APPROXIMATE METHOD OF THE SIMULTANEOUS ROTATION PROBLEM OF THE POROUS PLATE AND FLUID WITH ACCOUNT OF MAGNETIC FIELD AND HEAT TRANSFER IN CASE OF VARIABLE ELEQTROCONDUCTIVITY

L. Jikidze, V. Tsutskiridze

Department of Mathematics, Georgian Technical University
77 Kostava Str., Tbilisi 0175, Tbilisi, Georgia
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Abstract

In this paper by means of consistent approximation has been studied unsteady problem of the simultaneous rotation of the infinite porous plate and fluid with account of magnetic field and heat transfer in case of variable electroconductivity, when into the plate takes place injection of the same flow with $v_w(t)$ speed.

To determine the thickness of the dynamic and thermal boundary layers, differential equations are obtained and found the exact solutions in special cases when the injection velocity varies according to different laws and between the thicknesses of a functional dependence of the form $\delta_T(t) = \gamma\delta(t)$.

All physical characteristics of the flow are calculated.

Key words and phrases: flow, conductivity, injection velocity, magnetic field, porosity, heat transfer.

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1 Introduction

Injection of fluid through the plate is used to reduse the growth unstable perturbatoin in the boundary layer and delaying its separation. It can also serve as an effective means of intensifying prosseses that use heat transfer [1,2].

In [3] the method of successive approximation studied unsteady problem of rotation of a porous plate in a conducting fluid with allowance for heat transfer, and in [4] and [5] studied a similar problem, with the falling stream of the fluid on the plate with velocity components

$$v_r = ar, \quad v_\varphi = 0, \quad v_z = -2az.$$

And the simultaneous rotation of the porous plate and the fluid, taking into account the magnetic field and heat transfer.

2 Basic part

In this paper by means of consistent approximation a unsteady problem of the simultaneous rotation of the infinite porous plate and fluid with account of magnetic field and heat transfer in case of variable electroconductivity, when into the plate takes place injection of the same flow with $v_w(t)$ speed, has been studied.

Let the influence of dissipative effects on the fluid flow and heat transfer are negligible small and intensive injection leads to a significant reduction of the radial velocity of the fluid near the plate and the temperature difference in the main stream and the plate is relatively small.

With this in mind, to solve the problem, we use the following system of equations of unsteady motion of a conducting fluid in a uniform magnetic field and the energy equation

$$\begin{cases} \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\Delta v_r - \frac{v_r}{r^2}\right) - \frac{\sigma B_0^2}{\rho} v_r, \\ \frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_\varphi}{\partial r} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_\varphi v_r}{r} = \nu\left(\Delta v_\varphi - \frac{v_\varphi}{r^2}\right) - \frac{\sigma B_0^2}{\rho}(v_\varphi - \omega_2 r), \\ \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \Delta v_z, \\ \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0, \\ \lambda \frac{\partial^2 T}{\partial z^2} = \rho C_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z}\right), \end{cases} \tag{1}$$

where $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$; $v_r(t)$, $v_\varphi(t)$, $v_z(t)$-are the components of fluid velocity, T-temperature, c_p-is heat capacity for constant pressure, λ-thermal conductivity, μ-viscosity, σ-coefficient of electroconductivity, ρ-density, B_0-the magnetic field and $\omega_2(t)$-the angular velocity of the fluid.

System (1) must be integrated with the following initial and boundary conditions:

$$\begin{cases} t = 0, & v_r = v_\varphi = v_z = 0, \ \ T = T_w(z, 0), \\ t > 0, & z = 0, \ \ v_r = 0, \ \ v_\varphi = s\omega_1 r, \ \ v_z = -v_w(t), \ \ T = T_w(0, t), \\ & z = \infty, \ \ v_r = 0, \ \ v_\varphi = \omega_2 r, \ \ T = T_\infty. \end{cases} \tag{2}$$

