THE DIRICHLET BOUNDARY VALUE PROBLEM OF POROUS COSSERAT MEDIA WITH TRIPLE-POROSITY FOR THE CONCENTRIC CIRCULAR RING

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(Received: 07.01.2017; accepted: 22.06.2017)

Abstract

The purpose of this paper is to consider the two-dimensional version of the linear theory of elasticity for solids with triple-porosity in the case of an elastic Cosserat medium. Using the analytic functions of a complex variable and solutions of the Helmholtz equation the Dirichlet boundary value problem are solved explicitly for the concentric circular ring.

Key words and phrases: Triple-porosity, the elastic Cosserat medium, the Dirichlet boundary value problem, the concentric circular ring.

AMS subject classification: 74K20, 74F10, 74G05.

1 Introduction

The first mathematical formulation of flow through triple porosity media is introduced by Liu [1] and several new triple porosity models for single-phase flow in a fracture-matrix system are presented by Liu et al. [2], Abdassah and Ershaghi [3], Al Ahmadi and Wattenbarger [4], Wu et al. [5]. Recently, The full coupled linear theories of elasticity and thermoelasticity for triple porosity materials are presented in [6, 7]. It should be noted that all the papers mentioned above dealt with a classical (symmetric) medium. We consider the problem of elasticity for solids with triple-porosity in the case of an elastic Cosserat medium.

2 Basic equations

Let D be a bounded domain in the Euclidean two-dimensional space R^2 bounded by the contour S. Suppose that $S \in C^{1,\beta}$, $0 < \beta < 1$, i.e., S is a Lyapunow curve. Let $x = (x_1, x_2)$ is point of space, $\partial_\alpha = \dfrac{\partial}{\partial x_\alpha}$. Let the domain D be filled with an isotropic triple-porosity material.

The basic homogeneous system of equations for isotropic materials with triple porosity has the form [8]

$$
\begin{aligned}
&(\mu+\alpha)\Delta u_1 + (\lambda+\mu-\alpha)\partial_1\theta + 2\alpha\partial_2\omega - \partial_1(\beta_1 p_1 + \beta_1 p_3 + \beta_1 p_3) = 0,\\
&(\mu+\alpha)\Delta u_2 + (\lambda+\mu-\alpha)\partial_2\theta + 2\alpha\partial_1\omega - \partial_2(\beta_1 p_1 + \beta_1 p_3 + \beta_1 p_3) = 0,\\
&(\nu+\beta)\Delta\omega + 2\alpha(\partial_1 u_2 - \partial_2 u_1) - 4\alpha\omega = 0,\\
&\qquad\qquad \theta = \partial_1 u_1 + \partial_2 u_2,
\end{aligned}
\tag{1}
$$

where u_α are components of the displacement vector, ω is the component of the rotation vector, p_i $(i = 1; 2; 3)$ are the pressures in the fluid phase, λ and μ are the Lam parameters, α, β, μ are the constants characterizing the microstructure of the considered elastic medium, β_i $(i = 1; 2; 3)$ are the effective stress parameters, Δ is the 2D Laplace operator.

In the stationary case, the values $p = (p_1, p_2, p_3)^T$ satisfy the following equation

$$
\Delta p - Ap = 0, \quad A = \begin{pmatrix} b_1/a_1 & -a_{12}/a_1 & -a_{13}/a_1 \\ -a_{21}/a_2 & b_2/a_2 & -a_{23}/a_2 \\ -a_{31}/a_3 & -a_{32}/a_3 & b_3/a_3 \end{pmatrix} \tag{2}
$$

where $a_i = \frac{k_i}{\mu'}$ (for the fluid phase, each phase i carries its respectively permeability k_i, μ' is fluid viscosity), a_{ij} is the fluid transfer rate between phase i and phase j, $b_1 = a_{12} + a_{13}$, $b_2 = a_{21} + a_{23}$, $b_3 = a_{31} + a_{32}$.

