THE DIRICHLET BVP OF THE THEORY OF ELASTIC BINARY MIXTURES FOR A TRANSVERSALLY ISOTROPIC PLANE WITH CURVILINEAR CUTS

L.Bitsadze

I.Vekua Institute of Applied Mathematics
Tbilisi State University
0186 University Street 2, Tbilisi, Georgia

(Received: 17.02.09; accepted: 29.06.09)

Abstract

The Dirichlet boundary value problem of the theory of elastic binary mixtures for a transversally isotropic plane with curvilinear cuts is investigated. The solvability of the problem is proved by using the potential method and the theory of singular integral equations.

Key words and phrases: Binary mixtures for transversally isotropic solids, uniqueness theorem, potential method.

AMS subject classification: 74E30, 74G05.

Introduction

In the present paper the Dirichlet boundary value problem (BVP) of elastic binary mixture theory is investigated for a transversally-isotropic plane with curvilinear cuts. The boundary value problems of elasticity for anisotropic media with cuts were considered in [1,2]. In this paper we extend this approach to BVPs of elastic mixture for a transversally-isotropic elastic body. Here we shall be concerned with the plane problem of elastic binary mixture theory (it is assumed that the second components u_2' and u_2' of the three-dimensional partial displacement vectors $u'(u_1', u_2', u_3')$ and $u''(u_1'', u_2'', u_3'')$ are equal to zero, while the components u_1', u_3', u_1'', u_3'' depend only on the variables x_1, x_3).

1 Statement of the BVP

The basic homogeneous equations of statics of the transversally isotropic elastic binary mixtures theory in the case of plane deformation can be written in the form [3]

$$C(\partial x)U = \begin{pmatrix} C^{(1)}(\partial x) & C^{(3)}(\partial x) \\ C^{(3)}(\partial x) & C^{(2)}(\partial x) \end{pmatrix} U = 0, \qquad (1)$$

where the components of the matrix $C^{(j)}(\partial x) = \|C^{(j)}_{pq}(\partial x)\|_{2\times 2}$ read as

$$C^{(j)}_{pq}(\partial x) = C^{(j)}_{qp}(\partial x), \;\; p,q = 1,2, \;\; C^{(j)}_{11}(\partial x) = c^{(j)}_{11}\frac{\partial^2}{\partial x_1^2} + c^{(j)}_{44}\frac{\partial^2}{\partial x_3^2},$$

$$C^{(j)}_{12}(\partial x) = (c^{(j)}_{13} + c^{(j)}_{44})\frac{\partial^2}{\partial x_1 \partial x_3}, \;\; C^{(j)}_{22}(\partial x) = c^{(j)}_{44}\frac{\partial^2}{\partial x_1^2} + c^{(j)}_{33}\frac{\partial^2}{\partial x_3^2}, \; j = \overline{1,3},$$

$c^{(k)}_{pq}$ are constants, characterizing the physical properties of the mixture and satisfying certain inequalities caused by the positive definiteness of potential energy. $U = U^T(x) = (u', u'')$ is four-dimensional displacement vector-function, $u'(x) = (u'_1, u'_3)$ and $u''(x) = (u''_1, u''_3)$ are partial displacement vectors depending on the variables x_1, x_3. Throughout the paper the superscript "T" denotes transposition.

Let the plane be weakened by curvilinear cuts $l_j = a_j b_j, j = 1,2,..,p$. Assume that the cuts $l_j, j = 1,...,p$, are relatively simple nonintersecting open Lyapunov arcs. The direction from a_j to b_j is taken as the positive one on l_j. The normal to l_j will be drawn to the right relative to motion in the positive direction. Denote by D the infinite plane with curvilinear cuts $l_j, j = 1,2,..,p. l = \bigcup\limits_{j=1}^{p} l_j$. We suppose that D is filled with a transversally-isotropic mixture.

We introduce the notation $z = x_1 + ix_3, \zeta_k = y_1 + \alpha_k y_3, \tau_k = t_1 + \alpha_k t_3, \sigma_k = z_k - \varsigma_k$.

For equation (1) we pose the following boundary value problem of statics of the theory of elastic binary mixtures. Find in the domain D a regular solution $U(x)$ of equation (1) when the boundary values of the displacement vector are given on both edges of the arc l_j. Further, assume that at infinity we have the principal vector of external forces acting on l, stress vector and the rotation are equal to zero. It is required to define the deformed state of the plane.

