XXXIX International Enlarged Sessions of the Seminar of Ilia Vekua Institute of Applied Mathematics of Ivane Javakhisvili Tbilisi State University

Book of Abstracts

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The present book of abstracts contains abstracts of talks given at XXXIX Enlarged Sessions (April 23-25, 2025) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

ABSTRACTS OF OPENING TALKS

NONLINEAR DIRICHLET PROBLEM OF NON-LOCAL BRANCHING PROCESSES

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We present a method of solving a nonlinear Dirichlet problem with discontinuous boundary data and we give a probabilistic representation of the solution using the non-local branching process associated with the nonlinear term of the operator. Instead of the pointwise convergence of the solution to the given boundary data we use the controlled convergence which allows to have discontinuities at the boundary.

The talk is based on a joint work with Oana Lupascu-Stamate and Alexandra Teodor from Bucharest, in the frame of a project RESCI-ECO of the L'Agence Universitaire de la Francophonie (AUF).

ROGAVA'S THEOREM AND CONVERGENCE RATE OF THE LIE-TROTTER-KATO PRODUCT FORMULÆ

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Since 1875 it is known due to Sophus Lie that for any pair of (noncommutative) finite square matrices A and B as generators one has the norm estimate O(1/n) for convergence rate of the *exponential product formula*. In 1959 H. Trotter proved this formula on a Banach space for unbounded generators A and B of the strongly continuous semigroups in the *strong operator* topology. In 1978 T. Kato extended this result in the *strong operator* topology to the *non-exponential* product formula.

The breakthrough result in this direction was presented in the *Rogava theorem* (1993), saying that on a separable Hilbert space the exponential Trotter product formula may converge in the *operator-norm* topology with a rate of the order $O(\ln(n)/n^{1/2})$. This discovery initiated a number of papers addressed to study conditions on generators A and B with aiming to *optimise* the rate of convergence in Rogava's theorem.

Motivated by this discovery the *optimal rate* of convergence O(1/n) in the operator-norm topology under conditions of the Rogava theorem was proved only in 2001 (the ITTZ-theorem) for both the Trotter and the Trotter-Kato product formula. Under new *fractional* conditions on generators A and B the *optimal rate* of the Trotter-Kato product formula convergence in the operator-norm topology on a Hilbert space was established in the Ichinose-Neidhardt-Zagrebnov (INZ)-theorem (2004). For these and for some other recent results about the Lie-Trotter-Kato product formula on Hilbert and Banach spaces we refer to the ZNI-book 2024.

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ABSTRACTS OF SECTIONAL TALKS

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

FREE TEMPORAL GÖDEL ALGEBRA GENERATED BY CONSTANT ELEMENTS

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Modalized Heyting calculus *mHC*, introduced by Leo Esakia in [1], is the augmentation of the intuitionistic logic *Int* by a modal operator □. The temporal Heyting calculus *tHC* is defined based on *mHC* with additional axioms for the "adjoint" modality ◊, namely: t1) $p \rightarrow \Box\Diamond p$, t2) $\Diamond\Box p \rightarrow p$t3) $\Diamond(p \vee q) \rightarrow (\Diamond p \vee \Diamond q)$, t4) $\Diamond\bot \rightarrow \bot$ and an additional rule: $p \rightarrow q \,/\, \Diamond p \rightarrow \Diamond q$.

Algebraic models of *mHC* are *fHA*-algebras (frontal Heyting algebras). In [2] Castiglioni, Sagastume and San Martin have extended Heyting duality to the category **fHA**.

The variety of temporal Heyting algebras **tHA**, which represent algebraic models of temporal Heyting calculus *tHC* has been investigated with the following results [3]:

- Developing a theory of temporal Heyting algebras.
- Generalization of Heyting duality to the category **tHA**.
- Characterization of subdirectly irreducible and simple *tHA*-algebras.

We investigate the variety **tGA** of Temporal Gödel algebras, which are *tHA*-algebras with pre-linearity condition: $(x \rightarrow y) \vee (y \rightarrow x) = 1$. We describe 0-generated free algebras in the variety of **tGA.**

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SEVERAL NOTES ON MAZURKIEWICZ TYPE SETS WITH RESPECT TO THE CIRCLES

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In 1914, Mazurkiewicz proved the existence of a paradoxical point set in the Euclidean plane R^2, which intersects each straight line in this plane at exactly two points [1].

This theorem initiated deep researches where the existence and measurability properties of different versions of Mazurkiewicz type sets are studied. One version of Mazurkiewicz type sets is a Mazurkiewicz type set with respect to various families of circles in the plane.

In particular, a set $B \subset R^2$ is called a Mazurkiewicz type set with respect to the family of all circles in R^2 if $card(B \cap c) = n$ for each circle c lying in R^2.

Different geometric (topological) properties and the question of measurability of Mazurkiewicz type sets with respect to families of circles were studied in [2-5].

In the talk, some theorems regarding the existence and measurability of Mazurkiewicz type sets with respect to families of circles are presented.

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SOME PROPERTIES OF THE INVARIANT MEASURES IN THE INFINITE-DIMENSIONAL LINEAR POLISH SPACES

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It is well known, that in an infinite-dimensional linear Polish space X there exists non-zero σ-finite measure, which is invariant with respect to some dense subspace of X. (see. [1], [2]).

In the presented talk we consider some properties of the above-mentioned invariant measures.

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ON SOME VERSIONS OF ISOSCELES POLYHEDRA

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It is known that there are different notions of isoscelespolyhedra in combinatorial and discrete geometry (see, e.g., [1], [2]). In this presentation, we will consider some versions of isosceles polyhedra and present several problems that are closely related to combinatorial properties of convex isosceles polyhedra in the three-dimensional Euclidean space. In particular, certain connections of such polyhedral with the classical Euler formula will be discussed.

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NOTES ON CONJUNCTIVE REDUCIBILITY

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Let $A, B \subseteq \omega$, where ω denotes the set of natural numbers. A set A is conjunctively reducible (or c-reducible) to a set B (in symbols: $A \leq_c B$) (see [1]), if there exists a computable function f such that for all $x \in \omega$, $x \in A \Leftrightarrow D_{f(x)} \subseteq B$, where D_u is the finite set with a canonical index u. A set A is c-complete, if A is computably enumerable (c.e.) and $(\forall B)(B \text{ c.e.} \Rightarrow B \leq_c A)$.

Our notations and terminology are standard and can be found in [2] and [3]. In this talk we will present the following results:

Theorem 1. A c.e. set A is c-complete if and only if there exists a c.e. subset B of A and a strong array $\{D_{f(i)}\}_{i \in \omega}$ of disjoint nonempty sets such that

(i) $(\forall i)(D_{f(i)} - B \neq \emptyset)$;

(ii) $A = B \cup (\cup_{i \in I} D_{f(i)})$, where $I = \{i: D_{f(i)} \cap W_i \neq \emptyset\}$.

Theorem 2. There exists a c.e. set which is simultaneously Q-complete and bd-complete but not c-complete.

Acknowledgments. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number: N FR-22-16379. Algorithmic reducibilities and algebraic structures].

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INFINITE ASIMMETRIC TREES AND SOM OF THEIR APPLICATIONS

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The paper discusses the existence and number of asymmetric trees with any infinite cardinality (a tree is called asymmetric if it has only a trivial automorphism). Various examples

of constructions of asymmetric trees are provided [1], [2]. L. Babai's construction is examined in detail. It is proven that there exists a maximum number of pairwise non-isomorphic asymmetric trees with the cardinality of infinity that have vertices of only two prescribed degrees. The effective application of asymmetric trees in solving Ulam's problem and König's [3] problem is demonstrated, as well as their role in formulating equivalent sentences of the continuum and generalized continuum hypotheses.

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ON SOME APPLICATIONS OF KUNEN'S THEOREM

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Kenneth Kunen, in the research work (cf. [1]), has proved the well-known theorem, from which it follows that, there exists a set $X \subset R$ such that

$$card(X) < c \text{ და } \lambda^*(X) > 0. \quad \text{(cf. [2])}$$

By this result, we have proved that there exists an almost R-non-invariant set on R, whose measurable hull is not almost R-invariant (see [3]).