On the plane x_1x_2, we introduce the complex variable $z = x_1 + ix_2 = re^{i\vartheta}$, $(i^2 = -1)$ and the operators $\partial_z = 0.5(\partial_1 - i\partial_2)$, $\partial_{\bar z} = 0.5(\partial_1 + i\partial_2)$, $\bar z = x_1 - ix_2$, and $\Delta = 4\partial_z\partial_{\bar z}$.

The system (1) is written in the complex form

$$
\begin{aligned}
&2(\mu+\alpha)\partial_{\bar z}\partial_z u_+ + (\lambda+\mu-\alpha)\partial_{\bar z}\theta - 2\alpha i\partial_{\bar z}\omega\\
&-\partial_{\bar z}(\beta_1 p_1 + \beta_2 p_2 + \beta_3 p_3) = 0,\\
&2(\nu+\beta)\partial_{\bar z}\partial_z\omega + \alpha i(\theta - 2\partial_{\bar z}u_+) - 2\alpha\omega = 0,
\end{aligned}
\tag{3}
$$

where $u_+ = u_1 + iu_2$.

Equations (2) imply that

$$
p_i = f'(z) + \overline{f'(z)} + l_{i1}\chi_1(z,\bar z) + l_{i2}\chi_2(z,\bar z),
$$

where $f(z)$ is an arbitrary analytic functions of a complex variable z in the domain D and $\chi_\alpha(z,\bar{z})$ is an arbitrary solution of the Helmholtz equation

$$\Delta\chi_\alpha(z,\bar{z}) - \kappa_\alpha\chi_\alpha(z,\bar{z}) = 0,$$

κ_α are eigenvalues and (l_{11}, l_{21}, l_{31}), (l_{12}, l_{22}, l_{32}) are eigenvectors of the matrix A.

The general solution of the system of equations (3) is represented as follows [8, 9]:

$$2\mu u_+ = \varkappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} + \frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}(f(z) + z\overline{f'(z)})$$

$$+2i\partial_{\bar{z}}\chi(z,\bar{z}) + \frac{4\mu}{\lambda+2\mu}\partial_{\bar{z}}[\delta_1\chi_1(z,\bar{z}) + \delta_2\chi_2(z,\bar{z})],$$

$$2\mu\omega = \frac{2\mu}{\nu+\beta}\chi(z,\bar{z}) - \frac{\kappa+1}{2}i(\varphi'(z) + \overline{\varphi'(z)}),$$

where $\varkappa = \dfrac{\lambda+3\mu}{\lambda+\mu}$, $\delta_\alpha := \dfrac{l_{1\alpha}}{\kappa_\alpha}\beta_1 + \dfrac{l_{2\alpha}}{\kappa_\alpha}\beta_2 + \dfrac{l_{3\alpha}}{\kappa_\alpha}\beta_3$, $\varphi(z)$ and $\psi(z)$ are arbitrary analytic functions of a complex variable z in the domain V, $\chi(z,\bar{z})$ is an arbitrary solution of the Helmholtz equation

$$4\partial_z\partial_{\bar{z}}\chi(z,\bar{z}) - \xi^2\chi(z,\bar{z}) = 0,\ \ \xi^2 := \frac{2\mu\alpha}{(\nu+\beta)(\mu+\alpha)} > 0.$$

3 A problem for a circular ring

In this section, we solve a concrete boundary value problem for a concentric circular ring with radius R_1 and R_2 (see fig. 1). On the boundary of the considered domain the values of pressures p_1, p_2, p_3, the displacement and rotation vectors are given.

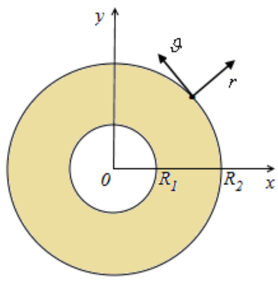

Fig. 1.

