERIES

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The approximation properties of the partial sums of trigonometric Fourier series for functions within the generalized variation classes $BV(p(n)\uparrow\infty,\varphi)$ and $B\Lambda(p(n)\uparrow\infty,\varphi)$ are investigated. The goal is to determine if these classes can provide better rates of uniform convergence compared to the classical Lebesgue estimate. The results show that under certain conditions, this classes offer improved convergence rates.

Additionally, the functions are constructed within the class $H^{\omega}\cap B\Lambda(p(n)\uparrow\infty,\varphi)$ (but not in $H^{\omega}\cap BV(p(n)\uparrow\infty,\varphi)$) whose Fourier series converges uniformly, emphasizing the advantage of the $B\Lambda(p(n)\uparrow\infty,\varphi)$ class.

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MODULI OF SMOOTHNESS OF CONJUGATE FUNCTIONS

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Moduli of smoothness play a fundamental role in harmonic analysis. For a given function f, they essentially measure the structure or smoothness of the function via the difference of some order. In the theory of conjugate functions, moduli of smoothness provide a natural framework for studying how smoothness is preserved, transferred, or modified under the conjugation operator. These problems are closely connected with the convergence and summability of Fourier series, as well as with the boundedness of conjugate operators in various function spaces.

In the present talk we study the behavior of moduli of smoothness of different orders of conjugate functions of several variables in the spaces *C* and *L* when the global smoothness of the original functions is known. The direct estimates are obtained and exactness of these estimates is established by proper examples.

SUMMABILITY OF THE LAPLACE SERIES BY THE GENERALIZED CESÀRO METHOD

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The summability of Laplace series by the (C, α) method was studied by Hobson (see [1]). He established sufficient conditions for summability by the (C,1) and (C, α) methods, for $\frac{1}{2} < \alpha < 1$, in a three-dimensional space. Subsequently, S. Topuria studied the summability of Fourier–Laplace series by the (C, α) method in spaces of arbitrary dimension. In particular, he proved analogues of Lebesgue's theorem for the case where α exceeds the critical index (see [2]).

As is known, Kaplan considered a certain generalization of the (C, α) method of summability for numerical series, namely the (C, α_n) method, where α_n is a sequence of real numbers with $\alpha_n > -1$. (see [3]). For trigonometric Fourier series, issues of summability by generalized Cesàro methods have been studied by Sh. Tetunashvili (see [4]), Th. Akhobadze (see [5]), and their students.

We studied the summability of Laplace series by the generalized Cesàro (C, α_n) method, where $\frac{1}{2} < \alpha_n < 1$. In particular, for the summability of Laplace series by the (C, α_n) method, we established:

1. an analogue of Hobson's theorem;
2. a certain generalization of Hobson's theorem;
3. an analogue of Lebesgue's theorem.

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ON THE STRONG $(C,1)$ SUMM ABILITY ALMOST EVERYWHERE OF SERIES WITH RESPECT TO BLOCK-ORTHONORMAL SYSTEMS

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Below a question connected with the problems of almost everywhere strong $(C,1)$ summability of series with respect to block-orthonormal systems are considered.

The series $\sum_{n=1}^{\infty} u_n$ is called strong $(C,1)$ summable to the number s if

$$\lim_{n\to\infty} \frac{1}{n+1} \sum_{k=0}^{n} \left(S_k - s\right)^2 = 0,$$

where S_k is a partial sums of series.

Let $\{N_k\}$ be increasing sequences of natural numbers and $\Delta_k = \left(N_k,\, N_{k+1}\right]$, $(k \geq 1)$ .Let $\{\varphi_n\}$ be a system of functions from $L^2(0,1)$. The system $\{\varphi_n\}$ will be called a Δ_k - orthonormal system if $\|\varphi_n\|_2 = 1$, $n = 1,2,...$ and $(\varphi_i, \varphi_j) = 0$, for $(i,j) \in \Delta_k$, $i \neq j$.

The conditions on $\{N_k\}$ are estabilished, when the series

$$\sum_{n=1}^{\infty} a_n \varphi_n(x) \tag{1}$$

is strong $(C,1)$ summable almost everywhere. In particular, it is established, that if $\{N_k\}$ are increasing sequences of natural numbers and

$$T_1 = N_1 = 1,\ T_i = \min\left\{\min\left\{N_q : N_q > T_{i-1},\ q \in N\right\}, \min\left\{2^k : 2^k > T_{i-1},\ k \in N\right\}\right\},\ i \geq 2,$$

then conditions $\sum_{n=1}^{\infty} \frac{1}{\omega(T_n)} < \infty$ and $\log_2^2 k = O(\omega(T_k))$, $k \to \infty$ with the condition $\sum_{n=1}^{\infty} a_n^2 \omega(n) < \infty$ guarantee the strong $(C,1)$ summability almost everywhere of series (1) with respect to Δ_k - orthonormal systems.

ON COEFFCIENTS OF UNIVERSAL SERIES

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It is shown in this talk that if there exists a universal series with respect to a system of measurable and almost everywhere finite functions $\Phi = \{\varphi_n(x)\}_{n=1}^{\infty}$, then for any sequence of numbers $\{c_n\}_{n=1}^{\infty}$, there exist two universal series with respect to the system Φ, with coefficients $\left\{\alpha_n^{(1)}\right\}_{n=1}^{\infty}$ and $\left\{\alpha_n^{(2)}\right\}_{n=1}^{\infty}$, such that for each natural number $n \geq 1$, the equality

$$c_n = \alpha_n^{(1)} + \alpha_n^{(2)}$$

holds.

