ASYMPTOTIC BEHAVIOR OF THE SOLUTION AND SEMI-DISCRETE FINITE DIFFERENCE SCHEME FOR ONE NONLINEAR INTEGRO-DIFFERENTIAL MODEL WITH SOURCE TERMS

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Abstract

One nonlinear integro-differential system with source terms is considered. The model arises at describing penetration of a magnetic field into a substance. Large time behavior of solution of the initial-boundary value problem is given. Corresponding semi-discrete finite difference scheme is studied as well.

Key words and phrases: Nonlinear integro-differential system, asymptotic behavior, semi-discrete scheme.

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1 Introduction

One system of nonlinear integro-differential equations is considered. Large time behavior of solution and semi-discrete finite difference scheme for the initial-boundary value problem is studied. The investigated system arises in mathematical modeling of the process of a magnetic field penetration into a substance. If the coefficient of thermal heat capacity and electroconductivity of the substance highly dependent on temperature, then the Maxwell's system [1], that describes above-mentioned process, can be rewritten in the following form [2]:

$$\frac{\partial H}{\partial t} = -rot\left[a\left(\int_0^t |rotH|^2 \, d\tau\right) rotH\right], \tag{1.1}$$

where $H = (H_1, H_2, H_3)$ is a vector of the magnetic field and the function $a = a(S)$ is defined for $S \in [0, \infty)$.

Note that (1.1) is complex. Special cases of such type models were investigated (see, for example, [2]-[12] and references therein). Investigations mainly are carried out for one-component magnetic field cases. The existence of global solutions for initial-boundary value problems of such models have been proven in [2]-[5],[11] by using the Galerkin and compactness methods [13],[14]. The asymptotic behavior of the solutions have been the subject of intensive research as well (see, for example, [11],[15],[16] and references therein).

The following one-dimensional system with two-component magnetic field is considered in many works as well (see, for example, [17]-[22]):

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{\partial}{\partial x}\left\{ a\left(\int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 \right] d\tau \right) \frac{\partial U}{\partial x} \right\}, \\
\frac{\partial V}{\partial t} &= \frac{\partial}{\partial x}\left\{ a\left(\int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 \right] d\tau \right) \frac{\partial V}{\partial x} \right\},
\end{aligned} \tag{1.2}$$

where $a = a(S)$ is a given function.

For the system (1.2) the convergence of the semi-discrete and full finite difference approximations of the initial-boundary value problem for the case $a(S) = 1 + S$ with first kind boundary conditions were studied in [22].

The aim of this note is to study asymptotic behavior of solution as $t \to \infty$ and to construct semi-discrete approximate solutions for one generalization of the system type (1.2) by adding monotonic nonlinear source terms.

2 Statement of Problem and Main Results

In the $[0,1]\times[0,\infty)$ let us consider following initial-boundary value problem:

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{\partial}{\partial x}\left\{ \left(1 + \int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 \right] d\tau \right)^p \frac{\partial U}{\partial x} \right\} - |U|^{q-2} U, \\
\frac{\partial V}{\partial t} &= \frac{\partial}{\partial x}\left\{ \left(1 + \int_0^t \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 \right] d\tau \right)^p \frac{\partial V}{\partial x} \right\} - |V|^{q-2} V,
\end{aligned} \tag{2.1}$$

$$U(0,t) = U(1,t) = V(0,t) = V(1,t) = 0, \tag{2.2}$$

$$U(x,0)=U_0(x), \quad V(x,0)=V_0(x), \tag{2.3}$$

where $0<p\le 1$, $q\ge 2$; $U_0=U_0(x)$ and $V_0=V_0(x)$ are given functions.

The following statement is true.

Theorem 1. *If* $0<p\le 1$, $q\ge 2$ *and* $U_0, V_0 \in H_0^1(0,1)$, *then problem (2.1) - (2.3) has not more than one solution and the following asymptotic property takes place*

$$\|U\|+\left\|\frac{\partial U}{\partial x}\right\|+\|V\|+\left\|\frac{\partial V}{\partial x}\right\|\le C\exp\left(-\frac{t}{2}\right).$$

Here $\|\cdot\|$ is the usual norm of the space $L_2(0,1)$ and C denotes positive constant independent of t.