We consider the following problem

$$p_j = \begin{cases} \sum\limits_{-\infty}^{+\infty} A'_{jn} e^{in\vartheta}, & |z| = R_1, \\ \sum\limits_{-\infty}^{+\infty} A''_{jn} e^{in\vartheta}, & |z| = R_2, \end{cases} \qquad j = 1, 2, 3 \tag{4}$$

$$u_+ = \begin{cases} \sum\limits_{-\infty}^{+\infty} D'_n e^{in\vartheta}, & |z| = R_1, \\ \sum\limits_{-\infty}^{+\infty} D''_n e^{in\vartheta}, & |z| = R_2, \end{cases} \qquad \omega = \begin{cases} \sum\limits_{-\infty}^{+\infty} E'_n e^{in\vartheta}, & |z| = R_1, \\ \sum\limits_{-\infty}^{+\infty} E''_n e^{in\vartheta}, & |z| = R_2. \end{cases} \tag{5}$$

The analytic function $f(z)$ and the metaharmonic functions $\chi_1(z, \bar{z})$, $\chi_2(z, \bar{z})$ are represented as the series

$$\begin{aligned} f(z) &= \alpha \ln z + \sum_{-\infty}^{+\infty} c_n z^n, \\ \chi_1(z, \bar{z}) &= \sum_{-\infty}^{+\infty} (\alpha_n I_n(r\kappa_1) + \beta_n K_n(r\kappa_1)) e^{in\vartheta}, \\ \chi_2(z, \bar{z}) &= \sum_{-\infty}^{+\infty} (\gamma_n I_n(r\kappa_2) + \delta_n K_n(r\kappa_2)) e^{in\vartheta}, \end{aligned} \tag{6}$$

where $I_n(r\zeta)$ and $K_n(r\zeta)$ are modified Bessel function of n-th order, $z = re^{i\vartheta}$, and are substituted in the boundary conditions (4) we have

$$\begin{aligned} &(\alpha + \bar{\alpha}) \ln R_1 + (\alpha - \bar{\alpha}) i\vartheta + \sum_{-\infty}^{+\infty} R_1^n \left(c_n e^{in\vartheta} + \bar{c}_n e^{-in\vartheta} \right) \\ &+ l_{j1} \sum_{-\infty}^{+\infty} (\alpha_n I_n(R_1\kappa_1) + \beta_n K_n(R_1\kappa_1)) e^{in\vartheta} \\ &+ l_{j2} \sum_{-\infty}^{+\infty} (\gamma_n I_n(R_1\kappa_2) + \delta_n K_n(R_1\kappa_2)) e^{in\vartheta} = \sum_{-\infty}^{+\infty} A'_{jn} e^{in\vartheta}, \\ &(\alpha + \bar{\alpha}) \ln R_2 + (\alpha - \bar{\alpha}) i\vartheta + \sum_{-\infty}^{+\infty} R_2^n \left(c_n e^{in\vartheta} + \bar{c}_n e^{-in\vartheta} \right) \\ &+ l_{j1} \sum_{-\infty}^{+\infty} (\alpha_n I_n(R_2\kappa_1) + \beta_n K_n(R_2\kappa_1)) e^{in\vartheta} \\ &+ l_{j2} \sum_{-\infty}^{+\infty} (\gamma_n I_n(R_2\kappa_2) + \delta_n K_n(R_2\kappa_2)) e^{in\vartheta} = \sum_{-\infty}^{+\infty} A''_{jn} e^{in\vartheta}, \end{aligned} \tag{7}$$

$$(j = 1, 2, 3).$$

From the condition of displacement uniqueness it follows that $\alpha - \bar{\alpha} = 0$. It is also assumed that c_0 is a real value; that is, $c_0 = \bar{c_0}$.

Comparison of terms independent of ϑ gives

$$\begin{array}{l} 2\alpha\ln R_1+2c_0+l_{j1}(\alpha_0 I_0(R_1\kappa_1)+\beta_0 K_0(R_1\kappa_1)) \\ +l_{j2}(\gamma_0 I_0(R_1\kappa_2)+\delta_0 K_0(R_1\kappa_2))=A'_{j0}, \\ 2\alpha\ln R_2+2c_0+l_{j1}(\alpha_0 I_0(R_2\kappa_1)+\beta_0 K_0(R_2\kappa_1)) \\ +l_{j2}(\gamma_0 I_0(R_2\kappa_2)+\delta_0 K_0(R_2\kappa_2))=A''_{j0},\ (j=1,2,3). \end{array} \tag{8}$$