This result generalizes Menshov's theorem, which states that every trigonometric series is the sum of two universal trigonometric series.

It is also shown that for any sequence of numbers $\{c_n\}_{n=1}^{\infty}$, there exist two universal series with respect to the system Φ, with coefficients $\left\{\beta_n^{(1)}\right\}_{n=1}^{\infty}$ and $\left\{\beta_n^{(2)}\right\}_{n=1}^{\infty}$, such that for each natural number $n \geq 1$, the equality

$$c_n = \beta_n^{(1)} \cdot \beta_n^{(2)}$$

holds.

THE PROBLEM OF CONVERGENCE OF GENERAL FOURIER SERIES FOR FUNCTIONS OF CLASS $Lip\alpha$ $(0 < \alpha < 1)$

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S. Banach [2] demonstrated that strong differentiability conditions on a function do not necessarily ensure the almost everywhere (a.e.) convergence of its Fourier series concerning arbitrary orthonormal systems. Conversely, it is well established that the Menshov-Rademacher Theorem provides sufficient criterion for the a.e. convergence of an orthonormal series.

The main aim of this paper is to investigate the convergence of general Fourier series of functions from the class $Lip\alpha$, where $0 < \alpha < 1$. We establish specific conditions on the orthonormal system under which the Fourier series of functions in $Lip\alpha$ is convergent. It is shown that these conditions are optimal in a certain sense.

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SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Chair: Grigory Giorgadze
Co-chair: George Akhalaia

SPECIAL NON-INTEGER VERTICES OF THE RELAXATION POLYTOPE OF THE LINEAR ORDERING POLYTOPE AND THEIR CUTTING FACETS

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In this talk, we consider those non-integer vertices of the relaxation polytope of the linear order problem that do not have a sufficient number of integer vertices to define facet. For such non-integer vertices, we find adjacent non-integer vertices whose integer adjacent vertices uniquely determine the facets. In this case, the facet cuts for non-integer adjacent vertices also constitutes the cutting facets for the special non-integer vertices.

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2. Bolotashvili G. Non-Integer Vertex of the Initial Relaxation Polytope of the Linear Ordering Problem and the Corresponding Cut-off Facets, an Example. *Reports of Enlarged Sessions of the Seminar of I. Vekua Institute of Applied Mathematics*, **39** (2025), 13-16.

ON THE DEFORMATION OF COMPLEX STRUCTURES OF VECTOR BUNDLES ON THE RIEMANN SPHERE

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In the presentation, we clarify the relationship between the topological structure of the space of matrix functions with fixed partial indices and the space of deformations of complex structures of vector bundles on the Riemann sphere. We show that both spaces are connected smooth manifolds and demonstrate that the approximation procedure for the factorization of matrix functions with given partial indices can be carried out along these manifolds.

Acknowledgements. The research supported by GNSF as part of grant No.FR-25-9781 project “Wiener-Hopf factorization of matrix functions, analytic and topological methods”.

UNITARITY OF MONODROMY MATRICES AND EQUATION PARAMETERS

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This report examines the unitarity of monodromy matrices and their dependence on equation parameters, illustrated by the Riemann and Lamé equations (see [1]). We derive

explicit calculation formulas and show how the unitarity condition governs the connection between these matrices.

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ON THE COULOMB T MATRIX IN THE WEAK SENSE

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The well-known integral representation of the Coulomb T matrix is analyzed. An exact analytic expression for the Coulomb T matrix of the continuous spectrum is obtained in the momentum space in the region $q \neq k,$ and an explicit analytic expression for this function in a small neighborhood of $q = k$ is also derived. It is shown that in the region of anomalous behavior the Coulomb T matrix exists in the sense of generalized functions and is equal to zero in the weak sense. The scenario described in the paper provides a rigorous justification of general considerations in the literature, according to which the Coulomb T matrix in a small neighborhood of $q = k$ is treated as a distribution. It is shown that this distribution, in the weak sense, yields the regular Coulomb function in an exact analytic form. The results obtained show that the integral formula for the Coulomb T matrix is important from the analytic point of view**,** since it gives an explicit analytic form of the Coulomb function of unclear nature in momentum space**.** The calculations are carried out within the framework of complex analysis using modern methods.

COMBINATORIAL PROPERTIES OF THE PARTIAL INDICES OF A MATRIX FUNCTION

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This report examines the properties of the deformation space of a matrix function with prescribed partial indices. The deformation space is formed by all matrix functions whose partial indices can be derived from those of a given matrix by elementary operations. This space is particularly relevant for the approximate factorization of matrix functions. The resulting multi-indices, obtained through these elementary operations on the partial indices, are described using combinatorial methods.

Acknowledgements. This work was supported by grant N FR-25-9781 from the ShotaRustaveli National Science Foundation.

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THIRD KIND TWO-DIMENSIONAL INTEGRAL EQUATIONS

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Sufficiently wide classes of integral equations of the third kind are considered on the complex plane, in which the integral operator of I. Vekua appears. In some sense complete analysis of homogeneous equation is obtained. Solvability criteria for the non-homogeneous equation are also obtained.