If we denote by $U^+(U^-)$ the limits of U on l from the left (right), then the boundary conditions of the problem will take the form

$$U^+ = f^+, U^- = f^-, \qquad (2)$$

where f^+, and f^- are known vector-functions on l of the Hölder class H, which have derivatives in the class H^* (for the definitions of the classes H

and H^* see[4]) and satisfying at the ends a_j and b_j of l_j, the conditions

$$f^+(a_j) = f^-(a_j), \quad f^+(b_j) = f^-(b_j).$$

It is obvious that displacement is not continuous across the cut l_j. Hence it is of interest for us to study the solution behavior in the neighborhood of the cuts.

First we investigate the question on the uniqueness of solutions of the above-mentioned problems.

2 Uniqueness Theorem

Theorem 1. The Dirichlet BVP (1),(2) has at most one regular solution in the domain D.

Proof. Let Dirichlet BVP has two regular solutions $U^{(1)}$ and $U^{(2)}$. we set $u = U^{(1)} - U^{(2)}$. Evidently, the vector u satisfies (1) and the boundary condition $u^\pm = 0$ on l. Note that if u is a regular solution of equation (1), we have the following Green's formula

$$\int_D\int E(u,u)d\sigma = \int_l [u^+(Tu)^+ - u^-(Tu)^-]ds,$$

where $E(u,u)$ is a potential energy [3]:

$$E(u,u) = E^{(1)}(u,u) + E^{(2)}(u,u) + 2E^{(3)}(u,u),$$

$$E^{(1)}(u,u) = c_{44}^{(1)} e_{13}^{(')2} + c_{33}^{(1)} e_{33}^{(')2} + c_{11}^{(1)} e_{11}^{(')2} + 2c_{13}^{(1)} e_{11}^{'} e_{33}^{'},$$

$$E^{(2)}(u,u) = c_{44}^{(2)} e_{13}^{('')2} + c_{33}^{(2)} e_{33}^{('')2} + c_{11}^{(2)} e_{11}^{('')2} + 2c_{13}^{(2)} e_{11}^{''} e_{33}^{''},$$

$$E^{(3)}(u,u) = c_{33}^{(3)} e_{33}^{'} e_{33}^{''} + c_{11}^{(3)} e_{11}^{'} e_{11}^{''} + c_{44}^{(3)} e_{13}^{'} e_{13}^{''} + c_{13}^{(3)} (e_{33}^{''} e_{11}^{'} + +e_{33}^{'} e_{11}^{''}),$$

Taking into account the fact, that the potential energy is positive definite, we conclude that $u = const, x \in D$. Since $u^\pm = 0$ on l, we have $u(x) = 0, \quad x \in D$. Thus the first BVP has in the domain D at most one regular solution.

3 Existence Theorem

We seek a solution of the problem in the form of double-layer potential ([5],[6])

$$U(z) = \frac{1}{\pi} Im \sum_{k=1}^{4} E_{(k)} \int_l \frac{\partial ln\sigma_k}{\partial s}[g(s) + ih(s)]ds + \sum_{k=1}^{p} V_j(z) + M^{p+1}, \quad (3)$$

where $E_{(k)} = \|A^{(k)}_{pq}A^{-1}\|_{4\times4}$ denotes a special matrix that reduces the Dirichlet BVP to a Fredholm integral equation of the second kind, $A^{(k)} = \|A^{(k)}_{pq}\|_{4\times4}$. The elements of the matrix $E_{(k)}$ and the matrix A^{-1} are defined as follows

$$A^{-1} = \frac{1}{\Delta_1\Delta_2}\begin{pmatrix} A_{33}\Delta_2 & 0 & -A_{13}\Delta_2; & 0 \\ 0 & A_{44}\Delta_1 & 0 & -A_{24}\Delta_1 \\ -A_{13}\Delta_2 & 0 & A_{11}\Delta_2 & 0 \\ 0 & -A_{24}\Delta_1 & 0 & A_{22}\Delta_1 \end{pmatrix},$$

$$A_{ij} = \sum_{k=1}^{4} A^{(k)}_{ij}, \quad A^{(k)}_{ij} = A^{(k)}_{ji},$$

$$A^{(k)}_{11} = d_k[c^{(2)}_{11}q_4 + \alpha^2_k t_{11} + \alpha^4_k t_{12} + c^{(2)}_{44}q_3\alpha^6_k],$$
$$A^{(k)}_{22} = d_k[q_1c^{(2)}_{44} + \alpha^2_k t_{44} + \alpha^4_k t_{42} + c^{(2)}_{33}q_4\alpha^6_k],$$