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THE PRINCIPLE OF TRANSFINITE INDUCTION AND SOME PARADOXICAL POINT SETS

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The existence of paradoxical point sets of various types, by using the principle of transfinite induction, is discussed (see [1], [2], [3], [4]). The main attention is concentrated on the complexity of corresponding transfinite constructions, which substantially depends on a form of AC used in the argument. Geometric and topological properties of such sets, the question of their measurability, and profound interrelations between them are considered. Besides, some examples of applications of paradoxical (pathological) point sets in mathematical analysis are presented.

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SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz (Austria)
Co-chair: Jemal Antidze, Besik Dundua, Mikheil Rukhaia

ABOUT CHALLENGES FOR INDUCTIVE LOGIC PROGRAMMING

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Inductive Logic Programming (ILP) is a research domain that lies at the intersection of Machine Learning and logic-based knowledge representation. ILP utilized logic programming as a unified representation language for examples, background knowledge, and learning hypotheses.

Over time, it has become a powerful tool for multi-relational learning and data mining from complex, structured data. ILP has also investigated various connections with statistical learning and probabilistic methods, greatly broadening its research scope.

Multiple papers presented at ILP conferences cover a wide range of topics, including theories, algorithms, representations, and languages, as well as ILP systems and applications.

They explore various aspects of learning in logic, relational learning, relational data mining, statistical relational learning, multi-relational data mining, relational reinforcement learning, graph mining, and connections to other learning paradigms, among others.

In this talk, we summarize various topics covered and the discussions held for Inductive Logic Programming throughout the years. Our focus is on the future research for ILP, including the challenges for Inductive Logic Programming and how these challenges can be solved through probabilistic methods and settings.

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INFERENCE IN UNRANKED PROBABILISTIC LOGIC

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Probabilistic logics combine logic's expressive power with the ability to reason with uncertainty. Several probabilistic logic languages have been proposed in the past, each with its own features.

Among them is the probabilistic constraint logic programming language, which combines constraint logic programming with probabilistic reasoning.

In this talk, we introduce the semantics of this language extended with unranked primitives. We present a resolution-based inference algorithm with the specified constraints and extend the examples from [1,2] to our new settings.

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ABOUT AI AND UNRANKED PROBABILISTIC LOGIC APPLICATIONS

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Recently, artificial intelligence (AI) has become the preoccupation of the entire world at all levels and ages. Of course, this matter has been exploited commercially to achieve financial gains by large companies. Many applications have been introduced that give you the impression that they can do anything and everything in the blink of an eye, just by pressing a button, and artificial intelligence (AI) has become like Aladdin's magic lamp.

The question is whether artificial intelligence (AI) is a blessing or a curse in our lives.

We asked artificial intelligence (AI) opinions about possible applications of unranked probabilistic logic. In this talk, we will explore its answers and discuss their feasibility.

HIGHER-ORDER PATTERN UNIFICATION MODULO SIMILARITY RELATIONS

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The combination of higher-order theories and fuzzy logic can be useful in decision-making tasks that involve reasoning across abstract functions and predicates, where exact matches are often rare or unnecessary. In this talk, we develop the integration of fuzzy similarity relations in higher-order pattern unification and study the termination, soundness, and completeness properties of the derived algorithm.

TRANSFORMING URBAN ENVIRONMENTS: AI SOLUTIONS FOR CITY PLANNING AND MANAGEMENT

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This research explores the integration of Artificial Intelligence (AI) into urban planning processes, with a special focus on developing smart city strategies for Tbilisi, Georgia. Recognizing the transformative potential of AI, machine learning, and data science, the study aims to establish a novel and practical framework to support strategic urban development, infrastructure planning, air pollution forecasting, and policy decision-making.

The interdisciplinary approach brings together experts from computer science, environmental science, urban planning, and business analytics. Using real-world datasets—including geospatial, demographic, environmental, and open data sources—the study applies advanced machine learning (ML) models to address key urban challenges. These include the optimal placement of city infrastructure, traffic flow optimization, and air pollution forecasting. Key contributions include an improved understanding of AI's role in infrastructure planning and management, as well as the development of a data-driven framework for smart urban decision-making, with a pilot application focused on the Tbilisi environment. By empowering policymakers with actionable insights, the research contributes to Georgia's long-term social, economic, and environmental development through the adoption of smart city practices.

Acknowledgements. This work is supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [FR-24-9597, "Smart City Planning and Management Using Artificial Intelligence"].

HOW ARTIFICIAL INTELLIGENCE HAS CHANGED THE JOB MARKET

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Artificial Intelligence (AI) has significantly transformed the job market by automating tasks, increasing efficiency, and changing work requirements. AI-powered technologies have replaced repetitive and manual jobs in industries such as production, customer service, and data entry, leading to job displacement. However, AI has also created new opportunities in fields like machine learning, cybersecurity, and AI ethics, requiring a job with advanced digital skills. The job market now requires adaptability, continuous learning, and collaboration between

humans and AI. While AI is growing productivity and innovation, it also raises concerns about job security and the need for upskilling to stay relevant in the evolving employment landscape.

Acknowledgements. This research work was supported by Georgian National University SEU.

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ONLINE IMPLEMENTATION OF A GEORGIAN↔ ENGLISH GRAMMAR DICTIONARY

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Most natural language processing (NLP) systems are implemented using supervised learning methods [1], [2], [3]. Since these systems require a large amount of annotated data, they are used for languages for which large text corpora have already been created. The small number of Georgian annotated corpora available today is not sufficient to implement NLP systems.

The paper considers the development of a Georgian-English grammatical online dictionary. A compilation system which relies on the modern Georgian and corresponding English grammatical dictionaries as the core of the base vocabulary was used for that. The system will respond to any word form provided by the user with the full paradigm of the corresponding lemma.

Acknowledgements. Supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG).

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VERIFICATION OF CYBER-PHYSICAL SYSTEMS IN MAUDE

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Cyber-Physical Systems, shortly CPS, are networks of controllers that interact or control physical environments. Some examples of CPS are cars, aircrafts, railway systems, smart traffic systems, and the like. It is very important, that CPS behaves correctly and securely, and only formal verification of the components can provide the guarantees. Lingua Franca (LF) is a coordination framework for designing and verification of CPS components.

Maude is a rewriting logic based programming language with strong formal verification capabilities. In this talk we will speak about programming LF models in Maude and about their formal verification.

Acknowledgements. This talk is part of the reasearch conducted under the NATO SPS project «Symbolic Rewriting Methods for Safety and Security of Critical Cyber-Physical Systems» (G6133) and I would like to thank all participants of the project.

THE ALGORITHMIC PROCESS TO ESTABLISH THE VALIDITY OF THE FORMULAE WITH SOME PROPERTY OF THE τ-LOGIC

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An algorithmic process to establish the validity of the formulae with some property of the τ-logic is constructed. It is directed to the automated theorem proving.

Some basic notes: The propositional meta-formula associated to the given formula, propositional tautology form, tautology form are introduced.

SECTION OF ALGEBRA, GEOMETRY AND NUMBER THEORY

Chairs: Mikhail Amaghlobeli, George Khimshiashvili, Teimuraz Vepkhvadze, Malkhaz Bakuradze
Co-chair: Ketevan Shavgulide

THE STRUCTURE OF THE FREE TWO-GENERATED R-GROUP OF NILPOTENCY CLASS 2, WHERE $R = Q[t]$

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Let R be an arbitrary associative ring with unity. Using this ring, we define a new category of R-groups in three different ways. To do this, we enrich the group language $L_{gr} = \{\bullet, {}^{-1}, e\}$ as follows: $L_{gr}^{R} = L_{gr} \cup \{f_\alpha(g) | \alpha \in R\}$, where $f_\alpha(g)$ is a unary operation denoted by $f_\alpha(g) = g^\alpha \ \forall g \in G$. A set G is called an R-group in the sense of Lyndon [1] if the operations $\bullet$, ${}^{-1}$, e, $f_\alpha(g)$ are defined on it and the following axioms hold:

1) group axioms;
2) $g^{\alpha+\beta} = g^\alpha g^\beta$, $g^{\alpha\beta} = (g^\alpha)^\beta$;
3) $(h^{-1} g h)^\alpha = h^{-1} g^\alpha h$.