SOLVABLE MODELS FOR INTERACTING PARTICLES WITH SPIN AND FUNDAMENTAL CONSTANTS

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Solvable models of interacting particles with spin are constructed. In connection with the Unruh effect, systems of units based on the fundamental constants of physics are considered.

ON COMPLEX SPACES

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One of the main objects of study of the function theory of several complex variables and complex analytic geometry is considered, namely, a complex space, which is a locally ringed space locally isomorphic to a complex analytic variety equipped with the structure sheaf. Some associated constructions are presented.

Acknowledgements. The research was supported by the ShotaRustaveli National Science Foundation grant number FR 22-354.

QUANTUM SIMULATION OF THE SCHRÖDINGER EQUATION

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In this presentation, we use the Qiskit library to simulate the time evolution of a quantum particle in one dimension. An initial state is evolved under different types of potentials, and the results are benchmarked against classical numerical solutions of the same system. We analyze how the accuracy of the quantum simulation depends on the number of qubits and the discretization of the system. Additionally, we study physical properties such as expectation values, probability distributions, and energy evolution over time to evaluate how well the quantum simulation reproduces the dynamics obtained from classical computational methods.

SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Chair: Tamaz Tadumadze
Co-chair: Tea Shavadze

Dedicated to the Blessed Memory of Professor Roman Koplatadze

SENSITIVITY ANALYSIS OF A CONTROLLED SEIR DIFFERENTIAL MODEL UNDER PERTURBATIONS OF DELAYS, INITIAL AND CONTROL FUNCTIONS

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Using local representation formulas for solving functional-differential equations with constant delays in phase coordinates and controls, a system of differential equations is established for the disease spread model, which is satisfied by the sensitivity coefficients.

FINDING PARETO OPTIMALITY POINTS FOR MARKOWITZ MEAN-VARIANCE PORTFOLIO OPTIMIZATION MODEL

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Portfolio optimization is the process of selecting and combining various assets to achieve the best possible outcome in terms of risk and return. The purpose of investors is to maximize expected return at an acceptable level of risk. We consider Markowitz mean-variance optimization financial portfolio model.[1]. For this model, we provide definitions of two criteria: average portfolio return and portfolio risk for asset random variables. We provide fundamental definitions in investment management: long and short position. We also offer definition of efficient frontier set as a Pareto optimal points [2]. For given input data, numerical results are given. Based on these results, the efficient frontier curve is constructed using a genetic algorithm. The types of utility functions are discussed: risk averse, risk neutral and risk accepting. We offer how to find the optimal portfolio for a given utility function.

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RPPI FORECASTING EXPERIENCE ON THE EXAMPLE OF GEORGIA AND NEW FORESACT ESTIMATES

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As is know that the Residential Property Price Index (RPPI) in Georgia reflects the dynamics of residential property prices. The index covers the newly built and under-construction housing segment of the real estate market in Tbilisi. Property price indices are used by central banks and other policy-making institutions of countries, as well as by analysts, researchers and other users, both within the country and abroad. Therefore, the appropriate accuracy of its forecast is very important.The point is that recently this market has become too (speculatively) attractive in Georgia as well. Which, of course, if it continues for a long time, can lead to a "bubble effect" - a sharp decrease in demand for real estate and, consequently, a fall in prices. Which, in turn, will deal a strong blow to both investors and the financial sector as a whole. Therefore, we continue to work on the problem of forecasting the RPPI index (in terms of quarters). Accordingly, the paper will analyze the dynamics of this index for 2020 - 2025. The accuracy of the forecast estimates obtained for 2025 will be considered and new forecast estimates for the current 2026 will be given. Finally, it should be noted that at each stage of the analysis and forecasting of the dynamics of this index, we used the capabilities of such a well-known computer package as Eviews.

CONSTRUCTING A KNOWLEDGE ASSESSMENT TEST USING MULTICRITERIA DISCRETE OPTIMIZATION

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An approach is proposed for the problem of determining the optimal structure of a test, which is based on models and algorithms of multicriteria discrete optimization. Such an approach is effective when several factors must be taken into account in the construction of the test.

APPLICATION OF ORDINARY DIFFERENTIAL EQUATIONS IN MEDICINE

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In the talk we consider the following questions: application of ordinary differential equations against cancer, the balance between efficacy and harm, Norton-Simon hypothesis, resistant cells and optimal control.

ARTIFICAL SATELLITES AND OPTIMAL CONTROL

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In the talk we considered the following questions: space technologies, Sputnik-1 and the founders of cosmonautics, Tsiolkovsky's equation, and optimal control.

NONLINEAR MODEL OF IMMUNE REGULATION IN RHEUMATOID ARTHRITIS: FROM DOSAGE TO EFFICACY

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Based on the equations proposed in [1], [2], and [3], we developed a new nonlinear mathematical model for the treatment of rheumatoid arthritis, which investigates the functional dynamics of cartilage degradation during the progression of the disease.The treatment process is carried out in a combined manner using two medications. In the model, the drug delivery dynamics are described by a function that determines the concentration of medications in the body over time and serves as the source from which active forms are generated, influencing treatment effectiveness. According to the model, the disease arises when the number of B cells exceeds a threshold level. The therapeutic agents involved in the equations promote the proliferation of target cells and/or reduce B lymphocytes to their threshold level. This is achieved through the functional impact of the drugs on Th and Treg cells, as well as on IL-6. To evaluate the dosage and effectiveness of the therapeutic agents, a Cauchy problem is formulated and solved.