$$A^{(k)}_{33} = d_k[c^{(1)}_{11}q_4 + \alpha^2_k t_{33} + \alpha^4_k t_{23} + c^{(1)}_{44}q_3\alpha^6_k],$$
$$A^{((k))}_{44} = d_k[q_1c^{(1)}_{44} + \alpha^2_k t_{55} + \alpha^4_k t_{52} + c^{(1)}_{33}q_4\alpha^6_k],$$

$$A^{(k)}_{12} = d_k\alpha_k[v_{12} + v_{11}\alpha^2_k + v_{13}\alpha^4_k],$$
$$A^{(k)}_{14} = d_k\alpha_k[w_{11} + w_{12}\alpha^2_k + w_{13}\alpha^4_k], k = 3, .., 4,$$

$$A^{(k)}_{23} = d_k\alpha_k[v_{22} + v_{21}\alpha^2_k + v_{23}\alpha^4_k],$$
$$A^{(k)}_{34} = d_k\alpha_k[w_{24} + w_{14}\alpha^2_k + w_{34}a^4_k], k = 3, ., 6,$$

$$A^{(k)}_{24} = d_k[-q_1c^{(3)}_{44} + \alpha^2_k t_{66} + \alpha^4_k t_{62} - c^{(3)}_{33}q_4\alpha^6_k],$$
$$A^{(k)}_{13} = d_k[-q_4c^{(3)}_{11} + \alpha^2_k t_{22} + \alpha^4_k t_{13} - c^{(3)}_{44}q_3\alpha^6_k],$$

$$\Delta_1 = \sqrt{a_1a_2a_3a_4}\Delta_2, \quad \Delta_2 = A_{22}A_{44} - A^2_{24}, \quad \Delta_1 = A_{11}A_{33} - A^2_{13},$$

$$t_{11} = c^{(2)}_{11}\delta_{22} + c^{(2)}_{44}q_4 - \alpha^{(3)2}_{13}c^{(2)}_{44} + 2c^{(3)}_{44}\alpha^{(2)}_{13}\alpha^{(3)}_{13} - \alpha^{(2)2}_{13}c^{(1)}_{44},$$

$$t_{12} = c^{(2)}_{44}\delta_{22} + c^{(2)}_{11}q_3 - \alpha^{(3)2}_{13}c^{(2)}_{33} + 2c^{(3)}_{33}\alpha^{(2)}_{13}\alpha^{(3)}_{13} - \alpha^{(2)2}_{13}c^{(1)}_{33},$$
$$t_{22} = -c^{(3)}_{11}\delta_{22} - c^{(3)}_{44}q_4 + \alpha^{(1)}_{13}\alpha^{(3)}_{13}c^{(2)}_{44} - c^{(3)}_{44}(\alpha^{(2)}_{13}\alpha^{(1)}_{13} + \alpha^{(3)2}_{13}) + \alpha^{(2)}_{13}\alpha^{(3)}_{13}c^{(1)}_{44},$$

$$t_{13} = -c^{(3)}_{44}\delta_{22} - c^{(3)}_{11}q_3 + \alpha^{(1)}_{13}\alpha^{(3)}_{13}c^{(2)}_{33} - c^{(3)}_{33}(\alpha^{(2)}_{13}\alpha^{(1)}_{13} + \alpha^{(3)2}_{13}) + \alpha^{(2)}_{13}\alpha^{(3)}_{13}c^{(1)}_{33},$$

$$t_{52} = t_{33} - c^{(1)}_{11}\delta_{22} + c^{(1)}_{33}\delta_{11}, t_{62} = t_{22} - c^{(3)}_{11}\delta_{22} + c^{(3)}_{33}\delta_{11},$$
$$t_{42} = t_{11} - c^{(2)}_{11}\delta_{22} + c^{(2)}_{33}\delta_{11},$$

$$t_{44} = c_{44}^{(2)}\delta_{11} + c_{33}^{(2)}q_1 - \alpha_{13}^{(3)2}c_{11}^{(1)} + 2c_{11}^{(3)}\alpha_{13}^{(2)}\alpha_{13}^{(3)} - \alpha_{13}^{(3)2}c_{11}^{(2)},$$