Here $v_w(t)$-injection velocity, s-parameter rotation, $\omega_1(t)$-angular velocity of the plate, T_w-temperature of the plate and T_∞-temperature of the fluid away from the plate. Solution of (1) is sought in the form:

$$\begin{cases} v_r = \omega_0 r f(\eta, t'), \quad v_\varphi = \omega_0 r q(\eta, t'), \quad v_z = \sqrt{\nu\omega_0}\, g(\eta, t'), \\ z = \sqrt{\frac{\nu}{\omega_0}}\eta, \quad t' = \omega_0 t, \quad \omega(t) = \omega_0 \omega'(t'), \quad v_w(t) = \sqrt{\nu\omega_0}\, v_w'(t'), \\ p = -\rho\nu\omega_0 p'(\eta, t'). \end{cases} \tag{3}$$

We assume that the electroconductivity is variable depending on the temperature, in the form of:

$$\sigma = \sigma_0 \frac{T}{T_\infty} \tag{4}$$

Using (3) and (4) into (1) , we obtain the following system of equations:

$$\begin{cases} \frac{\partial^2 f}{\partial \eta^2} - \frac{\partial f}{\partial t} - \frac{m^2}{T_\infty} fT = g\frac{\partial f}{\partial \eta} + f^2 - q^2 + \omega_2^2, \\ \frac{\partial^2 q}{\partial \eta^2} - \frac{\partial q}{\partial t} - \frac{m^2}{T_\infty} qT = 2fq + g\frac{\partial q}{\partial \eta} - m^2\omega_2, \\ \frac{\partial p}{\partial \eta} = -\frac{\partial^2 g}{\partial \eta^2} + \frac{\partial g}{\partial t} + g\frac{\partial g}{\partial \eta}, \\ \frac{\partial g}{\partial \eta} = -2f, \\ \frac{\partial^2 T}{\partial \eta^2} = P_r\left(\frac{\partial T}{\partial t} + g\frac{\partial T}{\partial \eta}\right) \end{cases} \tag{5}$$

where $m^2 = \frac{\sigma B_0^2}{\rho\omega}$, $P_r = \frac{\mu c_p}{\lambda}$-Prandtl's number and the primes over the symbols are omitted.

To determine the thicknesses of the dynamic and thermal boundary layers formed in the rotating plate, with the asymptotic layers consider the layers of finite thickness, which will change over time. To determine them we use the following terms:

$$\eta = \delta(t), \quad \frac{\partial q}{\partial \eta} = 0, \quad \eta = \delta_T(t), \quad \frac{\partial T}{\partial \eta} = 0. \tag{6}$$

Thus, for the solution of (5) we have the following initial and boundary conditions:

$$\begin{cases} t = 0, \quad f = q = g = 0, \quad T = T_w(\eta, 0), \quad \sigma(0) = 0, \quad \sigma_T = 0, \\ t > 0, \quad \eta = 0, \quad f = 0, \quad q = s\omega_1(t), \quad g = -v_w(t), \quad T = T_w(0, t), \\ \eta = \delta(t), \quad f = 0, \quad q = \omega_2(t), \quad \frac{\partial q}{\partial \eta} = 0, \\ \eta = \delta_T(t), \quad T = T_\infty, \quad \frac{\partial T}{\partial \eta} = 0. \end{cases} \tag{7}$$

Problem (5)–(7) will be solved by successive approximation and let us search solutions of this problem in the form of series:

$$f = \sum_{k=0}^{\infty} f_k(\eta, t), \quad q = \sum_{k=0}^{\infty} q_k(\eta, t), \quad g = \sum_{k=0}^{\infty} g_k(\eta, t), \quad T = \sum_{k=0}^{\infty} T_k(\eta, t). \tag{8}$$