Unfortunately, Lyndon's exponential R-groups are not always R-modules, which complicates the application of this concept in the case of non-free groups.

In the work [2], A.G. Myasnikov and V.N. Remeslennikov introduced a new category M_R (MR-groups) by adding a new series of axioms to Lyndon's axioms, namely the following series of quasi-identities:

(MR-axiom}) $\forall g, h \in G [g, h] = e \to (gh)^\alpha = g^\alpha h^\alpha, \alpha \in R,$ where $[g, h] = g^{-1} h^{-1} g h$.

Clearly, all R-modules over the ring R satisfy the (MR) axiom.

For nilpotent groups and binomial rings, P. Hall introduced in [3] a category of R-groups, which differs from the category M_R.

This talk is devoted to the study of nilpotent exponential R-groups (MR-groups) of nilpotency class 2 for the ring of polynomials in t with rational coefficients $R = Q[t]$. We give a description of free two-generated R-groups of nilpotency class 2 and compare it with Hall nilpotent R-groups of class 2. Furthermore, we show that there exist two-generated non-finitely presented R-groups [4] and study algorithmic problems for such groups.

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ON THE SPACES OF SPHERICAL POLYNOMIALS AND GENERALIZED THETA-SERIES FOR QUADRATIC FORMS OF SIX VARIABLES

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F. Gooding [1] constructed the spaces of spherical polynomials with respect to some positive definite binary quadratic forms and calculated the dimensions of the spaces of corresponding generalized theta-series. In [2], the upper bounds of the dimension of the space T(ν,Q) for some quadratic form of r variables are obtained. In this paper, some positive diagonal and non-diagonal quadratic forms of six variables are considered; the spaces of spherical polynomials with respect to these forms are constructed and corresponding generalized theta series are considered.

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THE MAIN DIFFERENCES BETWEEN CHORDAL AND RADIAL SECTIONS OF GML BODIES

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This report presents the fundamental differences between the traces (flat geometric shapes) on the radial section of GML bodies - chordal and radial cuts of these bodies. In the previous works [1-2], 3 types of chordal (VV, VS and SS) and 2 types of radial (OV and OS, names invented by Johan Gielis) cutting were analyzed in detail. Now we compare the geometric shapes of these e traces and notice that the Möbius phenomenon always takes place

in the case of radial cutting. A separate work by J. Geilis and mine was devoted to the number of different options for cutting the GML - bodies [3].

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TENSOR STRUCTURES OF TANGENTIAL FIBRATIONS OF T(Vn)

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Consider a tangent bundle $T\big(T(Vn)\big)$ with the local coordinates (x^i, y^i). Where x^i are the coordinates of the basis Vn, and y^i are those of the layer $T_z, z \in T(Vn)$, local coordinates of the point of the space $T(Vn)$ are transformed as follows:

$$\bar{x}^i = \bar{x}^i(x^k), \bar{y}^i = x^i_k y^k.$$

The following theorems are proved:

Theorem 1. If a GL(n,R)-vector field $\xi^i(x)$ and a GL(n,R)-covector field $\eta_j(x)$ are given in the basis Vn of a linearly connected space T(Vn) with Γ^i_k, then in the tangent fibration of T(Vn) there are two one-parameter cones of tensor structures, between which there are no complex structures.

Theorem 2. If a GL(n,R)-vector field $\xi^i(x, y)$ and a GL(n,R)-covector field $\eta_j(x, y)$ are given in the tangent fibration of T(Vn) with Γ^i_k, then in the tangent fibration of T(Vn) there are two one-parameter cones of tensor structures, between which there are no complex structures.

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REPRESENTING POSITIVE INTEGERS BY BINARY FORMS WITH EVEN DISCRIMINANT

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In [1], we worked out a full solution of defining singular series for arbitrary binary quadratic forms and proved that half of the sum of a generalized singular series that corresponds to a positive binary form is equal to the average number of representations of a positve integer by the genus containing this form. Morever, convinient formulas are obtained for calculating of values of this sum. It gives us the opportunity to characterize positive integers which can be represented by a positive binary form. In this paper we introduce this method in the case of binary forms with even discriminant belonging to multi-class genera.

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SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
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ON A GENERALIZATION OF BOUNDED VARIATION

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The generalized $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ of bounded variation classes [1] are studied. Relationship between $B\Lambda\left(p(n)\uparrow p,\varphi\right)$ classes for different φ functions are examined. The embedding $H^{\omega}\subset B\Lambda\left(p(n)\uparrow\infty,\varphi\right)$ criterion is established [2, 3] in the space of continues functions for ω modulus of continuity.

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CONJUGATE FUNCTIONS AND THE MODULUS OF SMOOTHNESS OF FRACTIONAL ORDER

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Moduli of smoothness play a basic role in approximation theory, Fourier analysis and their applications. For a given function f, they essentially measure the structure or smoothness of the functionvia the difference of some order.

In the present talk we study the behavior of the smoothness of fractional order of the conjugate functions of many variables in the space L if the global smoothness of the original functions are known. The direct estimates are obtained and exactness of the seestimates are established by proper examples.

ON THE CONVERGENCE AND SUMMABILITY OF GENERAL FOURIER SERIES

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The sufficient conditions are obtained for the almost everywhere convergence, summability by the Cesaro method and unconditional convergence of general Fourier series. They are equivalent to the conditions in the well-known theorems of Menshov-Rademacher, Menshov and Orlich.

ON THE ABSOLUTE CONVERGENCE OF FOURIER SERIES

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Let M be the class of all continuous 2π-periodic functions f for which there exists a partitioning of the segment $[0,2\pi]$ by the points $0 = x_1(f) < \cdots < x_{m+1}(f) = 2\pi$ such that f is convex or concave on each segment $[x_k(f), x_{x+1}(f)], k = 1, \dots m$.

We have obtained the following result:

Theorem 1. Let $f \in M$ and for any adjacent intervals $(x_k(f), x_{k+1}(f))$ and $(x_{k+1}(f), x_{k+2}(f))$, let the function f be convex on one of them and concave on the other, then the convergence of the series $\sum_{n=1}^{\infty} \omega\left(\frac{1}{n}; f\right)\frac{1}{n}$ is the necessary and sufficient condition for the Fourier series of the function f to converge absolutely.

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ON THE REARRANGED DOUBLE BLOCK-ORTHONORMAL SYSTEMS

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Let $\{M_k\}$ and $\{N_k\}$ be increasing sequences of natural numbers and $\Delta_{p,q} = (M_p, M_{p+1}] \times (N_q, N_{q+1}]$, $(p,q \geq 1)$. Let $\{\varphi_{mn}\}$ be a system of functions from $L^2((0,1)^2)$. The system $\{\varphi_{mn}\}$ will be called a $\Delta_{p,q}$-orthonormal system if $\|\varphi_{mn}\|_2 = 1$, $m = 1,2,..., n = 1,2,...$ and $(\varphi_{ij}, \varphi_{kl}) = 0$, for $(i,j),(k,l) \in \Delta_{p,q}$, $(i,j) \neq (k,l)$, $(p,q \geq 1)$.

In [1] Statements connected with the $(C,1,1)$ almost everywhere summability of series

$$\sum_{m,n=1}^{\infty} a_{mn}\varphi_{mn}(x,y) \tag{1}$$

with respect to $\Delta_{p,q}$-orthonormal system $\{\varphi_{mn}\}$ is given. In particular it is stated that if the condition $\sum_{p,q=1}^{\infty} \frac{1}{(\log\log M_p)^2 (\log\log N_q)^2} < \infty$ is fulfilled, then for every $\Delta_{p,q}$ ONS $\{\varphi_{mn}\}$ the condition

$$\sum_{m,n=1}^{\infty} a_{mn}^2 (\log\log(m+3))^2 (\log\log(n+3))^2 < \infty \tag{2}$$

guarantees the summability by the $(C,1,1)$ method a. e. on $(0,1)^2$ of the series (1);

If however $\sum_{p,q=1}^{\infty} \frac{1}{(\log\log M_p)^2 (\log\log N_q)^2} = \infty$, then there exists the numbers a_{mn} and $\Delta_{p,q}$-ONS $\{\varphi_{mn}\}$ such that the condition (2) is fulfilled though the arithmetic means $\sigma_{mn}^{11}(x,y)$ of series (1) diverge everywhere on $(0,1)^2$.