$$t_{66} = -c_{44}^{(3)}\delta_{11} - c_{33}^{(3)}q_1 + \alpha_{13}^{(2)}\alpha_{13}^{(3)}c_{11}^{(1)} - c_{11}^{(3)}(\alpha_{13}^{(2)}\alpha_{13}^{(1)} + \alpha_{13}^{(3)2}) + \alpha_{13}^{(1)}\alpha_{13}^{(3)}c_{11}^{(2)},$$

$$t_{23} = c_{44}^{(1)}\delta_{22} + c_{11}^{(1)}q_3 - \alpha_{13}^{(1)2}c_{33}^{(2)} + 2c_{33}^{(3)}\alpha_{13}^{(1)}\alpha_{13}^{(3)} - \alpha_{13}^{(3)2}c_{33}^{(1)},$$
$$t_{33} = c_{11}^{(1)}\delta_{22} + c_{44}^{(1)}q_4 - \alpha_{13}^{(1)2}c_{44}^{(2)} + 2c_{44}^{(3)}\alpha_{13}^{(1)}\alpha_{13}^{(3)} - \alpha_{13}^{(3)2}c_{44}^{(1)},$$

$$t_{55} = c_{44}^{(1)}\delta_{11} + c_{33}^{(1)}q_1 - \alpha_{13}^{(1)2}c_{11}^{(2)} + 2c_{11}^{(3)}\alpha_{13}^{(1)}\alpha_{13}^{(3)} - \alpha_{13}^{(3)2}c_{11}^{(1)},$$

$$v_{11} = \alpha_{13}^{(2)}(\alpha_{13}^{(2)}\alpha_{13}^{(1)} - \alpha_{13}^{(3)2}) - \alpha_{13}^{(1)}(c_{44}^{(2)2} + c_{11}^{(2)}c_{33}^{(2)}) - \alpha_{13}^{(2)}(c_{44}^{(3)2} + c_{11}^{(3)}c_{33}^{(3)})$$
$$+\alpha_{13}^{(3)}(2c_{44}^{(2)}c_{44}^{(3)} + c_{11}^{(2)}c_{33}^{(3)} + c_{11}^{(3)}c_{33}^{(2)}),$$

$$w_{12} = -\alpha_{13}^{(2)}(\alpha_{13}^{(2)}\alpha_{13}^{(1)} - \alpha_{13}^{(3)2}) - \alpha_{13}^{(3)}(c_{44}^{(1)}c_{44}^{(2)} + c_{11}^{(2)}c_{33}^{(1)} + c_{44}^{(3)2} + c_{11}^{(3)}c_{33}^{(3)})$$
$$+\alpha_{13}^{(2)}(c_{44}^{(1)}c_{44}^{(3)} + c_{11}^{(3)}c_{33}^{(3)}) + \alpha_{13}^{(1)}(c_{44}^{(2)}c_{44}^{(3)} + c_{11}^{(2)}c_{33}^{(3)}),$$

$$v_{21} = -\alpha_{13}^{(3)}(\alpha_{13}^{(2)}\alpha_{13}^{(1)} - \alpha_{13}^{(3)2}) - \alpha_{13}^{(3)}(c_{44}^{(2)}c_{44}^{(1)} + c_{11}^{(3)}c_{33}^{(3)} + c_{44}^{(3)2} + c_{11}^{(1)}c_{33}^{(2)})$$
$$+\alpha_{13}^{(2)}(c_{44}^{(1)}c_{44}^{(3)} + c_{11}^{(1)}c_{33}^{(3)}) + \alpha_{13}^{(1)}(c_{44}^{(2)}c_{44}^{(3)} + c_{11}^{(3)}c_{33}^{(2)}),$$

$$w_{14} = \alpha_{13}^{(1)}(\alpha_{13}^{(2)}\alpha_{13}^{(1)} - \alpha_{13}^{(3)2}) + \alpha_{13}^{(3)}(2c_{44}^{(1)}c_{44}^{(3)} + c_{11}^{(3)}c_{33}^{(1)} + c_{44}^{(3)2} + c_{11}^{(1)}c_{33}^{(3)})$$
$$-\alpha_{13}^{(1)}(c_{44}^{(3)2} + c_{11}^{(3)}c_{33}^{(3)}) - \alpha_{13}^{(2)}(c_{44}^{(1)2} + c_{11}^{(1)}c_{33}^{(1)}),$$