The coefficients $\alpha,\ c_0,\ \alpha_0,\ \beta_0,\ \gamma_0,\ \delta_0$ are found by solving (8).
Comparison of terms involving $e^{in\vartheta}$ for $n=\pm1,\pm2,...$ gives

$$\begin{array}{l} R_1^n c_n+R_1^{-n}\bar{c}_{-n}+l_{j1}(\alpha_n I_n(R_1\kappa_1)+\beta_n K_n(R_1\kappa_1)) \\ +l_{j2}(\gamma_n I_n(R_1\kappa_2)+\delta_n K_n(R_1\kappa_2))=A'_{jn}, \\ R_2^n c_n+R_2^{-n}\bar{c}_{-n}+l_{j1}(\alpha_n I_n(R_2\kappa_1)+\beta_n K_n(R_2\kappa_1)) \\ +l_{j2}(\gamma_n I_n(R_2\kappa_2)+\delta_n K_n(R_2\kappa_2))=A''_{jn},\ (j=1,2,3). \end{array} \tag{9}$$

The coefficients $c_n,\ \alpha_n,\ \beta_n,\ \gamma_n,\ \delta_n$ are found by solving (9).
The analytic functions $\varphi(z)$, $\psi(z)$ and the metaharmonic function $\chi(z,\bar{z})$ are represented as series

$$\varphi(z)=\beta\ln z+\sum_{-\infty}^{\infty}a_n z^n,\quad \psi(z)=\gamma\ln z+\sum_{-\infty}^{\infty}b_n z^n,$$

$$\chi(z,\bar{z})=\sum_{-\infty}^{+\infty}(\alpha'_n I_n(r\kappa_1)+\beta'_n K_n(r\kappa_1))e^{in\vartheta},$$

and are substituted in the boundary conditions (5) we have

$$(\varkappa\beta-\bar{\gamma})\ln r+(\varkappa\beta+\bar{\gamma})i\vartheta+\sum_{-\infty}^{\infty}(\varkappa a_n r^n e^{in\vartheta}-n\bar{a}_n r^n e^{-i(n-2)\vartheta}-\bar{b}_n r^n e^{-in\vartheta})$$

$$-\bar{\beta}e^{2i\vartheta}+i\xi\sum_{-\infty}^{+\infty}(\alpha'_n I_{n+1}(r\zeta)-\beta'_n K_{n+1}(r\zeta))e^{i(n+1)\vartheta}=\begin{cases}\sum\limits_{-\infty}^{+\infty}B'_n e^{in\vartheta},\ |z|=R_1,\\ \sum\limits_{-\infty}^{+\infty}B''_n e^{in\vartheta},\ |z|=R_2,\end{cases}$$

$$\frac{\varkappa+1}{2}i\left(\frac{\bar{\beta}}{r}e^{i\vartheta}-\frac{\beta}{r}e^{-i\vartheta}+\sum_{-\infty}^{\infty}nr^{n-1}\left(\bar{a}_n e^{-i(n-1)\vartheta}-a_n e^{i(n-1)\vartheta}\right)\right)$$

$$+\frac{2\mu}{\nu+\beta}\sum_{-\infty}^{+\infty}(\alpha'_n I_n(r\zeta)+\beta'_n K_n(r\zeta))e^{in)\vartheta}=\begin{cases}\sum\limits_{-\infty}^{+\infty}C'_n e^{in\vartheta},\ |z|=R_1,\\ \sum\limits_{-\infty}^{+\infty}C''_n e^{in\vartheta},\ |z|=R_2,\end{cases}$$

where

$$B_n = D_n - \frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}\left((n+1)r^n c_{n+1} - (n-1)r^{-n}\bar{c}_{1-n}\right)$$

$$-\frac{4\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1}{2}\left(\alpha_{n-1}I_n(\kappa_1 r) - \beta_{n-1}K_n(\kappa_1 r)\right)\right.$$