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MODELING GPU CLUSTER DYNAMICS VIA NONLINEAR MACKEY-GLASS-TYPE DELAY DIFFERENTIAL EQUATIONS

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In this paper, we propose a mathematical model for GPU cluster load dynamics based on nonlinear delay differential equations of Mackey–Glass type (see [1]). The model captures how past system states, such as task scheduling and resource allocation, affect current load through delayed feedback. Building on the theoretical results obtained in [2], we establish conditions for the existence and stability of equilibrium states, providing insight into congestion formation and load fluctuations in large-scale GPU clusters. The model is validated using real production data from the Alibaba Lingjun GPU cluster, showing that accounting for delays improves prediction of system load and scheduling delays compared to models without delay. This framework offers both a theoretically grounded and practically useful tool for monitoring, forecasting, and managing GPU workloads efficiently.

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THE GLOBAL REPRESENTATION FORMULAS OF SOLUTIONS FOR THE NONLINEAR CONTROLLED FUNCTIONAL-DIFFERENTIAL EQUATIONS WITH SEVERAL DELAYS IN THE PHASE COORDINATES AND CONTROLS CONSIDERING VARIATION OF THE INITIAL MOMENT AND CONTINIOUS OF THE INITIAL CONDITION

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The global analytic representation formulas of solutions are proved for the nonlinear controlled functional-differential equations with several delays in the phase coordinates and controls. In the formulas, the effects of perturbations of the initial moment, the initial function, the control function, delays parameters contained in the phase coordinates, as well as the effect of the continuous initial condition are revealed. The representation formula of the solution is used when investigating optimization problems, finding an approximate solution of the perturbed functional-differential equation and carrying out a sensitivity analysis of mathematical models.

SECTION OF PARTIAL DIFFERENTIAL EQUATIONS AND MECHANICS OF CONTINUA

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili, George Jaiani
Co-chair: Zurab Kiguradze, Natalia Chinchaladze

Dedicated to the 110th Birthday of Professor Andro Bitsadze

INVESTIGATION OF HIGH-ORDER APPROXIMATIONS OF THREE- PHASE-LAG MODEL OF THERMOELASTIC SOLIDS

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In the present paper, dynamic nonclassical three-dimensional models of thermoelastic solids that depend on three phase-lag parameters are studied. The considered models are various high-order approximations of the generalisation of the classical model of thermoelastic solids, where the Fourier law of heat conduction is replaced by the three-phase-lag heat conduction model proposed by S.K. Roy Choudhuri [1], in which one phase-lag corresponds to the temperature gradient, the second one corresponds to the heat flux, and the third one corresponds to the thermal displacement gradient introduced by A.E. Green and P.M. Naghdi in [2]. The models studied in this paper are generalisations of the Roy Choudhuri model [1] with three phase-lags, where the orders of approximations for the heat flux, the temperature gradient and the thermal displacement gradient are 2, 1 and 1, respectively, which in turn is a modification and generalisation of the Chandrasekharaiah-Tzou model [3] with two phase-lag of heat flux and temperature gradient. Variational formulations of the general three-dimensional initial-boundary problems with mixed boundary conditions in corresponding spaces of vector-valued distributions with respect to the time variable with values in Sobolev spaces are obtained, and the existence and uniqueness of solutions, energy equalities, and continuous dependence of the solutions on the given data are proved.

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A BOUNDARY VALUE PROBLEM OF THE COUPLED THEORY OF PLANE ELASTICITY FOR THE CIRCULAR RING WITH DOUBLE-POROSITY

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In the present talk the linear model of elastic double-porosity materials in which the coupled phenomenon of the concepts of Darcy's law and the volume fractions is considered [1]. For the plane deformation the corresponding system of differential equations is written in a complex form and its general solution is presented with the use of three analytic functions of a complex variable and three solutions of the Helmholtz equation [2]. A boundary value problem has been solved analytically for the circular ring using the obtained solutions.

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SOME BOUNDARY VALUE PROBLEMS OF THE THEORY OF CONSOLIDATION WITH DOUBLE POROSITY

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In this paper, the boundary value problems of statics of consolidation theory with double porosity for a half-space are considered. On the boundary of the half-space, the limiting values of normal component of the displacement vector, the tangent components of the stress vector, and the normal derivatives of the fluid pressure in the primary and secondary pores, or the limiting values of the normal component of the stress vector, the tangent components of the displacement vector, and the fluid pressure in the primary and secondary pores are given. Using general formulas for representing the solution of a system of differential equations of consolidation theory and the Fourier transform, the solutions of the boundary value problems are constructed in an explicit form in quadratures.