$$v_{12} = -\alpha_{13}^{(1)}c_{44}^{(2)}c_{11}^{(2)} + \alpha_{13}^{(3)}(c_{11}^{(2)}c_{44}^{(3)} + c_{11}^{(3)}c_{44}^{(2)}) - \alpha_{13}^{(2)}c_{11}^{(3)}c_{44}^{(3)},$$

$$v_{13} = -\alpha_{13}^{(1)}c_{44}^{(2)}c_{33}^{(2)} + \alpha_{13}^{(3)}(c_{33}^{(2)}c_{44}^{(3)} + c_{33}^{(3)}c_{44}^{(2)}) - \alpha_{13}^{(2)}c_{33}^{(3)}c_{44}^{(3)},$$

$$w_{11} = \alpha_{13}^{(1)}c_{44}^{(3)}c_{11}^{(2)} - \alpha_{13}^{(3)}(c_{11}^{(2)}c_{44}^{(1)} + c_{11}^{(3)}c_{44}^{(3)}) + \alpha_{13}^{(2)}c_{11}^{(3)}c_{44}^{(1)},$$

$$w_{13} = \alpha_{13}^{(1)}c_{44}^{(2)}c_{33}^{(3)} - \alpha_{13}^{(3)}(c_{33}^{(1)}c_{44}^{(2)} + c_{33}^{(3)}c_{44}^{(3)}) + \alpha_{13}^{(2)}c_{44}^{(3)}c_{11}^{(1)},$$

$$v_{22} = \alpha_{13}^{(1)}c_{44}^{(2)}c_{11}^{(3)} - \alpha_{13}^{(3)}(c_{11}^{(1)}c_{44}^{(2)} + c_{11}^{(3)}c_{44}^{(3)}) + \alpha_{13}^{(2)}c_{11}^{(1)}c_{44}^{(3)},$$

$$v_{23} = \alpha_{13}^{(1)}c_{44}^{(3)}c_{33}^{(2)} - \alpha_{13}^{(3)}(c_{33}^{(2)}c_{44}^{(1)} + c_{33}^{(3)}c_{44}^{(3)}) + \alpha_{13}^{(2)}c_{33}^{(3)}c_{44}^{(1)},$$

$$w_{24} = -\alpha_{13}^{(1)}c_{44}^{(3)}c_{11}^{(3)} + \alpha_{13}^{(3)}(c_{11}^{(3)}c_{44}^{(1)} + c_{11}^{(1)}c_{44}^{(3)}) - \alpha_{13}^{(2)}c_{11}^{(1)}c_{44}^{(1)},$$

$$w_{34} = -\alpha_{13}^{(1)}c_{44}^{(3)}c_{33}^{(3)} + \alpha_{13}^{(3)}(c_{33}^{(1)}c_{44}^{(3)} + c_{33}^{(3)}c_{44}^{(1)}) - \alpha_{13}^{(2)}c_{33}^{(1)}c_{44}^{(1)}.$$

α_k, $k = 1, .., 4$, are the roots of the characteristic equation ([6]). g and h are the unknown real vectors from the Hölder class that have derivatives of the class H^*.

We introduce the following vector

$$V_j(z) = \frac{1}{\pi} Im \sum_{k=1}^{4} A^{(k)} \frac{(z_k - b_j^{(k)}) ln(z_k - b_j^{(k)}) - (z_k - a_j^{(k)}) ln(z_k - a_j^{(k)})}{b_j^{(k)} - a_j^{(k)}} M^j,$$

$$a_j^{(k)} = Rea_j + \alpha_k Ima_j, \;\; b_j^{(k)} = Reb_j + \alpha_k Imb_j,$$

$M^{(j)}, j = 1, .., p+1$, are the unknown real constant vectors to be defined.