Now we established that if $\overline{\lim_{p\to\infty}}(M_{p+1} - M_p) = \infty$ and $\overline{\lim_{q\to\infty}}(N_{q+1} - N_q) = \infty$, then for the arbitrary $\Delta_{p,q}$-orthonormal system $\{\varphi_{mn}\}$ there exists a rearrangement of functions of the system $\{\varphi_{mn}\}$ such that the condition (2) guarantees the summability by the $(C,1,1)$ method a. e. on $(0,1)^2$ of the series (1) with respect to the rearranged system $\{\varphi_{\mu(m)\nu(n)}\}$; And if $\overline{\lim_{p\to\infty}}(M_{p+1} - M_p) < \infty$ or $\overline{\lim_{q\to\infty}}(N_{q+1} - N_q) < \infty$, then there exists $\Delta_{p,q}$-orthonormal system $\{\psi_{mn}\}$ such that for every rearrangement of functions of the system $\{\psi_{mn}\}$ the $(\log\log(m+3))^2 (\log\log(n+3))^2$ is not the Weyl multiplier for the summability by the $(C,1,1)$ method a. e of corresponding series (1).

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ON THE EXISTENCE AND APPROXIMATING PROPERTIES OF UNIVERSAL REPRESENTING FUNCTIONS FOR CERTAIN CLASSES OF FUNCTIONS

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In [1], the notions of a set of uniqueness and a universal representing function for a class *A* of real-valued functions defined over (0,1) are introduced.

We say that a set $E \subset (0,1)$ is a set of uniqueness for the class A if, for any two functions $f(x)$ and $g(x)$ of A, whenever $f(x) = g(x)$ for each $x \in E$, it follows that $f(x) = g(x)$ for each $x \in (0,1)$. Also, we say that a function $F(x)$ is a universal representing function for the class A if, for each function $f(x) \in A$, there exists a set of uniqueness E for A such that $F(x) = f(x)$ for each $x \in E$.

In the present talk, the following classes of functions defined over (0,1) are considered: the class of all continuous functions, denoted by $C(0,1)$, the class of all functions of the first Baire class, denoted by $B_1(0,1)$, the class of all summable functions, denoted by $L_1(0,1)$, and the class of all Lebesgue measurable and almost everywhere finite functions, denoted by $L_0(0,1)$.

The question of the existence of a universal representing function for each of the above-mentioned classes is considered.

Namely, in [1], the existence of a universal representing function $F(x)$ for the class $C(0,1)$ is announced.

In the talk, a theorem is proved which implies the nonexistence of a universal representing function for each of the following classes: $B_1(0,1)$, $L_1(0,1)$, and $L_0(0,1)$.

In addition, it is shown that there exists a universal representing function $F(x)$ for the class $C(0,1)$ such that each function $f(x)$ of the class $B_1(0,1)$ can be approximated by using $F(x)$ at each point of (0,1), each function $f(x)$ of the class $L_1(0,1)$ can be approximated by using $F(x)$ at each Lebesgue point of $f(x)$, and each function $f(x)$ of the class $L_0(0,1)$ can be approximated by using $F(x)$ at almost every point of (0,1).

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THE PROBLEM OF CONVERGENCE OF GENERAL FOURIER SERIES FOR FUNCTIONS OF CLASS $Lip\alpha\ (0<\alpha<1)$

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S. Banach [1] demonstrated that strong differentiability conditions on a function do not necessarily ensure the almost everywhere (a.e.) convergence of its Fourier series with respect to arbitrary orthonormal systems (ONS). Conversely, it is well established that the Menshov-Rademacher Theorem provides a sufficient criterion for the a.e. convergence of an orthonormal series.

The main aim of this paper is to investigate the convergence of the general Fourier series $Lip\alpha$, when $0<\alpha<1$. In particular, we found special conditions for functions of the orthonormal system, for which the Fourier series of functions of class $Lip\alpha$ is convergence. It is established that the resulting conditions are the best possible in a certain sense

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SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Chair: Grigory Giorgadze
Co-chair: George Akhalaia

ON APPROXIMATION OF MATRIX FUNCTIONS AND DEFORMATION OF COMPLEX STRUCTURES OF VECTOR BUNDLES

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In the report, we consider the topological structure of the space of matrix functions with fixed partial indices and the space of deformations of complex structures of vector bundles on the Riemann sphere. We show that both spaces are connected smooth manifolds and demonstrate that the approximation procedure for the factorization of matrix functions with given partial indices can be carried out along these manifolds. The main result is given in [1].

Acknowledgements. This work was supported by grants N FR 22-354 from the Shota Rustaveli National Science Foundation.

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REGULAR COULOMB FUNCTIONS

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The two-particle regular Coulomb functions of the continuous spectrum are described. It is rigorously shown that they exist in the sense of the generalized functions, and in the weak representation belong to the set of the Hilbert space functions $\left(L_2\right)$. The above functions satisfy the two-particle homogeneous integral equations of the perturbation theory.

Due to their simple analytic structures and correct properties, the functions under consideration is expected to be very useful for studying the standard singular integral equations of the scattering theory of several charged particles.

The section of the complex analysis and its application.

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CONNECTION BETWEEN HYPERGEOMETRICAL AND HEUN'S GENERAL DIFFERENTIAL EQUATIONS

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The Heun equation is a Fuchsian differential equation with four singular points. Linearly independent solutions of this equation define a mapping from the upper half-plane to curvilinear quadrilaterals. In this work, we establish a connection between our previous results on quadrilaterals and the hypergeometric differential equation, which leads to the derivation of a special case of the Heun equation. The main result is given in [1].

Acknowledgements. This work was supported by grants N FR 22-354 from the Shota Rustaveli National Science Foundation.

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ON ONE GENETIC INVARIANT OF ELLIPTIC SYSTEMS IN THE PLANE

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Determining those properties of an object that are invariant under one or another type of its transformation is undoubtedly interesting from the point of view of studying this object itself. In this work we study the classical and generalized entire analytic functions of a complex variable. The "natural state" of an "alive" classical entire analytic function is to be unbounded, otherwise it is "dead" being constant. A generalized entire analytic function is characterized by somewhat similar properties. It is quite natural to study whether the transformed "alive" entire analytic function (by restricting it to a particular area of the complex plane) inherits the (as noted, one of the most important) property of its "ancestor" - unboundedness.

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FRACTAL CALCULUS AND INTEGRABLE SYSTEMS

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Integrable motion equations of the hydrodynamics and their complex extensions; fractal integro-differential, discrete and continuous [1] calculus are considered.

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HIRZEBRUCH-RIEMANN-ROCH THEOREM

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For a holomorphic vector bundle $p : E \to X$ on a compact complex manifold X the Hirzebruch-Riemann-Roch theorem relates the Euler-Poincaré characteristic of the bundle to its Chern classes of it and the Todd classes of the holomorpic tangent bundle of the manifold, it states:

$$\chi(X, E) = \int \mathrm{ch}(E)\mathrm{td}(X).$$

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SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Chairs: Roman Koplatadze, Tamaz Tadumadze
Co-chair: Tea Shavadze

OPTIMIZATION OF THE DISBALANCE FOR ONE DIFFERENTIAL MODEL OF MARKET RELATIONS WITH A NON-LINEAR FUNCTIONAL AND DELAY IN CONTROL

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It is known that, market relations are characterized by the functions of demand and supply. Typically, the price of a product changes over time and it is obvious that the supply and demand for that product changes. Achieving equality between supply and demand is problematic for many reasons. We call the difference between supply and demand functions the disbalance index. If the disbalance index is zero, we call it equilibrium. In the present work, an optimal control problem for the market relations model with the constant delay in control is considered. In the model it is assumed that demand at any point in time will be satisfied trough pre ordering. For considered optimal control problem the existence theorem of an optimal control and the necessary optimality condition of control is formulated. For illustration a linear problem is considered. We note that for a practical situation supply and demand can be described in the form of linear functions by using linear regression.