$$\left.+\frac{\delta_2\kappa_2}{2}\left(\gamma_{n-1}I_n(\kappa_2 r) - \delta_{n-1}K_n(\kappa_2 r)\right)\right],$$

$$(n=\pm 1,\ -2,\ \pm 3, ...),$$

$$B_1 = D_1 - \frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}\left(2rc_2 + \frac{\alpha}{r}\right) - \frac{4\mu}{\lambda+2\mu}$$

$$\times\left[\frac{\delta_1\kappa_1}{2}\left(\alpha_0 I_1(\kappa_1 r) - \beta_0 K_1(\kappa_1 r)\right) + \frac{\delta_2\kappa_2}{2}\left(\gamma_0 I_1(\kappa_2 r) - \delta_0 K_1(\kappa_2 r)\right)\right],$$

and $C_n = E_n$.

From the condition of displacement uniqueness it follows that

$$\varkappa\beta + \bar{\gamma} = 0.$$

Comparison of terms independent of ϑ gives

$$\begin{cases} 2\varkappa \ln R_1 \beta - 2R_1^2\bar{a}_2 + i\xi\left(\alpha'_{-1}I_0(\xi R_1) - \beta'_{-1}K_0(\xi R_1)\right) \\ +\varkappa a_0 - \bar{b}_0 = B'_0, \\ 2\varkappa \ln R_2 \beta - 2R_2^2\bar{a}_2 + i\xi\left(\alpha'_{-1}I_0(\xi R_2) - \beta'_{-1}K_0(\xi R_2)\right) \\ +\varkappa a_0 - \bar{b}_0 = B''_0. \end{cases} \tag{10}$$

Comparison of terms involving $e^{in\vartheta}$ for $n = \pm 1, \pm 2, ...$ gives

$$\begin{cases} \varkappa R_1^2 a_2 - \bar{\beta} - R_1^{-2}\bar{b}_{-2} + i\xi\left(\alpha'_1 I_2(\xi R_1) - \beta'_1 K_2(\xi R_1)\right) = B'_2, \\ \varkappa R_2^2 a_2 - \bar{\beta} - R_2^{-2}\bar{b}_{-2} + i\xi\left(\alpha'_1 I_2(\xi R_2) - \beta'_1 K_2(\xi R_2)\right) = B''_2, \end{cases} \tag{11}$$

$$\begin{cases} \varkappa R_1^n a_n + (n-2)R_1^{2-n}\bar{a}_{2-n} - R_1^{-n}\bar{b}_{-n} \\ +i\xi\left(\alpha'_{n-1}I_n(\xi R_1) - \beta'_{n-1}K_n(\xi R_1)\right) = B'_n, \\ \varkappa R_2^n a_n + (n-2)R_2^{2-n}\bar{a}_{2-n} - R_2^{-n}\bar{b}_{-n} \\ +i\xi\left(\alpha'_{n-1}I_n(\xi R_2) - \beta'_{n-1}K_n(\xi R_2)\right) = B''_n, \\ (n=\pm 1,\ -2,\ \pm 3, ...), \end{cases} \tag{12}$$

$$\begin{cases} \dfrac{2\mu\left(\alpha'_1 I_1(\xi R_1) + \beta'_1 K_1(\xi R_1)\right)}{\nu+\beta} - \dfrac{\varkappa+1}{2}i\left(2R_1 a_2 - \dfrac{\beta}{R_1}\right) = C'_1, \\ \dfrac{2\mu\left(\alpha'_1 I_1(\xi R_2) + \beta'_1 K_1(\xi R_2)\right)}{\nu+\beta} - \dfrac{\varkappa+1}{2}i\left(2R_2 a_2 - \dfrac{\beta}{R_2}\right) = C''_1, \end{cases} \tag{13}$$

$$\begin{cases} \dfrac{2\mu}{\nu+\beta}\left(\alpha_n' I_n(\xi R_1)+\beta_n' K_n(\xi R_1)\right) \\ -\dfrac{\varkappa+1}{2} i\left((n+1)R_1^n a_{n+1}+(n-1)R_1^{-n}\bar{a}_{1-n}\right)=C_n', \\ \dfrac{2\mu}{\nu+\beta}\left(\alpha_n' I_n(\xi R_2)+\beta_n' K_n(\xi R_2)\right) \\ -\dfrac{\varkappa+1}{2} i\left((n+1)R_2^n a_{n+1}+(n-1)R_2^{-n}\bar{a}_{1-n}\right)=C_n'', \\ (n=0,\ -1,\ \pm 2,\ \pm 3, ...). \end{cases} \tag{14}$$