On [0,1] let us introduce a net with mesh points denoted by $x_i=ih$, $i=0,1,\dots,M$, with $h=1/M$. The boundaries are specified by $i=0$ and $i=M$. The semi-discrete approximation at (x_i,t) is designed by $u_i=u_i(t)$ and $v_i=v_i(t)$. The exact solution to the problem at (x_i,t) is denoted by $U_i=U_i(t)$ and $V_i=V_i(t)$. At points $i=1,2,\dots,M-1$, the integro-differential equation will be replaced by approximation of the space derivatives by a forward and backward differences. We will use the following known notations:

$$r_{x,i}(t)=\frac{r_{i+1}(t)-r_i(t)}{h}, \quad r_{\bar{x},i}(t)=\frac{r_i(t)-r_{i-1}(t)}{h}.$$

Using usual methods of construction of discrete analogs (see, for example, [26]) let us construct the following semi-discrete finite difference scheme for problem (2.1) - (2.3):

$$\begin{aligned}
\frac{du_i}{dt}&=\left\{\left(1+\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p u_{\bar{x},i}\right\}_x-|u_i|^{q-2}u_i,\\
\frac{dv_i}{dt}&=\left\{\left(1+\int_0^t\left[(u_{\bar{x},i})^2+(v_{\bar{x},i})^2\right]d\tau\right)^p v_{\bar{x},i}\right\}_x-|v_i|^{q-2}v_i,\\
&\qquad i=1,2,\dots,M-1,
\end{aligned} \tag{2.4}$$

$$u_0(t)=u_M(t)=v_0(t)=v_M(t)=0, \tag{2.5}$$

$$u_i(0)=U_{0,i}, \quad v_i(0)=U_{0,i}, \quad i=0,1,\dots,M. \tag{2.6}$$

The following statement takes place.

Theorem 2. *If* $0<p\le 1$, $q\ge 2$ *and the initial-boundary value problem (2.1) - (2.3) has the sufficiently smooth solution* $U=U(x,t)$, $V=V(x,t)$,

then the semi-discrete scheme (2.4) - (2.6) converges and the following estimate is true

$$\|u(t) - U(t)\|_h + \|v(t) - V(t)\|_h \leq Ch.$$

Here $\|\cdot\|_h$ is a discrete analog of the norm of the space $L_2(0,1)$ and C is a positive constant independent of h.

3 Convergence of the Semi-discrete Scheme

In section 2 we constructed Cauchy problem for nonlinear system of ordinary integro-differential equations (2.4) - (2.6) as semi-discrete analog for problem (2.1) - (2.3). The aim of the present section is the proof of the Theorem 2.

Introduce inner products and norms:

$$(r,g)_h = h\sum_{i=1}^{M-1} r_i g_i, \quad (r,g]_h = h\sum_{i=1}^{M} r_i g_i,$$

$$\|r\|_h = (r,r)_h^{1/2}, \quad \|r]|_h = (r,r]_h^{1/2}, \quad \|r\|_{q,h}^q = h\sum_{i=1}^{M-1} |r_i|^q.$$

After multiplying scalarly corresponding equations in system (2.4) by $u(t) = (u_1(t), u_2(t), \ldots, u_{M-1}(t))$ and $v(t) = (v_1(t), v_2(t), \ldots, v_{M-1}(t))$ and using discrete analog of integrating by part we get:

$$\frac{d}{dt}\|u(t)\|_h^2 + h\sum_{i=1}^{M}\left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p (u_{\bar{x},i})^2 + \|u(t)\|_{q,h}^q = 0,$$

$$\frac{d}{dt}\|v(t)\|_h^2 + h\sum_{i=1}^{M}\left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p (v_{\bar{x},i})^2 + \|v(t)\|_{q,h}^q = 0.$$

From these relations we obtain the following inequalities:

$$\begin{aligned} &\|u(t)\|_h^2 + \int_0^t \|u_{\bar{x}}]|_h^2 d\tau + \int_0^t \|u(t)\|_{q,h}^q d\tau \leq C, \\ &\|v(t)\|_h^2 + \int_0^t \|v_{\bar{x}}]|_h^2 d\tau + \int_0^t \|v(t)\|_{q,h}^q d\tau \leq C. \end{aligned} \tag{3.7}$$

The a priori estimates (3.7) guarantee the global solvability of the problem (2.4) - (2.6).

Proof of Theorem 2. For $U = U(x,t)$ and $V = V(x,t)$ we have:

$$\frac{dU_i}{dt} - \left\{ \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p U_{\bar{x},i} \right\}_x + |U_i|^{q-2} U_i = \psi_{1,i}(t),$$

$$\frac{dV_i}{dt} - \left\{ \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p V_{\bar{x},i} \right\}_x + |V_i|^{q-2} V_i = \psi_{2,i}(t), \tag{3.8}$$

$$i = 1, 2, \ldots, M-1,$$

$$U_0(t) = U_M(t) = V_0(t) = V_M(t) = 0, \tag{3.9}$$

$$U_i(0) = U_{0,i}, \quad V_i(0) = V_{0,i}, \quad i = 0, 1, \ldots, M, \tag{3.10}$$