ON ONE SYSTEM OF NONLINEAR TWO-DIMENSIONAL PARTIAL DIFFERENTIAL EQUATIONS

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Many processes are described by systems of nonlinear partial differential equations that contain equations of different orders and are strongly interconnected. This circumstance necessitates the use of appropriate research methods for each specific system, since the general theory is still incompletely developed even in the linear case. One such two-dimensional system arises in the process of vein formation in plant leaves [1]. Naturally, a parabolic regularization of this model arises, along with the study of the corresponding initial-boundary value problems. This system has been discussed in many works (see, for example, [2-4] and the literature cited therein). The construction, investigation, and numerical implementation of approximation algorithms are essential.

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ON A GENERAL PAPKOVICH–NEUBER-TYPE REPRESENTATION FOR THERMOELASTIC BODIES WITH VOIDS INCORPORATING MICROTEMPERATURE EFFECTS

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This report considers the linear static Cowin-Nunziato model [1] for thermoelastic bodies with voids, accounting for the microtemperature effect [2,3]. The corresponding system of equations is expressed in invariant form, and general Papkovich-Neuber-type representations are derived. The general representation for the solution to the thermal balance system (including temperature change and the microtemperature vector) consists of one arbitrary harmonic function, a general solution to the scalar Helmholtz equation, and a solenoidal solution to the vector Helmholtz equation. Based on these, the general representation for the thermoelasticity system - which involves the displacement vector, volume fraction change, and temperature change - is given by one arbitrary harmonic vector function and a solution to the scalar

Helmholtz equation. Each function in these representations depends on three variables; through their appropriate selection, eight independent boundary conditions can be satisfied.

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ON THE SOLVABILITY OF THE CHARACTERISTIC PROBLEM FOR ONE HIGH-ORDER NONLINEAR EQUATION OF COMPOSITE TYPE

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The characteristic problem in the conical domain for a high-order nonlinear equation of composite type is considered. The problem is equivalently reduced to a nonlinear functional equation on a certain subspace of the Sobolev space. Under certain conditions imposed on the nonlinear term, theorems on the existence, uniqueness, and nonexistence of a solution to this problem are proved.

A NUMERICAL METHOD FOR SOLVING THE TWO-DIMENSIONAL KIRCHHOFF-TYPE EQUATION

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The paper considers a two-dimensional nonlinear Kirchhoff-type integro-differential equation with Dirichlet-type boundary conditions. An algorithm for constructing an approximate solution using the Chipot [1] approach is presented. A numerical example is provided to test the effectiveness of the proposed method.

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ON THE SOLVABILITY OF A MIXED PROBLEM FOR ONE CLASS OF SECOND-ORDER NON-STRICTLY HYPERBOLIC SYSTEMS

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A mixed problem in a half-band is considered for one class of second-order non-strictly hyperbolic systems. In the case of normal hyperbolicity of the system, the existence and uniqueness of a solution to the mixed problem are proven, and in the case of violation of normal hyperbolicity, an example of a hyperbolic system is given for which the posed mixed problem is not correct, and for this system it is shown what kind of change should be made to the boundary conditions in order for the problem to become correct.

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THE NONSTATIONARY FLOW OF INCOMPRESSIBLE ELECTRICALLY CONDUCTING FLUID IN BETWEEN TWO PLANAR PARALLEL PLATES

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In the paper the stationary flow of electro-conductive viscous incompressible liquid between two isothermal walls is studied. The action of external homogeneous magnetic field on heat exchange during constant drop of pressure and constant loss of liquid is shown. The essential relationship between, the value of external magnetic field and electric- conductivity of the walls of a flat pipe is revealed.

The physical characteristics of fluid flow are found.

IVESTIGATION OF STRESS-STRAIN STATE OF AN INFINITE ELASTIC PLANE CONTAINING A SYMMETRIC CIRCULAR MOON-SHAPED HOLE

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The stress-strain state of an infinite elastic plane containing a symmetric circular moon-shaped hole (area bounded by the arcs of two mutually intersecting circles) is studied. For this purpose, analytical (exact) solutions of two-dimensional static boundary value problems of elasticity are constructed in bipolar coordinates for homogeneous isotropic bodies bounded by the coordinate lines of the bipolar coordinate system. In particular, boundary value problems on the stretching of an infinite elastic plane with a hole in the shape of a symmetric circular moon are

posed and solved. At the same time, longitudinal, transverse or all-round uniform stretching caused by external forces applied at infinity is considered. The posed problems are solved in terms of stresses. The main formulas are expressed via the stress function in bipolar coordinates. The problems are solved analytically. The analytical solutions are presented in the form of Fourier integrals. Numerical results of some boundary value problems on the tension of an infinite elastic plane attenuated by a symmetric circular moon-shaped hole are obtained and the corresponding graphs are presented.

SECTION OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Chairs: Elizbar Nadaraya, Omar Purtukhia
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ON A NOTION OF A SUBGAUSSIAN RANDOM SEQUENCE

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The concept of sub-Gaussian random sequences [1] is discussed, along with its relationship to the concept of sub-Gaussian random variables [2] introduced by the renowned French mathematician J.P. Kahane.