To determine the unknown functions required only the first two approximations. Options f_0, q_0, T_0, f_1, q_1, T_1 respectively are solutions

$$\frac{\partial^2 f_0}{\partial \eta^2} = 0, \quad \frac{\partial^2 q_0}{\partial \eta^2} = 0, \quad \frac{\partial^2 T_0}{\partial \eta^2} = 0,$$

$$\begin{cases} \eta = 0, \quad f_0 = 0, \quad q_0 = s\omega_1(t), \quad T_0 = T_w(0,t), \\ \eta = \delta(t), \quad f_0 = 0, \quad q_0 = \omega_2(t), \\ \eta = \delta_T(t), \quad T_0 = T_\infty, \end{cases}$$

$$\begin{cases} \frac{\partial^2 f_1}{\partial \eta^2} = g_0 \frac{\partial f_0}{\partial \eta} + \frac{\partial f_0}{\partial t} + \frac{m^2}{T_\infty} f_0 T_0 + f_0^2 - q_0^2 + \omega_2^2, \\ \frac{\partial^2 q_1}{\partial \eta^2} = g_0 \frac{\partial q_0}{\partial \eta} + \frac{\partial q_0}{\partial t} + \frac{m^2}{T_\infty} q_0 T_0 + 2 f_0 q_0 - m^2 \omega_2, \\ \frac{\partial^2 T_1}{\partial \eta^2} = P_r \Big(\frac{\partial T_0}{\partial t} + g_0 \frac{\partial T_0}{\partial \eta} \Big), \end{cases}$$

$$\begin{cases} \eta = 0, \quad f_1 = 0, \quad q_1 = 0, \quad T_1 = 0, \\ \eta = \delta(t), \quad f_1 = 0, \quad q_1 = 0, \\ \eta = \delta_T(t), \quad T_1 = 0 \end{cases}$$

and functions g_0 and g_1 determined from the expressions

$$g_0 = -2 \int_0^\eta f_0 d\zeta - v_w(t), \quad g_1 = -2 \int_0^\eta f_1 d\zeta.$$

Functions f_0, q_0, g_0, T_0, f_1, q_1, g_1, T_1 are as follows:

$$\begin{aligned}
f_0 =& 0, \quad q_0 = \frac{\omega_2 - s\omega_1}{\delta} \eta + s\omega_1, \quad g_0 = -v_w(t), \quad T_0 = \frac{\theta}{\delta_T} \eta + T_w, \\
f_1 =& - \frac{(\omega_2 - s\omega_1)^2}{12\delta^2} (\eta^4 - \delta^3 \eta) - \frac{s\omega_1(\omega_2 - s\omega_1)}{3\delta} (\eta^3 - \delta^2 \eta) + \\
&+ \frac{\omega_2^2 - s^2\omega_1^2}{2} (\eta^2 - \delta\eta), \\
q_1 =& \frac{m^2 \theta (\omega_2 - s\omega_1)}{T_\infty \delta \delta_T} \Big(\frac{\eta^4}{12} - \frac{\delta^3}{12} \eta \Big) + \\
&+ \Big[\Big(\frac{\omega_2 - s\omega_1}{\delta} \Big)' + \frac{m^2 s\omega_1 \theta}{T_\infty \delta_T} + \frac{m^2 (\omega_2 - s\omega_1) T_w}{T_\infty \delta} \Big] \Big(\frac{\eta^3}{6} - \frac{\delta^2}{6} \eta \Big) + \\
&+ \Big[- \frac{v_w(\omega_2 - s\omega_1)}{\delta} + s\omega_1' - m^2 \omega_2 + \frac{s\omega_1 m^2 T_w}{T_\infty} \Big] \Big(\frac{\eta^2}{2} - \frac{\delta}{2} \eta \Big), \\
g_1 =& \frac{(\omega_2 - s\omega_1)^2}{6\delta^2} \Big(\frac{\eta^5}{5} - \frac{\delta^3}{2} \eta^2 \Big) + (\omega_2^2 - s^2\omega_1^2) \Big(\frac{\eta^3}{3} - \frac{\delta}{2} \eta^2 \Big) + \\
&+ \frac{s\omega_1(\omega_2 - s\omega_1)}{3\delta} \Big(\frac{\eta^3}{2} - \delta^2 \eta^2 \Big), \\
T_1 =& P_r \Big[\frac{\partial}{\partial t} \Big(\frac{\theta}{\delta_T} \Big) \Big(\frac{\eta^3}{6} - \frac{\delta_T^2}{6} \Big) + \Big(\frac{\partial T_w}{\partial t} - \frac{v_w \theta}{\delta_T} \Big) \Big(\frac{\eta^2}{2} - \frac{\delta_T}{2} \eta \Big) \Big],
\end{aligned}$$

where $\theta = T_\infty - T_w$.