EXPERIENCE IN FORECASTING RESIDENTIAL REAL ESTATE PRICE INDEX IN GEORGIA AND FORECAST ESTIMATES

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As is known, the Residential Real Estate Price Index (RPPI) in Georgia reflects the dynamics of residential real estate prices. The index covers the segment of newly built and under-construction housing in the real estate market in Tbilisi. Property price indices are used by central banks of countries and other policy-making institutions, as well as by analysts, researchers and other users, both within the country and abroad. As is known, the real estate market is not risk-free. To demonstrate this, it is enough to recall the great financial and economic crisis that was triggered by the US real estate market. In particular, in 2007-2009, US GDP decreased by 4.3%, real estate prices decreased by 30%, and the S&P 500 index of the

largest US stock companies decreased by 57%. It is noticeable that this market is becoming excessively (speculatively) attractive in Georgia as well. In particular, its growth rate is much higher than the growth rate of the entire economy. In addition, real estate prices are increasing at a very high rate. If this process continues for a long time, it will have a "bubble effect", leading to a sharp decrease in demand for real estate and, consequently, a fall in prices, which, in turn, will deal a strong blow to both investors and the entire financial sector and may end in a deep economic crisis. Therefore, we first of all considered the problem of forecasting the RPPI index (in terms of quarters) for 2024, based on data for 2020 - 2023, which turned out to be quite accurate, already for the first quarter of 2024. Further, our forecast estimates of this indicator for the last three quarters of 2024 turned out to be quite accurate. Finally, we conducted an analysis of the dynamics of this index for 2020 - 2024 and, based on the mathematical models we built, found its forecast estimates for 2025.

REPRESENTATION FORMULAS OF SOLUTION FOR CONTROLLED FUNCTIONAL-DIFFERENTIAL EQUATIONS WITH SEVERAL CONSTANT DELAYS TAKING INTO ACCOUNT VARIATION OF THE INITIAL MOMENT AND CONTINUITY OF THE INITIAL CONDITION

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The global representation formulas of solution are obtained for the nonlinear perturbed controlled functional-differential equation with several constant delays in phase coordinates and controls. In formulas the effects of the continuous initial condition and variation of the initial moment and delays containing in phase coordinates are revealed. The representation formula of solution plays an important role in the investigation of optimization problems, allows one to get an approximate solution of the perturbed equation and to carry out a sensitivity analysis of mathematical models.

NONLINEAR MATHEMATICAL MODELS OF IMMUNOPATHOGENESIS AND COMBINATION TREATMENT OF RHEUMATOID ARTHRITIS CONSIDERING IL6

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Based on the equations described in [1, 2], we created a new mathematical model of the immunopathogenesis of rheumatoid arthritis, which takes into account the role of the important protein IL6. This model is the main prerequisite for developing a treatment model for rheumatoid arthritis. Based on the immunopathogenesis model, we have developed a new nonlinear mathematical model for the treatment of rheumatoid arthritis, which studies the functional dynamics of cartilage destruction during disease progression. The Cauchy problem is posed and the exact solution of the system of equations is found.

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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF THE HIGHER ORDER ORDINARY LINEAR DIFFERENTIAL EQUATION

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A linrear n-th order ordinary differential equation is considered. Sufficient conditions of the existence of vanishing,unbounded and oscillatory solutions are established which are a generalization of previously well-known results.

THE LOCAL REPRESENTATION FORMULAS OF SOLUTION FOR CONTROLLED FUNCTIONAL-DIFFERENTIAL EQUATIONS WITH SEVERAL DELAYS TAKING INTO ACCOUNT VARIATION OF THE INITIAL MOMENT AND DISCONTINUITY OF THE INITIAL CONDITION

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The local analytic representation formulas of solution are established for the nonlinear perturbed controlled functional-differential equation with several constant delays in the phase coordinates and controls. In formulas the effects of the discontinuous initial condition and variation of the initial moment and, also variation of delays containing in phase coordinates are revealed. The representation formula of solution plays an important role in the investigation of optimal control problems, allows one to get an approximate solution of the perturbed equation and to carry out a sensitivity analysis of mathematical models. Finally, for the sensitivity coefficient of the controlled Hatchinson model with several delays the form of the corresponding functional-differential equation is established.

SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili
Co-chair: Zurab Kiguradze

THE DIRICHLET CHARACTERISTIC PROBLEM FOR ONE CLASS OF HIGH-ORDER NONLINEAR HYPERBOLIC SYSTEMS

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For one class of fourth-order nonlinear hyperbolic systems with an iterated wave operator in the principal part, the Dirichlet problem in the characteristic rectangle is considered. Depending on the structure and behavior in the neighborhood of infinity of the nonlinear terms included in the system, theorems of existence, uniqueness and absence of solutions to this problem are proved.

ON THE FOURTH-ORDER NONLINEAR PARABOLIC INTEGRO-DIFFERENTIAL EQUATION

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The initial-boundary value problem for the fourth-order nonlinear parabolic integro-differential equation is considered. The discussed equation is based on the integro-differential models proposed in [1] and studied in many works (see, for example, [2-5] and references therein). Extensive references to the mentioned models are given in the following monographs [4,5]. In our work, a constructed discrete analog is studied. The stability and uniqueness of the solution for this problem are investigated in [6].

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APPROXIMATE SOLUTION OF ONE SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS USING MACHINE LEARNING

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Initial-boundary value problems for one type of Maxwell's system of nonlinear partial differential equations [1] are considered. Numerical experiments have been conducted using decomposition algorithms [2, 3] and machine learning methods [4]. The obtained results are analyzed, and computer experiments are compared with theoretical conclusions.

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SOME BOUNDARY VALUE PROBLEMS OF THE THEORY OF CONSOLIDATION WITH DOUBLE POROSITY

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In this paper we consider boundary value problems of statics of the theory of consolidation with double porosity for a half-space. On the boundary of the half space there are given limiting values of the normal component of displacement vector, the tangent components of the stress vector, and the normal derivates of fluid pressure within primary and secondary pores or limiting values of the normal component of stress vector, the tangent components of displacement vector, and the fluid pressure within primary and secondary pores. Using general representation formulas of solution of the system of differential equations of consolidation theory and the Fourier transform, solutions of the boundary value problems are constructed in an explicit form in quadratures, in the form similar to Poisson's formula.

ITERATION METHOD FOR THE NEUMANN PROBLEM IN THE CASE OF THE HELMHOLTZ EQUETION

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Based on the potential method, we construct a convergent recurrence scheme for the solution of the three-dimensional Neumann boundary value problem in the case of the Helmholtz equation. By the single layer potential, the Neumann problem is reduced to the uniquely solvable second kind Fredholm integral equation on the boundary. The integral equation is reduced to the equivalent first kind integral equation with symmetric compact positive operator. First, we construct a sequence of successive approximations which converges to the solution of the boundary integral equation in the appropriate Bessel potential spaces of functions defined on the boundary. Afterwards, using these approximations, we construct another iteration which converges to the solution of the Neumann boundary value problem in the appropriate Sobolev spaces of functions defined in the three-dimensional region under consideration. We assume that the boundary surface belongs to the Lipschitz class.

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ON ONE SYSTEM OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Nonlinear systems of partial differential equations describing various processes are the subject of investigation for many scientists. The main features of many such models are expressed in the fact that they contain equations of different kinds, which are strongly connected to each other. The mentioned fact for each concrete system determines the usage of respective methods of research, because general theory is incompletely developed for such systems even in the linear case. One of such two-dimensional systems arises in the process of vein formation of young leaves [1]. Naturally arises the questions of investigation of these problems and its multidimensional analogous. The mentioned system has been discussed in many works (see, for example, [2-6] and references therein). The essential difficulties arise in the processes of constructing, investigating and realizing the numerical algorithms for the considered model. It is very important to reduce this problem to a set of one-dimensional ones. Our aim is to continue the study of this system in these directions.