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CONSTRAINED BAYESIAN METHOD FOR TESTING EQUI-CORRELATION COEFFICIENT OF A STANDARD SYMETRIC MULTIVARIATE NORMAL DISTRIBUTION

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The problem of testing equi-correlation coefficient of a standard symmetric multivariate normal distribution is considered. Constrained Bayesian and classical Bayes methods, using the maximum likelihood estimation and Stein's approach, are examined. For the investigation of the obtained theoretical results and choosing the best among them, different practical examples are computed using computer codes developed on MATLAB. Simulation results showed that CBM using Stein's approach gives opportunities to make decisions with higher reliability for testing hypotheses concerning equi-correlation coefficient than Bayes method. Besides, the

conditional Bayesian method with Stein's approach when using the distribution laws of linear combinations of chi-square random variables gives better results compared to the use of integrated distribution laws as in the number of used observations, providing necessary reliabilities, as well in the convenience of the practical use. Chi-square random variables are obtained by the summation of the squares of the vectors resulted as the Helmert's orthogonal transformation of uncorrelated multivariate normal vectors. Recommendation for the use of the developed methods at solving practical problems are given.

MINIMUM ENTROPY MARTINGALE MEASURE IN A TWO-DIMENSIONAL FINANCIAL SCHEME

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On the probability space $(\Omega, \mathcal{F}, P)$ we consider financial market with two risky assets, where evolution of these risky assets price is described by the trinomial stochastic sequence

$$S = (S_n^{(1)}, S_n^{(2)})$$

$$S_n^{(1)} = S_{n-1}^{(1)}\left(1 + \rho_n^{(1)}\right), \quad S_n^{(2)} = S_{n-1}^{(2)}\left(1 + \rho_n^{(2)}\right),$$

where $\rho_n^{(1)}$ and $\rho_n^{(2)}$ are the sequences of independent identically distributed random variables that take three possible values with known probabilities. For this scheme, the martingale probability measure Q for sequence S is constructed and this measure minimizes the relative entropy $E_P\left(\frac{dQ}{dP} ln \frac{dQ}{dP}\right)$.

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A METHOD OF PRICING FOR SPREAD OPTIONS

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This paper intoroduces a closed-form formula for pricing spread option. We determine the notion of implied volatility for spread option and present the methods for its computation [1]. Then we equalize the spread option price to a basket option price and look for a stock with stochastic volatility to mimic the behavour of the basket option. This allows us to estimate the

stochastic volatility parameter using the Black Scholes option pricing formula. The resulting parameter value is interpreted as the implied stochastic volatility of the spread option.

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ON THE STRONG LAW OF LARGE NUMBERS FOR A SEQUENCE WITH CHAIN DEPENDENCE

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On the probability space (Ω, F, P), a two-component stationary (in the narrow sense) sequence $\{\xi_i, Y_i\}_{i\geq 1}$ is considered. The control sequence $\{\xi_l\}_{l\geq 1}$ $(\xi_i : \Omega \to \{s_1, s_2, \mathrm{K}, s_r\})$ is a finite, homogeneous regular Markov chain. Sufficient conditions are obtained for a sequence with chain dependence $\{Y_i\}_{i\geq 1}$ to satisfy the strong law of large numbers.

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ON THE CONVERGENCE OF EXPONENTIAL MARTINGALES

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The necessary conditions for the mean square convergence of exponential martingales are established in terms of predictable characteristics of basic martingales.The results are used to prove the closedness of the set of martingale measures.

ON TESTING HYPOTHESIS OF EQUALITY TWO POISSON REGRESSION FUNCTIONS

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The limiting distribution of the integral square deviation between two nonparametric estimatorsof Poisson regression functions is established. Based on this result, the test for the hypothesistesting of equality of two Poisson regression functions is constructed. The question of consistency is studied for the constructed test.

INTEGRAL REPRESENTATION OF STOCHASTICALLY NON-SMOOTH FUNCTIONAL OF THE RUNNING MAXIMUM OF BROWNIAN MOTION

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Let's consider the functional

$$F = I_{\{B_T \le b, M_T > a\}},\ a < b.$$

It is known that the running maximum of Brownian motion $M_t = \overset{max}{_{0\le s\le t}} B_s$ is stochastically smooth. In particular, $D_u M_t = I_{\{u\le\tau_t\}}$, where τ_t is the moment of reaching the maximum $\tau_t := arg\overset{max}{_{0\le s\le t}} B_s$.

However, since the event indicator I_A is stochastically smooth if and only if the probability of this event $P(A)$ is zero or one ([1]), the functional F is stochastically non-smooth and its stochastic integral representation cannot be obtained using the well-known Clark-Ocone formula. Furthermore, it turns out that its conditional mathematical expectation (with respect to the natural filtration of Brownian motion) is not stochastically smooth either.

Theorem. The following representation holds:

$$E[I_{\{B_T\le b, M_T>a\}}|\Im_t^B] = \Phi(\tfrac{b-B_t}{\sqrt{T-t}}) - I_{\{M_t\le a\}}[2\Phi(\tfrac{a-B_t}{\sqrt{T-t}}) - 1]. \quad (1)$$

Accordingly, we cannot use the Glonti-Purtukhia representation ([2]) either, because the second term of representation (1) contains an indicator as a co-product.