where

$$\psi_{k,i}(t) = O(h), \quad k = 1,\ 2.$$

Let $z_i(t) = u_i(t) - U_i(t)$ and $w_i(t) = v_i(t) - V_i(t)$. From (2.4) - (2.6) and (3.8) - (3.10) we have:

$$\frac{dz_i}{dt} - \left\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p u_{\bar{x},i} - \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p U_{\bar{x},i} \right\}_x + |u_i|^{q-2} u_i - |U_i|^{q-2} U_i = -\psi_{1,i}(t),$$

$$\frac{dw_i}{dt} - \left\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p v_{\bar{x},i} - \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p V_{\bar{x},i} \right\}_x + |v_i|^{q-2} v_i - |V_i|^{q-2} V_i = -\psi_{2,i}(t), \tag{3.11}$$

$$z_0(t) = z_M(t) = w_0(t) = w_M(t) = 0,$$

$$z_i(0) = w_i(0) = 0.$$

Multiplying scalarly on $z(t) = (z_1(t), z_2(t), \ldots, z_{M-1}(t))$ the first equation of system (3.11), using the discrete analogue of the formula of integration by parts we get

$$\frac{1}{2}\frac{d}{dt}\|z\|^2 + h\sum_{i=1}^{M}\left\{\left(1+\int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p u_{\bar{x},i}\right.$$
$$\left. - \left(1+\int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2\right] d\tau\right)^p U_{\bar{x},i}\right\} z_{\bar{x},i}$$
$$+h\sum_{i=1}^{M-1}\left(|u_i|^{q-2}\,u_i - |U_i|^{q-2}\,U_i\right)(u_i - U_i) = -h\sum_{i=1}^{M-1}\psi_{1,i}z_i.$$

Analogously,

$$\frac{1}{2}\frac{d}{dt}\|w\|^2 + h\sum_{i=1}^{M}\left\{\left(1+\int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p v_{\bar{x},i}\right.$$
$$\left. - \left(1+\int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2\right] d\tau\right)^p V_{\bar{x},i}\right\} w_{\bar{x},i}$$
$$+h\sum_{i=1}^{M-1}\left(|v_i|^{q-2}\,v_i - |V_i|^{q-2}\,V_i\right)(v_i - V_i) = -h\sum_{i=1}^{M-1}\psi_{2,i}w_i.$$

Using monotonicity of the function $f(r) = |r|^{q-2}r$, from these two equalities we have

$$\frac{1}{2}\frac{d}{dt}\left(\|z\|^2 + \|w\|^2\right) + h\sum_{i=1}^{M}\left\{\left(1+\int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p u_{\bar{x},i}\right.$$
$$\left. - \left(1+\int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2\right] d\tau\right)^p U_{\bar{x},i}\right\} z_{\bar{x},i} \qquad (3.12)$$
$$+h\sum_{i=1}^{M}\left\{\left(1+\int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2\right] d\tau\right)^p v_{\bar{x},i}\right.$$

$$- \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p V_{\bar{x},i} \Bigg\} w_{\bar{x},i}$$

$$\leq -h \sum_{i=1}^{M-1} \left(\psi_{1,i} z_i + \psi_{2,i} w_i \right).$$

Note that,

$$\Bigg\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p u_{\bar{x},i}$$

$$- \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p U_{\bar{x},i} \Bigg\} (u_{\bar{x},i} - U_{\bar{x},i})$$

$$+ \Bigg\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p v_{\bar{x},i}$$

$$- \left(1 + \int_0^t \left[(U_{\bar{x},i})^2 + (V_{\bar{x},i})^2 \right] d\tau \right)^p V_{\bar{x},i} \Bigg\} (v_{\bar{x},i} - V_{\bar{x},i})$$

$$= \int_0^1 \frac{d}{d\xi} \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^p$$

$$\times [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})] \, d\xi \, (u_{\bar{x},i} - U_{\bar{x},i})$$

$$+ \int_0^1 \frac{d}{d\xi} \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^p$$

$$\times [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})] \, d\xi \, (v_{\bar{x},i} - V_{\bar{x},i})$$

$$= 2p \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p-1}$$

$$\times \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})] (u_{\bar{x},i} - U_{\bar{x},i}) + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})] (v_{\bar{x},i} - V_{\bar{x},i}) \right\} d\tau$$

$$\times [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})] \, d\xi \, (u_{\bar{x},i} - U_{\bar{x},i})$$

$$+ \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^p$$

$$\times (u_{\bar{x},i} - U_{\bar{x},i})\, d\xi\, (u_{\bar{x},i} - U_{\bar{x},i})$$

$$+2p \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p-1}$$

$$\times \int_0^t \{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]\,(u_{\bar{x},i} - U_{\bar{x},i}) + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]\,(v_{\bar{x},i} - V_{\bar{x},i}) \}\, d\tau$$