To determine the unknown thicknesses $\delta(t)$ and $\delta_T(t)$ use the condition (6) continuous transition of velocity and temperature boundary layer velocity and temperature of the external flow, assuming that they are functions of time only.

For determining the thicknesses of the dynamic and thewrmal boundary layers, we obtain the following system of equations:

$$\begin{cases} (\delta^2)' - A_1\frac{\delta^3}{\delta_T} - [A_2 + 2\ln'(\omega_2 - s\omega_1)]\delta^2 + 3v_w\delta = 6, \\ (\delta_T^2)' - \Big[2(\ln\theta)' + \frac{3}{\theta}\frac{\partial T_w}{\partial t}\Big]\delta_T^2 + 3v_w\delta_T = \frac{6}{P_r}, \end{cases} \tag{9}$$

where we introduced the following notations:

$$A_1 = \frac{m^2(3\omega_2 + s\omega_1)\theta}{2T_\infty(\omega_2 - s\omega_1)}, \quad A_2 = \frac{T_w m^2(2\omega_2 + s\omega_1)}{T_\infty(\omega_2 - s\omega_1)} - \frac{3(m^2\omega_2 - s\omega_1')}{\omega_2 - s\omega_1}.$$

Let us consider some special cases, when it will be possible to obtain an expression $\delta_T(t)$ explicitly.

Let $v_w(t) = \beta_T\delta_T(t)$, where $\beta_T = \text{const}$. Then from the second equation of (9), we obtain the following differential equation

$$(\delta_T^2)' - \Big[2(\ln\theta)' + \frac{3}{\theta}\frac{\partial T_w}{\partial t} - 3\beta_T\Big]\delta_T^2 = \frac{6}{P_r}.$$

The solution of this equation can be written as:

$$\delta_T^2(t) = \frac{6\theta^2}{P_r}e^{\int_0^t(\frac{3}{\theta}\frac{\partial T_w}{\partial\tau} - 3\beta_T)d\tau}\int_0^t \frac{1}{\theta^2(\tau)}e^{-\int_0^t(\frac{3}{\theta}\frac{\partial T_w}{\partial\alpha} - 3\beta_T)d\alpha}d\tau.$$

In particular, if $\theta = \text{const}$, than $\delta_T(t) = \sqrt{\frac{2}{P_r\beta_T}(1 - e^{-3\beta_T t})}$.

If $\beta_T = \frac{1}{\theta(t)}\frac{\partial T_w}{\partial t} + \frac{2}{3}[\ln\theta(t)]'$, then for any $\theta(t)$ we have: $\delta_T(t) = \sqrt{\frac{6}{P_r}t}$.

If the injection velocity is chosen as

$$v_w(t) = \frac{2(\ln\theta)' + \frac{3}{\theta}\frac{\partial T_w}{\partial t}}{3}\delta_T(t), \tag{10}$$

then to determine the thickness of the thermal boundary layer we obtain a simple equation $(\delta_T^2(t))' = \frac{6}{P_r}$, whence $\delta_T(t) = \sqrt{\frac{6}{P_r}t}$.