Theorem. The following stochastic integral representation holds true

$$F = \Phi(\tfrac{b}{\sqrt{T-t}}) - 2\Phi(\tfrac{a}{\sqrt{T-t}}) + 1 + \int_0^T \tfrac{1}{\sqrt{T-t}}[2I_{\{M_t\le a\}}\varphi(\tfrac{a-B_t}{\sqrt{T-t}}) - \varphi(\tfrac{b-B_t}{\sqrt{T-t}})]dB_t.$$

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THE M/G/1 QUEUEING SYSTEM WITH AN UNRELIABLE SERVERA NEW SOLUTION

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This paper investigates an M/G/1-type queueing system with an unreliable server. The server is subject to failures occurring with a constant intensity, while the repair times are modeled as a random variable with a general distribution. A semi-Markov representation of the system is developed using the supplementary variables technique.The analysis is carried out employing a purely probabilistic approach, without recourse to partial differential equations (specifically, Kolmogorov's forward equations). The stationary behavior of the system is examined using operational methods, including Laplace transforms and probability generating functions.The derived results enable the determination of key performance measures of the system, such as the queue length, waiting time, and sojourn time. The proposed methodology is applicable to a broad class of semi-Markov queueing systems.It is demonstrated that the suggested approach simplifies the analysis of semi-Markov queueing models and can be effectively utilized in various applications within reliability theory and service system modeling.

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ON GAUSSIAN STATISTICAL STRUCTURES

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Necessary and sufficient conditions for the existence of consistent estimators of unknown parameters and consistent criterion for hypothesis testing in multidimensional linear Gaussian systems are established.

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SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS

Chairs: Teimuraz Davitashvili, Jemal Rogava, Tamaz Vashakmadze
Co-chair: Archil Papukashvili

COMPUTER MODELING OF THE PROCESS OF INTERACTION AMONGGEORGIAN, LAZ, MINGRELIAN, AND SVAN POPULATIONS

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Previously, we presented a scenario for the transformation of the Proto-Kartvelian population into Georgian, Laz, Mingrelian, and Svan populations. The mathematical and computer models for the second and third periods have been discussed in [1–3]. The descriptive mathematical model for the fourth period is formalized as a four-dimensional nonlinear dynamical system, where each unknown function represents the population dynamics of one of the groups (Georgians, Laz, Mingrelians, Svans). First integrals have been found for this nonlinear dynamical system. These first integrals allow the four-dimensional system to be reduced to a two-dimensional system. The Poincaré–Bendixson theorem is applied to study the two-dimensional dynamical system. Several theorems are proved concerning changes in the sign of the divergence of the vector field in a domain located in the first quadrant of the phase plane. These results confirm the existence of closed integral trajectories, indicating that none of the groups undergoes complete assimilation and that all four linguistic groups achieve stable coexistence within the same geographical space [4].

The thesis presents computer modeling of the fourth period in the case of variable coefficients, where the Colchian population has already split into Laz and Mingrelian populations, and the interaction among the four populations takes place. Computational simulations have yielded interesting results that effectively corroborate the theoretical conclusions.

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STUDY OF THE TRANSPORT PATHWAYS OF AEOLIAN DUST AEROSOLS OVER THE CAUCASUS (GEORGIA) USING AN ENSEMBLE OF MODERN NUMERICAL MODELS

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Fine dust aerosol particles are hazardous to human health and are one of the main source of modern climate variability [1]. Indeed. the role of Aeolian Desert Dust (ADD) in the Earth's climate system is strengthened by its active participation in all scales of physical, chemical, and biogeochemical processes [2]. Since dust aerosol is one of the main pollutants in Georgia, it is important to further study the frequency and routs of ADD incursion into Georgia from neighboring deserts. The main purpose of this work is to study the frequency and routes of transfer of ADD clouds to the territory of the Caucasus (Georgia) from the surrounding deserts in the conditions of complex orography of the Caucasus and against the background of regional climate changes using modern scientific methods. To achieve this objective, numerical modeling using satellite and in situ data is used. Namely, Weather Research and Forecasting WRF-chem v.4.5.1 in combination with the GOCART dust module, Copernicus Atmosphere Monitoring Service (CAMS) and Hybrid Single-Particle Lagrangian Integrated Trajectory (HYSPLIT) models are used to study the ADD transport from deserts to the territory of Georgia. Some results of the numerical study are presented and analyzed in this paper.

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REVIEW OF THE LINK BETWEEN THE SPACE WEATHER AND THE EARTH'S ATMOSPHERIC WINDS

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The features of space weather and the role of solar activity in its formation are analyzed. Various scenarios of the impact of space weather on the processes taking place in the Earth's atmosphere are discussed. The solar wind, which is carried by a strong magnetic field, interacts with the geomagnetic field and causes its disturbance, exchanging energy in large-scale processes, which, in turn, are transformed into small-scale high-frequency electromagnetic wave structures under the influence of displacement currents generated by instabilities and transfer energy to low latitudes of the atmosphere and cause changes in atmospheric parameters. The solar magnetic wind also has a thermal effect on the Earth's atmosphere, which causes changes in pressure, temperature and affects local weather. The paper analyzes a mathematical model describing the above processes. Experimental data on solar activity and the characteristic field of the geomagnetic field obtained at the Dusheti Observatory have been studied using various machine learning methods, and a strong connection between them has been identified.