$$\times [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]\, d\xi\, (v_{\bar{x},i} - V_{\bar{x},i})$$

$$+ \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p}$$

$$\times (v_{\bar{x},i} - V_{\bar{x},i})\, d\xi\, (v_{\bar{x},i} - V_{\bar{x},i})$$

$$= 2p \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p-1}$$

$$\times \int_0^t \{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]\,(u_{\bar{x},i} - U_{\bar{x},i}) + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]\,(v_{\bar{x},i} - V_{\bar{x},i}) \}\, d\tau$$

$$\times \{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]\,(u_{\bar{x},i} - U_{\bar{x},i}) + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]\, d\xi\, (v_{\bar{x},i} - V_{\bar{x},i}) \}\, d\xi$$

$$+ \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p}$$

$$\times \left[(u_{\bar{x},i} - U_{\bar{x},i})^2 + (v_{\bar{x},i} - V_{\bar{x},i})^2 \right] d\xi$$

$$= p \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p-1}$$

$$\times \frac{d}{dt} \Bigg(\int_0^t \{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]\,(u_{\bar{x},i} - U_{\bar{x},i})$$

$$+ \; [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]\,(v_{\bar{x},i} - V_{\bar{x},i}) \}\, d\tau)^2\, d\xi$$

$$+ \int_0^1 \left(1 + \int_0^t \left\{ [U_{\bar{x},i} + \xi(u_{\bar{x},i} - U_{\bar{x},i})]^2 + [V_{\bar{x},i} + \xi(v_{\bar{x},i} - V_{\bar{x},i})]^2 \right\} d\tau \right)^{p}$$

$$\times\left[(u_{\bar{x},i}-U_{\bar{x},i})^2+(v_{\bar{x},i}-V_{\bar{x},i})^2\right]d\xi.$$

After substituting this equality in (3.12), integrating received equality on $(0,t)$ and using formula of integrating by parts we get

$$\|z\|^2+\|w\|^2+2h\sum_{i=1}^{M}\int_0^t\int_0^1\left(1+\int_0^{t'}\left\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})]^2\right.\right.$$

$$\left.\left.+\ [V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})]^2\right\}d\tau'\right)^p\left[(u_{\bar{x},i}-U_{\bar{x},i})^2+(v_{\bar{x},i}-V_{\bar{x},i})^2\right]d\xi d\tau$$

$$+2ph\sum_{i=1}^{M}\int_0^1\left(1+\int_0^t\left\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})]^2+[V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})]^2\right\}d\tau\right)^{p-1}$$

$$\times\left(\int_0^t\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})](u_{\bar{x},i}-U_{\bar{x},i})\right.$$

$$+\ [V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})](v_{\bar{x},i}-V_{\bar{x},i})\}d\tau)^2d\xi$$

$$-2p(p-1)h\sum_{i=1}^{M}\int_0^1\int_0^t\left(1+\int_0^{t'}\left\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})]^2\right.\right.$$

$$\left.\left.+\ [V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})]^2\right\}d\tau'\right)^{p-2}$$

$$\times\left\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})]^2+[V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})]^2\right\}$$

$$\times\left(\int_0^{t'}\{[U_{\bar{x},i}+\xi(u_{\bar{x},i}-U_{\bar{x},i})](u_{\bar{x},i}-U_{\bar{x},i})\right.$$

$$\left.+\ [V_{\bar{x},i}+\xi(v_{\bar{x},i}-V_{\bar{x},i})](v_{\bar{x},i}-V_{\bar{x},i})\}d\tau'\right)^2d\xi d\tau$$

$$=-2h\sum_{i=1}^{M-1}\int_0^t(\psi_{1,i}z_i+\psi_{2,i}w_i)\,d\tau.$$

Taking into account relation $0 < p \leq 1$ from last equality we have

$$\|z(t)\|_h^2 + \|w(t)\|_h^2 \leq \int_0^t \left(\|z(\tau)\|_h^2 + \|w(\tau)\|_h^2\right) d\tau + \int_0^t \left(\|\psi_1\|_h^2 + \|\psi_2\|_h^2\right) d\tau. \tag{3.13}$$

From (3.13) using Gronwall's inequality we get validity of the Theorem 2.

Note that investigated semi-discrete scheme (2.4) - (2.6) is using for numerical solution of the problem (2.1) - (2.3) by natural discretisation of time derivative and integral as it are given for example in [23], [24] for the case $p = 1$. Solving the obtaining finite difference scheme we use a algorithm analogical to [25]. So, it is necessary to use Newton iterative process [27]. According to this method the great numbers of numerical experiments are carried out. These experiments agree with the theoretical results given in the Theorems 1 and 2.

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