The vector $V_j(z)$ satisfies the following conditions:

1. $V_j(z)$ has the logarithmic singularity at infinity

$$V_j = \frac{1}{\pi} Im \sum_{k=1}^{4} A^{(k)}(-\ln z_k + 1) M^j + O(z_k^{-1}).$$

2. By V_j is meant a branch, which is uniquely defined on the plane cut along l_j.

3. V_j is continuously extendable on l_j from the left and from the right, and

$$V_j^+(a_j) = V_j^-(a_j), \;\; V_j^+(b_j) = V_j^-(b_j),$$

$$V_j^+ - V_j^- = 2Re \sum_{k=1}^{4} A^{(k)} \frac{\tau_k - a_j^{(k)}}{b_k - a_j^{(k)}} M^j, j = 1, ..p.$$

To define the unknown density, by virtue of (2)-(3), we obtain a system of singular integral equations of normal type

$$\pm g(\tau) + \frac{1}{\pi} Im \sum_{k=1}^{4} E_{(k)} \int_l \frac{\partial ln(\tau - \varsigma}{\partial s} (g + ih) ds$$

$$+ \sum_{j=1}^{p} V_j^{\pm} + M_j^{p+1} = f^{\pm}(\tau).$$

This system implies

$$2g(\tau) = f^+ - f^- - Re \sum_{j=1}^{p} \sum_{k=1}^{4} A^{(k)} \frac{\tau_k - a_j^{(k)}}{b_k - a_j^{(k)}} M^j,$$

$$\frac{1}{\pi} \int_l \frac{h(\varsigma) ds}{\varsigma - \tau} + \frac{1}{\pi} \int_l K(\tau, \varsigma) h(\varsigma) ds = \Omega(\tau), \tag{4}$$

where

$$K(\tau,\varsigma)=-i\frac{\partial\theta}{\partial s}E+Re\sum_{k=1}^{4}E_{(k)}\frac{\partial}{\partial s}ln(1+\lambda_k\frac{\overline{\tau}-\overline{\varsigma}}{\tau-\varsigma}),\lambda_k=\frac{1+i\alpha_k}{1-i\alpha_k},$$

$$\theta=arg(\tau-\varsigma),\quad \Omega(\tau)=\frac{1}{2}(f^{+}+f^{-})-\frac{1}{2}\sum_{j=1}^{p}(V_j^{+}+V_j^{-})$$

$$-M^{p+1}-\frac{1}{\pi}Im\sum_{k=1}^{4}E_{(k)}\int\limits_{l}\frac{\partial ln(\tau_k-\varsigma_k)}{\partial s}gds.$$

Thus we have defined the vector g on l. It is not difficult to verify that $g\in H, g'\in H^{*}, g(a_j)=g(b_j)=0, \Omega\in H, \Omega'\in H^{*}$. Equality (4) is a system of singular integral equations of the normal type with respect to the vector h. The points a_j and b_j are nonsingular, while the total index of the class h_{2p} is equal to $-4p$ (for the definition of the class h_{2p} see [4]).

A solution of system (4), if it exists, will be expressed by a vector of the Hölder class that vanishes at the points a_j, b_j, and has derivatives of the class H^{*}.

Next we shall prove that the homogeneous system of equations corresponding to (4) admits only a trivial solution in the class h_{2p}. Let the contrary be true. Assume $h^{(0)}$ to be a nontrivial solution of the homogeneous system corresponding to (4) in the class h_{2p} and construct the potential

$$U_0(z)=\frac{1}{\pi}Im\sum_{k=1}^{4}E_{(k)}\int\limits_{l}\frac{\partial ln(z_k-\varsigma_k)}{\partial s}h^{(0)}(s)ds.$$

Clearly, $U_0^{+}(z)=U_0^{-}(z)=0$ and by the uniqueness theorem we have $U_0(z)=0, \quad z\in D$. Then $\quad TU_0(z)=0, \quad z\in D$ and

$$(TU_0(z))^{+}-(TU_0(z))^{-}=A\frac{\partial h^{0}}{\partial s}=0,\ \ A=\sum_{k=1}^{4}A^{(k)}.$$

Therefore, since $h^{(0)}(a_j)=0$, we obtain $h^{(0)}(z)=0$, which completes the proof. Thus the homogeneous system adjoint to (4) will have $4p$ linearly independent solutions σ_j, $j=1,..,4p$, in the adjoint class and the condition for system (4) to be solvable will be written as

$$\int\limits_{l}\Omega\ \sigma_j\ ds=0,\quad j=1,..,4p. \tag{5}$$

Taking into account the latter conditions and that

$$\int_l [(TU)^+ - (TU)^-]ds = 0 = -2\sum_{k=1}^{p} M^{(k)}, \tag{6}$$

we obtain a system of $4p+4$ algebraic equations with the same number of unknowns with respect to the components of the unknown vector M^j.