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PROBABILISTIC METHODS IN SOLVING OF ANALYTICAL PROBLEMS

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It is generally acknowledged that semi-Markov models, where the supplementary variables technique is used, result in a non-classical boundary value problem of mathematical physics with non-local boundary conditions. These models always contain partial differential equation(s) (Kolmogorov forward or backward equation(s)), an integro-differential equation or a system of such equations (finite or infinite). Besides, both an ordinary differential equation or a system of such equations (finite or infinite). Traditionally, the solution to this problem is obtained using the Laplace transforms, which often involves a highly complex process. However, deep probabilistic reasoning about the considered systems resolves this issue. We can derive a solution to this problem by bypassing Kolmogorov equations and using a purely probabilistic method. This new probabilistic approach was initially introduced by Kakubava in the context of queuing theory. Furthermore, this approach can be applied to both semi-Markovian and non-semi-Markovian models where the supplementary variables technique is used.

ITERATION METHOD FOR THE DIRICHLET PROBLEM OF THE ELASTICITY THEORY OF ANISOTROPIC BODIES

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We construct a convergent recurrence scheme for a solution of the three-dimensional Dirichlet boundary value problem of the elasticity theory of anisotropic bodies. By the potential method, the Dirichlet problem is reduced to the uniquely solvable Fredholm integral equation of the first kind with a weakly singular boundary integral operator generated by the single layer potential. We assume that the elastic body under consideration has a Lipschitz boundary.

First, we construct a sequence of successive approximations which converges to the solution of the boundary integral equation in appropriate Bessel-potential spaces of functions defined on the boundary. Afterwards, using these approximations as densities of the single layer potential, we formulate another iteration which converges to the solution of the Dirichlet boundary value problem in the appropriate Sobolev-Slobodetskii spaces of functions defined in the region occupied by the elastic body.

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SOME GENERALIZATIONS AND APPLICATIONS OF DUHAMEL PRINCIPLE

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Some generalizations of Duhamel principle for certain classes of partial differential equations, namely for high-order hyperbolic equations with constant coefficients, are given in the paper. Based on the solution of the Cauchy problem effectively obtained earlier for the corresponding homogeneous equation, the solution of the Cauchy problem for the nonhomogeneous equation is constructed in quadratures.

SECTION OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Chairs: Elizbar Nadaraya, Omar Purtukhia
Co-chair: Petre Babilua

ON ONE NONPARAMETRIC ESTIMATION OF THE POISSON REGRESSION FUNCTION

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The limiting distribution of the integral square deviation of a nonparametric kernel-type estimator for the Poisson regression function is established. A criterion for testing the hypothesis on the Poisson regression function is constructed. The power asymptotic of the constructed criterion is studied for certain types of close alternatives.

SOLVING LINEAR SECOND ORDER BACKWARD STOCHASTIC DIFFERENTIAL EQAUATIONS (2BSDE) IN THE MARKOWIAN CASE

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We will study second order backward stochastic differential equations (2BSDE) in the Markovian case. At first we will define 2BSDE and its solution. Then we will solve it using the solution of an ordinary stochastic differential equation: We will insert that solution in a special function and obtain the solution of 2BSDE as a triple of stochastic processes.

ON THE EMPIRICAL PROCESS OF REGRESSION

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In regression problems, an approach based on appropriate empirical processes is as important as in classical hypothesis testing problems. Empirical regression processes can be constructed in various ways. Their properties may turn out to depend not on the regression model but on the distribution of errors, which should have a secondary effect.

We show how significant this relationship is and how the power of statistical tests based on different processes can differ in distinguishing between seemingly identical regression models.

CONSTRAINED BAYESIAN METHOD FOR TESTING COMPOSITE HYPOTHESES CONCERNING NORMAL DISTRIBUTION WITH EQUAL PARAMETERS

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The problem of testing composite hypotheses with respect to the equal parameters of a normal distribution using the constrained Bayesian method is discussed. Hypotheses are tested using maximum likelihood and Stein's methods. The optimality of our decision rule is shown by the criteria: the mixed directional false discovery rate; the false discovery rate; the Type I and Type II errors, under the conditions of providing a desired level of constraint. The algorithms for implementing the proposed methods and the computational tools for their application are included. Simulation results show validity of the theoretical results along with their superiority over classical Bayesian method.

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THE LAW OF LARGE NUMBERS FOR SOME CLASSES OF DEPENDENT RANDOM VARIABLES

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On the probability space (Ω, F, P), a stationary, in the narrow sense, two-component sequence $\{\xi_i, Y_i\}_{i\geq 1}$ is considered. The control sequence $\{\xi_?\}_{i\geq 1}$ $(\xi_k : \Omega \to \Xi)$ satisfies the condition: for each function f $(f : \Xi \to R)$, for which $Ef(\xi_1) < \infty$ at $n \to \infty$ with probability 1 the convergence

$$\frac{1}{n}\sum_{k=1}^{n} f(\xi_k) \to Ef(\xi_1) \text{ a.e.}$$

holds.

$\{Y_k\}_{k\geq 1}$ $(Y_k : \Omega \to R)$ is a conditionally independent sequence or a sequence with chain dependence (when $\{\xi_?\}_{i\geq 1}$ is a Markov chain). $\{Y_k\}_{k\geq 1}$ $(Y_k : \Omega \to R)$ is a conditionally independent sequence or a chain-dependent sequence (when $\{\xi_?\}_{i\geq 1}$ is a Markov chain). Sufficient conditions are established under which these sequences satisfy the law of large numbers. An inequality is obtained which is used to establish the accuracy of the Rosenblatt-Parzen type kernel density estimator.

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CONSTRUCTIVE MARTINGALE REPRESENTATION OF A ONE NON-SMOOTH PATH-DEPENDENT BROWNIAN FUNCTIONAL

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As is known, the class of functionals to which the Clark-Ocone formula ([1]) is applicable is limited by the condition of their differentiability in the Malliavin sense. Later, it turned out that the requirement for the smoothness of a functional can be weakened. It is known that if a random variable is stochastically differentiable in the sense of Malliavin, then its conditional mathematical expectation is also differentiable (Proposition 1.2.8 [2]). In particular,

if $F \in D_{1,2}$, then $E(F \mid \Im_s^B) \in D_{1,2}$ and $D_t[E(F \mid \Im_s^B)] = E(D_t F \mid \Im_s^B) I_{[0,s]}(t)$. On the other hand, the conditional mathematical expectation may be smooth even if the random variable is not stochastically smooth. For example, it is known that for any real constant c: $I_{\{B_T<c\}} \notin D_{1,2}$ (the event indicator I_A is Malliavin differentiable if and only if the probability $P(A)$ equals zero or one (Proposition 1.2.6 [2])), but for all $t \in [0,T)$: $E[I_{\{B_T \le c\}} \mid \Im_t^B] = \Phi[(c - B_t)/\sqrt{T-t}] \in D_{1,2}$, where Φ is the standard normal distribution function. Glonti and Purtukhia ([3]) generalized the Clark-Ocone formula to the case when the functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable, and proposed a method for finding the integrand. Here we consider a non-smooth path-dependent Brownian functional and propose a different method for obtaining a constructive stochastic integral representation.

For any $0 \le a < b \le T$ and a fixed constant c, consider the Brownian functional $F(a,b) = \exp\{\int_a^b I_{\{B_u \le c\}} du\}$.

Theorem. The function $V(t,x) := E[F(t,T) \mid B_t = x] = E[\exp(\int_t^T I_{\{N(0,u-t) \le c-x\}} du)]$ satisfies the requirement of Ito's formula ($V(\cdot,\cdot) \in C^{1,2}$) and the following stochastic integral representation is valid:

$$F(0,T) = E[F(0,T)] + \int_0^T F(0,t) V_x'(t, B_t) dB_t \qquad (P\text{-a.s.}).$$

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ABOUT STATISTICAL STRUCTURES

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We define consistent estimators of parameters, consistent criteria for hypothesis testing, a consistent estimators of any parametric function, consistent criterion for hypothesis testing of any parametric function, an unbiassed estimators of any parametric function, an unbiassed criterion for hypothesis testing.