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THE INFLUENCE OF VARIOUS METEOROLOGICAL PARAMETERS ON MBLA THROUGH ITS NUMERICAL MODEL

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Our group has developed a two-dimensional (X–Z vertical plane) model of the mesoscale boundary layer of the atmosphere (**MBLA**), which qualitatively describes the object of study satisfactorily [1-5]. In this paper, we will consider the influence of such important parameters on atmospheric processes as atmospheric stratification, ambient wind, relative humidity, and others using this model. This research is very relevant for local weather forecasting, studying the characteristics of humid fields, and implementing artificial influence on some atmospheric phenomena. The results of numerical calculations physically describe

quite adequately the influence of the above-mentioned parameters on the processes occurring in the **MBLA**.

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ON A METHOD FOR SOLVING THE GENERALIZED KIRCHHOFF EQUATION DESCRIBING THE STATIC STATE OF A BEAM

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The paper considers a boundary value problem for a Kirchhoff-type integro-differential equation that describes the static state of a beam. A method for finding an approximate solution to the problem is proposed. A test case is solved to assess the effectiveness of the proposed method.

ON THE STOKES AXI-SYMMETRIC FLOW IN THE INFINITE AREAS

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In the paper the Stokes incompressible steady axisymmetric flow over the convex body is studied in the infinite area. The corresponding system of linear Stokes equations (STS) with the continuity equation is considered. STS is studied for the certain pressure and with the suitable boundary conditions. The exact solutions are obtained. The profile ofvorticity is constructed by means of MAPLE.

ON THE APPROXIMATE SOLUTION OF THE J. BALL'S BEAM EQUATIONIN THE CASE OF EXTERNAL DAMPING

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The presented article is a direct continuation of the articles [1]-[6] that considered an initial-boundary value problem for the J. Ball integro-differential equation, which describes the dynamic state of a beam. In the articles [1]-[2] the algorithm has been approved by tests. In the article [3]-[6] and this paper we present the approximate solution of one practical problem, particularly, the results of numerical computations of the initial-boundary value problem for an iron beam. In the present paper we discuss the case when an external impact acts on a beam. The results of numerical calculations qualitatively satisfactorily describe the process under consideration.

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A COMBINED NUMERICAL APPROACH FOR SOLVING A NONLINEAR DYNAMIC TIMOSHENKO BEAM SYSTEM

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In this talk, we consider a nonlinear, dynamic, spatially one-dimensional Timoshenko beam coupled system. For its numerical treatment, a symmetric three-layer semi-discrete time-stepping scheme is developed. The proposed scheme yields second-order linear ordinary differential equations for each unknown at every time level, enabling parallel computation of the solution.

The convergence of the scheme is analyzed, and second-order accuracy in time is established. For spatial discretization, a Legendre–Galerkin spectral approximation is employed, in which differences of Legendre polynomials are used as basis functions. Within this combined framework, the original problem is reduced to subsystems of linear equations whose coefficient matrices are sparse, symmetric, and positive definite. Moreover, these subsystems can be efficiently decoupled.

The convergence of the Galerkin method is also investigated. Several numerical experiments are conducted on carefully selected test problems, and the results are consistent with the developed theoretical findings. This talk is based on arXiv:2602.02068 (see 1).

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APPROXIMATE SOLUTION OF SCREENED LAPLACE EQUATION WITH GREEN-LAGRANGE COEFFICIENT METHOD UNDER MIXED BOUNDARY CONDITIONS

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We generalize the apparatus developed in [1, Chapter III] for constructing approximate solutions to one-dimensional problems to two-dimensional screened Poisson equations with mixed boundary conditions in a rectangle, using Green's function representation of the solution and the two-dimensional Lagrange interpolation formula. With such an approach, it is possible to pre-calculate the tensor coefficients that can be reused. Then, we determine the solution to the differential problem by a system of linear algebraic equations approximating the boundary value problem, the corresponding matrix of which is simplyassembled by tensor coefficients. If the coefficients of the initial differential equation are polynomials and we use high-order

tensor coefficients, then the error obtained by the above algorithm has machine accuracy. The numerical results obtained on the test examples are presented in a tabular format.

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ON THE CONSTRUCTION AND FOUNDATION OF REFINED THEORIES AND HIERARCHICAL MODELS FOR THIN-WALLED STRUCTURES

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The report is devoted to the issues of mathematical foundation and construction of refined theories and hierarchical models for thin-walled structures in the linear case (with essential notes for the nonlinear case). Taking into account the fundamental work of I.Vekua [1], we develop a certain approach using orthogonal polynomials in [2], constructing hierarchical models for anisotropic inhomogeneous, but not only elastic, for example, magneto-elastic, piezo-elastic and poro-elastic bodies. The essential emphasis of the research is shifted to the study of the well-known problem of satisfying boundary conditions when determining generalized stress vectors on the surfaces of elastic plates and shells. In the nonlinear case, where the bending and compression-expansion processes are not decoupled, we present the exact structure of the system of differential equations, which is constructed without *ad hoc* assumptions. The models constructed in this way represent a two-dimensional (with respect to the thickness variable) correctly posed boundary value problem, the solution of which is obtained by the recursive (factorization) method - by solving simple 3x3 differential problems. For all models, the boundary conditions on the faces are satisfied. The above creates a basis not only for the study of thin-walled structures, but also for the study of Truesdell-Ciarlet models. For the sake of completeness, we note that the methodology presented for thin-walled structures directly transfers to obtaining an approximate solution of the boundary value problems corresponding to the three-dimensional theory of elasticity, when the thickness is finite or semi-infinite in one direction.

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