We shall show that system (5)-(6) is solvable. Assume that the homogeneous system obtained from (5)-(6) has a nontrivial solution $M_j^0 = \left(M_{1j}^0, M_{2j}^0, M_{3j}^0, M_{4j}^0\right)$, $j = 1,..,p+1$, and construct the potential

$$U_0(z) = \frac{1}{\pi} Im \sum_{k=1}^{4} E_{(k)} \int_l \frac{\partial ln(z_k - \varsigma_k)}{\partial s}(g^0 + h^0)(s)ds + \sum_{k=1}^{p} V_j^0(z) + M_{p+1}^0,$$

where

$$g_0(\tau) = -Re \sum_{j=1}^{p}\sum_{k=1}^{4} A^{(k)} \frac{\tau_k - a_j^{(k)}}{b_k - a_j^{(k)}} M_j^0,$$

$$V_j^0(z) = \frac{1}{\pi} Im \sum_{k=1}^{4} A^{(k)} \frac{(z_k - b_j^{(k)})ln(z_k - b_j^{(k)}) - (z_k - a_j^{(k)})ln(z_k - a_j^{(k)})}{b_j^{(k)} - a_j^{(k)}} M_j^0,$$

It is obvious that $U_0^+ = U_0^- = 0$. Applying the formulas $\sum\limits_{k=1}^{p} M_{(k)}^0 = 0$ and $\int\limits_{lj}[TU_0(z) - TU_0(z)]ds = -2M_j^0 = 0, j = 1,..,p, U_0(\infty) = M_j^{p+1} = 0$, we obtain $M_j^0 = 0$, which contradicts the assumption. Therefore system (5)-(6) has a unique solution.

For M_j^0 system (4) is solvable in the class h_{2p}. The solution of the problem posed is given by potential (3) which is constructed using the solution h of system (4) and the vector g.

We have thereby proved the following theorem:

Theorem 2. If $f \in H$, then a solution of problem (2) exists, is unique and represented by the potential (3), where g and h are solutions of the integral equations (4), which is always solvable for an arbitrary vector f.

Let us consider a particular case with a rectilinear cut along the segment $[a, b]$ of the real axis. Assuming that the principal vector of external forces vanishes at infinity, we obtain

$$U_0(z) = \frac{1}{2\pi} Im \sum_{k=1}^{4} E_{(k)} \int_l \frac{u^+ - u^-}{t - z_k} ds + \frac{1}{2\pi} Im \sum_{k=1}^{4} E_{(k)} X(z_k) \int_l \frac{u^+ - u^-}{X(t)(t - z_k)} dt,$$

where $X(z_k) = \sqrt{(z_k - a)(b - z_k)}$, $\quad X(t) = \sqrt{(t - a)(b - t)}$.

ACKNOWLEDGEMENT.The designated project has been fulfilled by financial support of Georgian National Science Foundation (Grant N GNSF/ST06/3-033). Any idea in this publications is possessed by the author and may not represent the opinion of Georgian National Science Foundation itself.

References

1. Zazashvili Sh., Mixed Boundary Value Problem for an Infinite Plane with a Rectilinear Cut, (Russian),Reports of Enlarged Sessions of the Seminar of I. Vekua Inst.of Appl. Math., 4(1989), No.2, 95-98.

2. Zazashvili Sh., Contact Problem for two Anisotropic Half-plane with a Cut along the Contact Line, (Russian), Bull. Acad. Sci. Georgia 145(1992), No.2, 283-285.

3. Rushchitski Ya. Ya. , Elements of Mixture Theory, (Russian), Naukova Dumka, Kiev (1991).

4. Muskhelishvili N. I. , Singular Integral Equations. Boundary Problems of the Theory of Functions and some their Applications in Mathematical Physics. (Russian) 3rd ed. Nauka, Moscow, 1968.

5. Bitsadze L., The Basic BVPs of Statics of the Theory of Elastic Transversally Isotropic Mixtures. Reports Seminar of I.Vekua Inst.of Appl. Math. Reports v. 26-27. N 1-3 (2000-2001), 79-87.

6. Bitsadze L., On Some Solutions of Statics of the Theory of Elastic Transversally Isotropic Mixtures. Reports of Enlarged Session of the Seminar of I. Vekua Inst.of Appl. Math. v. 16. N 2, (2001), 41-45.