Let us assume that $v_w(t) = \beta\delta(t)$ and thicknesses between $\delta(t)$ and $\delta_T(t)$ there is a functional relationship of the form $\delta_T(t) = \gamma\delta(t)$, where β and γ are constants. Then, to determine the dinamic boundary layer from the first equation of (9) we obtain the following differential equation

$$(\delta^2)' - \Big[2\ln'(\omega_2 - s\omega_1) + \frac{A_1}{\gamma} + A_2 - 3\beta\Big]\delta^2 = 6,$$

whose solution can be written as follows

$$\delta^2(t) = 6[\omega_2(t) - s\omega_1(t)]^2 e^{\int_0^t(\frac{A_1}{\gamma} + A_2 - 3\beta)d\tau}\times$$

$$\times\int_0^t \frac{1}{[\omega_2(\tau)-s\omega_1(\tau)]^2} e^{-\int_0^t(\frac{A_1}{\gamma}+A_2-3\beta)d\alpha} d\tau.$$

If $\omega_1(t)$, $\omega_2(t)$ and $\theta(t)$ are constants, then

$$\delta(t)=\sqrt{\frac{6}{3\beta-\frac{A_1}{\gamma}-A_2}\Big[1-e^{-(3\beta-\frac{A_1}{\gamma}-A_2)}t\Big]},$$

If $\omega_1(t)$, $\omega_2(t)$ and $\theta(t)$ are constants and $\beta=\frac{1}{3}\Big(\frac{A_1}{\gamma}+A_2\Big)$, then $\delta(t)=\sqrt{6t}$.

If the obtained expressions $\delta(t)$ and $\delta_T(t)$ calculate circumferencial component of shear stress-$\tau_{z\varphi}$, moment of resistance to rotation of the plate-M, moment coefficient of resistance-C_M and heat transfer coefficient-N, we have

a) for the district component of shear stress:

$$\tau_{z\varphi}=r\rho\omega_0\sqrt{\nu w_0}\Big\{\frac{(2+v_w\delta)(\omega_2-s\omega_1)}{2\delta}-$$
$$-\Big[\frac{m^2\theta(\omega_2+s\omega_1)}{2T_\infty\delta_T}+\Big(\frac{\omega_2-s\omega_1}{\delta}\Big)'\Big]\frac{\delta^2}{6}-$$
$$-\Big[\frac{m^2T_w(\omega_2+2s\omega_1)}{6T_\infty}+\frac{s\omega_1'-m^2\omega_2}{2}\Big]\delta\Big\},$$

b) for the moment of resistance of the plate:

$$M=-\frac{\pi\rho\omega_0 s\sqrt{\nu\omega_0}R^4}{2}\Big\{\frac{(2+v_w\delta)(\omega_2-s\omega_1)}{2\delta}-$$
$$-\Big[\frac{m^2\theta(\omega_2+s\omega_1)}{2T_\infty\delta_T}+\Big(\frac{\omega_2-s\omega_1}{\delta}\Big)'\Big]\frac{\delta^2}{6}-$$
$$-\Big[\frac{m^2T_w(\omega_2+2s\omega_1)}{6T_\infty}+\frac{s\omega_1'-m^2\omega_2}{2}\Big]\delta\Big\},$$

c) for the torque coefficient of resistance

$$C_M=\frac{2\pi s}{\omega_0\sqrt{R_e}}\Big\{\frac{(2+v_w\delta)(\omega_2-s\omega_1)}{2\delta}-$$
$$-\Big[\frac{m^2\theta(\omega_2+s\omega_1)}{2T_\infty\delta_T}+\Big(\frac{\omega_2-s\omega_1}{\delta}\Big)'\Big]\frac{\delta^2}{6}-$$
$$-\Big[\frac{m^2T_w(\omega_2+2s\omega_1)}{6T_\infty}+\frac{s\omega_1'-m^2\omega_2}{2}\Big]\delta\Big\},$$

d) for the heat transfer coefficient:

$$N=-\frac{r}{T_w}\Big\{\frac{\theta}{\delta_T}-P_r\Big[\frac{\delta_T^2}{6}\frac{\partial}{\partial t}\Big(\frac{\theta}{\delta_T}\Big)+\frac{\delta_T}{2}\frac{\partial T_w}{\partial t}-\frac{v_w\theta}{2}\Big]\Big\}.$$

3 Conclusion

From the above formulas we can easily discern the influence of the magnetic field, velocity suction fluid, angular velocity of the plate, Reynolds and Prandtl's numbers on the physical characteristics of the flow and heat transfer.

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