The necessary and sufficient conditions for the existence of such estimators and criteria are given for Statistical Structures.

Theorem 1. A countable statistical structure $\{E, S, \mu_i, i \in I\}$, $cardI = \chi_0$, admits a consistent estimator of parameters if and only if this statistical structure is orthogonal.

Theorem 2. In order for the Borel orthogonal statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c$, to admit a consistent criterion for hypotheses testing in the theory of (ZFC) & (MA) it is necessary and sufficient that this statistical structure be strongly separable.

Theorem 3. In order for the Borel orthogonal statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c$, to admit a consistent estimators of parameter in the theory of $(ZFC)\&(MA)$ it is necessary and sufficient that this statistical structure be strongly separable.

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SECTION OF MECHANICS OF CONTINUA

Chairs: George Jaiani
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INVESTIGATION OF HIGH-ORDER APPROXIMATIONS OF DUAL- PHASE-LAG MODEL OF THERMOELASTIC SOLIDS

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In the present paper various nonclassical dynamic three-dimensional models of thermo-elastic solids, which depend on two phase-lag parameters, are studied. The considered models are various high-order approximations of the generalized model of thermoelastic solids where the classical Fourier law of heat conduction is replaced by the dual-phase-lag heat conduction model proposed by D. Tzou [1], in which one phase-lag corresponds to the temperature gradient and is caused by micro-structural interactions such as phonon scattering or phonon-electron interactions and the second one corresponds to the heat flux and it is interpreted as the relaxation time due to fast-transient effects of thermal inertia. The models studied in this paper are generalizations of the Chandrasekharaiah-Tzou model [2] with two phase-lags, which in turn is a modification and generalization of the Lord-Shulman model [3] with only one phase-lag of heat flux. Variational formulations of the general three-dimensional initial-boundary problems with mixed boundary conditions in corresponding spaces of vector-valued distributions with respect to the time variable with values in Sobolev spaces are obtained, the existence and uniqueness of solutions, energy equalities, and continuous dependence of the solutions on the given data are proved.

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BOUNDARY VALUE PROBLEMS FOR THE CIRCULAR RING WITH TRIPLE VOIDS

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The present talk deals with the basic boundary value problems of the plane theory of elasticity for the circular ring with triple voids [1, 2]. The general solution of the governing system of equations of the plane strain is represented by means of two analytic functions of the complex variable and three solutions of Helmholtz equations. Using the obtained solutions, the problems for the circular ring are solved analytically.

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HIERARCHICAL MODELS FOR THE THERMOELASTIC DEFORMATION OF CHIRAL POROUS PRISMATIC SHELLS

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Applying the I. Vekua's dimension reduction method ([1], [2]), the present paper is devoted to construction of hierarchical models for thermoelastic deformation of chiral porous prismatic shells. Special attention is paid to the case, when the prismatic shell considered as a 3D body occupies a spatial angular domain and to the study of consequent mathematical and physical peculiarities, since by dimension reduction the geometrical 3D singularity will be transferred into singularity of 2D governing partial differential equations and exclusiveness of well-posedeness of BVPs will be needed to be investigated. For field equations we use the strain gradient theory [3].

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ON THE ELASTIC EQUILIBRIUM OF POROUS DISKS ROTATING WITH A CONSTANT ANGULAR VELOCITY

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The paper examines the elastic equilibrium of porous disks rotating with a constant angular velocity in their own plane around their center. Analytical solutions of the corresponding boundary value problems for disks of circular shape and the shape of a concentric circular ring are obtained. Next, an algorithm for constructing approximate solutions to boundary value problems of the type under consideration is formulated, on the basis of which several problems are solved. Two-dimensional working equations of equilibrium for plates with voids are obtained by reducing the three-dimensional Cowin-Nunziato model [1] by the method of successive differentiation developed by I. Vekua [2]. To obtain analytical and approximate solutions of the problems, the method of N. Muskhelishvili [3] and the method of fundamental solutions [4-6] are used, respectively.

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THE PROBLEM OF FINDING AN EQUALLY RIGID CONTOUR FOR A RHOMBUS IN THE PLANE THEORY OF ELASTICITY

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The problem of finding an equally strong contour for a rhombus, on the boundary of which stamps act with normal stresses with a known principal vector, is considered, while the sought contour is free from external loads and the condition of its equally strong implies the constancy of the tangential normal stress on it.

Based on the theory of conformal reflections and analytic functions, the problem is reduced to the Keldysh-Sedov problem, and by effectively solving the latter, the equation of the sought contour is constructed in an analytical form. The shape of the contour is given in the first approximation.

PULSATION FLOW OF INCOMPRESSIBLE ELECTRICALLY CONDUCTING LIQUID WITH HEAT TRANSFER

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In the last years heat phenomena in pipes with action of extermal magnetic field caused special interest. In the paper stationary flow of electro-conductive viscous incompres-sible liquid between two isothermal walls with heat exchange is studied. The action of external homogeneous magnetic field on heat exchange during constant drop of pressure and constant loss of liquid is studied. The essential relationship between the temperature of liquid flow, Nuselt number of heat exchange, the value of external magnetic field and electric- conductivity of the walls of flat pipe is revrated.

The physical characteristics of fluid flow are found.

STUDY OF STRESS-STRAIN STATE OF AN ECCENTRIC CIRCULAR RING

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Analytical (exact) solutions of 2D statics problems of elasticity are constructed in bipolar coordinates for homogeneous isotropic bodies bounded with coordinate lines of a bipolar coordinate system. In particular, some boundary problems for an eccentric circular ring are considered. The system of equilibrium equations and Hooke's Law are written with bipolar coordinates. The work solves the plane problem of the theory of elasticity in terms of stresses: a) basic formulas are given in bipolar coordinates as a function of stress; b) the boundary problems for an eccentric circular semi-ring, when symmetry or antisymmetry conditions are given on the diametrical boundary, are considered. The exact solutions are obtained by the method of separation of variables. The corresponding graphs of numerical results of some boundary value problems for an eccentric circular semi-ring are presented.

SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS

Chairs: Teimuraz Davitashvili, Jemal Rogava, Tamaz Vashakmadze
Co-chair: Archil Papukashvili

NUMERICAL ANALYSIS OF THE DEFORMED STATE OF A STRUCTURE COMPOSED OF LAYERED ROTATIONAL SHELL ELEMENTS WITH DIFFERENT CONFIGURATIONS

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The numerical analysis of the deformation problem of an orthotropic layered structure, which is constructed from spherical and cylindrical shell elements, caused by the influence of a surface force, is considered based on different theories. The considered theories are built on the basis of taking into account the hypothesis of cracks, both in the case of linear and nonlinear deformations.

A particular example of the deformation of a layered structure is given. In this example, the deformation problem of an orthotropic three-layer structure rigidly fixed by its extremities under the influence of a normal surface force is considered.

The numerical implementation of the given example is carried out based on different theories. Comparison of the obtained numerical results provides a basis for comparing the deformation process of a layered structure.

NON-INTEGER VERTEX OF THE INITIAL RELAXATION POLYTOPE OF THE LINEAR ORDERING PROBLEM AND THE CORRESPONDING CUT-OFF FACETS, AN EXAMPLE

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If we reduce the NP-hard linear ordering problem to an integer linear programming problem, then the study of non-integer vertices of the initial relaxation polytope and the corresponding cut-off facets play an important role in solving the problem. In this paper, we consider an example of a non-integer vertex of the initial relaxation polytope and show how we construct the corresponding cut-off facets of the linear ordering polytope.

COMPUTER MODELING OF THE INTERACTIONS BETWEEN THE GEORGIAN, COLCHIAN, AND SVAN POPULATIONS

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Previously, we presented a scenario for the transformation process of the Proto-Kartvelian population into Georgian, Megrelian, Laz, and Svan populations, which considered four periods of this process (from 5000 BC to the present). The results of the mathematical and computer modeling for the first two periods of the transformation were published earlier [1-3].