From (14), dividing the first equation of (12) by R_1^n, and second by R_2^n, and subtracting, one obtains the first of the following formulas:

$$\begin{cases} T_n a_n + S_n \bar{a}_{-n+2} = G_n, \\ S_{-n+2} a_n + T_{-n+2}\bar{a}_{-n+2} = \bar{G}_{-n+2}, \end{cases} \tag{15}$$

where

$$G_n = R_2^n B_n'' - R_1^n B_n' - \frac{i\xi(\nu+\beta)(R_2^n I_n(\xi R_2) - R_1^n I_n(\xi R_1))}{2\mu(I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1))}$$

$$\times(C_n' K_{n-1}(\xi R_2) - C_n'' K_{n-1}(\xi R_1)) + \frac{i\xi(\nu+\beta)(R_2^n K_n(\xi R_2) - R_1^n K_n(\xi R_1))}{2\mu}$$

$$\times\frac{(C_n'' I_{n-1}(\xi R_1) - C_n' I_{n-1}(\xi R_2))}{I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1)},$$

$$T_n = \varkappa(R_2^{2n} - R_1^{2n}) - \frac{\xi(\varkappa+1)(\nu+\beta)n(R_2^n I_n(\xi R_2) - R_1^n I_n(\xi R_1))}{4\mu(I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1))}$$

$$\times(R_1^{n-1} K_{n-1}(\xi R_2) - R_2^{n-1} K_{n-1}(\xi R_1)) + \frac{\xi(\varkappa+1)(\nu+\beta)n}{4\mu}$$

$$\times\frac{(R_2^n K_n(\xi R_2) - R_1^n K_n(\xi R_1))(R_2^{n-1} I_{n-1}(\xi R_1) - R_1^{n-1} I_{n-1}(\xi R_2))}{I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1)},$$

$$S_n = (n-2)\left[R_2^2 - R_1^2 - \frac{\xi(\varkappa+1)(\nu+\beta)n(R_2^n I_n(\xi R_2) - R_1^n I_n(\xi R_1))}{4\mu(I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1))}\right.$$

$$\times(R_1^{-n+1} K_{n-1}(\xi R_2) - R_2^{-n+1} K_{n-1}(\xi R_1)) + \frac{\xi(\varkappa+1)(\nu+\beta)n}{4\mu}$$

$$\times\frac{(R_2^n K_n(\xi R_2) - R_1^n K_n(\xi R_1))(R_2^{n-1} I_{n-1}(\xi R_1) - R_1^{n-1} I_{n-1}(\xi R_2))}{I_{n-1}(\xi R_1)K_{n-1}(\xi R_2) - I_{n-1}(\xi R_2)K_{n-1}(\xi R_1)}.$$

The second equation (15) is obtained from the first by replacing n by $-n+2$ and going the conjugate complex expression.

The coefficients a_n $(n = \pm 1,\ -2,\ \pm 3, ...)$ are found by solving (15). The coefficients α'_n and β'_n may be found from (14). The coefficients b_n may be found from one of the two formulae (12). Analogous, from (10), (11) and (13), we can found $\varkappa a_0 - b_0$, a_2, b_{-2}, β, γ, α'_1.

It is easy to prove the absolute and uniform convergence of the series obtained in the circular ring (including the contours) when the functions set on the boundaries have sufficient smoothness.

Acknowledgment

The designated project has been fulfilled by financial support of the Shota Rustaveli National Science Foundation (Grant SRNSF/FR /358/5-109/14).

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