For the third period (X BC – I BC), the mathematical model is described by a three-dimensional nonlinear dynamical system. Numerical results were obtained through computational experiments, considering demographic, self-limitation, and assimilation coefficients (exponential and trigonometric functions), and were divided into three groups:

1. During this period, the Colchian population grew from 2 to 2.2 million, then decreased to 1.8–1.7 million; the Georgian population increased from 1.5 to 2–2.07 million, reaching parity with the Colchian population (in the III–II BC); the Svan population, due to assimilation, decreased from 0.3 to 0.24 million.

2. The Colchian population declined from 2 to 1.7 million; the Georgian population grew from 1.5 to 2–2.04 million, matching the Colchian population (by the III BC); the Svan population decreased from 0.3 to 0.23 million.

3. The Colchian population increased from 2 to 2.18 million; the Georgian population grew rapidly from 1.5 to 2.4–2.45 million, matching the Colchian population in the 4th–3rd centuries BC; the Svan population decreased from 0.3 to 0.235 million.

Based on historical sources, we suggest that the first group is the most likely, aligning with the mathematical model's analytical results.

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APPLICATION OF NONLINEAR SOIL THERMAL CONDUCTIVITY THEORY TO STUDY PROCESSES OCCURRING DURING DROUGHT

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A well-designed model is considered a powerful tool for studying the problem set in specific areas of knowledge and making correct rational decisions. In order to identify the thermal and advection-dynamic factors responsible for drought in specific regions of Georgia, in this study we use the nonlinear theory of soil thermal conductivity to study the processes occurring during drought in the easternmost and southernmost regions of Georgia. With the help of a mathematical model of drought, which is based on the integration of nonlinear partial differential equations with appropriate initial and boundary conditions, the thermal and advection-dynamic factors responsible for drought are studied using both analytical and numerical integration methods. The model showed that the nonlinear thermal process occurring in the soil causes its structural change, and in order to prevent this, it is necessary to take measures that will reduce the solar radiation load on the soil and the quenching of the greenhouse effect in its active layer.

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ANALYZING GEOMAGNETIC DATA FROM DUSHETI OBSERVATORY USING MACHINE LEARNING MODELS DURING THE INTENSE MAGNETIC STORMS OF 2024

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Geomagnetic storms, intense disturbances in Earth's magnetosphere, pose risks to both technology and human health. In this study we analyzed data from the Dusheti Observatory during the intense geomagnetic storms of 2024, including a storm on May 11. We used techniques such as cross-correlation, wavelet coherence, and detrended fluctuation analysis, and investigated the relationship between the interplanetary magnetic field (IMF), plasma parameters, and the H-component of the geomagnetic field. Our results reveal significant correlations and coherence between the IMF and geomagnetic field, with notable coherence

observed before the storm. Plasma β analysis, incorporating a 12-hour lag, suggests its potential role as a predictor of geomagnetic storms. Additionally, detrended fluctuation analysis highlights regime changes in the Hurst exponent, indicating self-organization prior to storms. These findings emphasize the importance of localized studies for understanding impacts of geomagnetic storms on Georgia.

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THE PERTURBATION ALGORITHM FOR THE REALIZATION OF A HIGH-ORDER FINITE DIFFERENCE APPROXIMATION FOR AN EVOLUTIONARY EQUATION

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The Cauchy problem for the evolution equation is considered. Based on the formulas of high-order finite difference approximation for the derivative, a perturbation algorithm for the given problem has been constructed (for the perturbation algorithm see the works [1]-[4]). For the proposed algorithm the issues of approximation and convergence are studied.

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SOLUTION METHOD FOR AN INITIAL AND TIME DEPENDENT BOUNDARY VALUES PROBLEM FOR A DYNAMIC BEAM KIRCHHOFF TYPE NONLINEAR DIFFERENTIAL EQUATION

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An initial boundary value problem for the Kirchhoff type beam nonlinear integro-differential equation is considered in the case of inhomogeneous boundary conditions. The problem is reduced to a solution of some more complex equation than the original one, but when the homogeneous boundary conditions hold and the type of initial conditions remain the same. To find an approximate solution of the received problem, a numerical method that is a combination of the Galerkin method, Crank-Nicolson difference scheme and Jacobi-Cardano iteration process is used. Outlines how to obtain an approximate solution of the original problem using the solution of the auxiliary problem.

ON THE PERFECT FLUID FLOW IN THE INFINITE RESERVOIR WITH THE POLYGONAL BOTTOM

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We study the steady 2D perfect fluid flow in the infinite reservoir with the polygonal bottom. One vertex of the polygon lies at infinity and other angles are known. The velocity of the flow is given in terms of the complex potential [1, 2]. By means of the conformal mapping method we define the complex potential in the explicit form [3, 4]. The obtained formula shows the dependence of the velocity on the angles of the polygon. Several examples are given and the velocity and streamlines are constructed by means of "MAPLE". The 2D viscous fluid flow was studied by the author in [5, 6].

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ON THE APPROXIMATE SOLUTION OF THE J. BALL'S BEAM EQUATIONIN THE CASE OF PRESSURE DEPENDENCE OF EFFECTIVE VISCOSITY

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The presented article is a direct continuation of the articles [1]-[5] that considered an initial-boundary value problem for the J. Ball integro-differential equation, which describes the dynamic state of a beam. In the articles [1]-[2] the algorithm has been approved by tests. In the article [3]-[5] and this paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam. In the presented article the case where the effective viscosity depends on the pressure is discussed. The results of numerical calculations qualitatively satisfactorily describe the process under consideration.

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COMPUTER VERIFICATION OF A METHOD FOR SOLVING NONLINEAR INITIAL AND TIME-DEPENDENT BOUNDARY VALUE PROBLEMS FOR A STRING AND A BEAM

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In [1] and [2] nonlinear integro-differential equations with initial and time-dependent boundary conditions for a string and a beam, respectively, are considered. Algorithms for an approximate solution of the indicated problems are constructed. In the present work, a computer verification of these algorithms is carried out on test examples.

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AN APPROXIMATE SOLUTION SCHEME FOR A SYSTEM OF NONLINEAR ABSTRACT HYPERBOLIC EQUATIONS

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In the Hilbert space, the Cauchy problem is considered for a system of nonlinear abstract hyperbolic equations associated with the well-known Timoshenko model of nonlinear beam vibrations. To approximate this problem, a locally linear, symmetric, three-layer, semi-discrete scheme is constructed. The stability and convergence of this scheme are investigated. It is shown that the order of convergence in the class of smooth solutions is quadratic in the time step.

ALTERNATIVE METHOD OF THE ASYMPTOTIC EXPANSION

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Based on [1,2], we consider an analogue of the asymptotic expansion method for linear operator differential equations using orthogonal polynomials. Classically, the asymptotic method presents the solution as a power series of the parameter around a specific point, while using orthogonal polynomials we obtain an interval approximation of the parameter, which is derived from explicit formulas using recurrence relations. Approximate solutions computed with various orthogonal polynomials are presented in a tabular form and a comparative analysis is provided with respect to the classical method.

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ON THE CONSTUCTION OF A CLASS OF REFINED THEORIES FOR SHELLS

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We previously constructed [3] a class of so-called “refined theories" of continuum power for plates, not only in the case of elastic bodies. In the case of bending, such a quantity was determined by the introduction of a parameter dependent on two variables for this class, the simplest selection of which yields all known ones in the literature, including the classical Sophie Germain theory. Such a multitude of theories for elastic plates was due to the introduction of a large number of simplifying hypotheses of a geometric and physical nature, which were excluded for elastic shells. At this stage, there is some theory of elastic shells (see, for example, the monographs of F. Ciarlet [1] and I.Vekua [4]). In the report, using the results of S. Lukasiewicz [2] and I. Vekua [4], the will be presented a class of parameter-dependent two-dimensional models for bending and tension-compression problems, as long as the equations describing these phenomena do not break down for the shell.

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CONTENT

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