THE CALCULATION OF SHELL TYPE THIN-WALLED COMPOSITE CONSTRUCTIONS WITH THE MODEL OF A PLASTIC-RIGID BODY

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Abstract

The purpose of this work is to investigate the estimation of composite plastic shell's carrying ability, for which are adapted such materials of shells, which are subjected to the ideally plastic-rigid model of diagram "stress-deformation".

Key words and phrases: Slightly curved shell, composite shell, monolithic shell, intermediate couplings.
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Introduction

In the last half century, the efforts of engineers and scientists were directed toward overcoming shortcomings in the constructions of those concluded from the basic building materials, the large dead weight and the significant labor expense of their assembly. The cardinal solution of the problem concerning reduction in the labor expense of assembly is to come over to the composite constructions, as far as the decrease of dead weight is concerned, it is achieved by the use of three-dimensional constructions of the type of shells.

The wide use of composite shells places to the urgent the research of the methods of their scientifically substantiated correct calculation both in the elastic stage and beyond elastic limits.

1 Basic dependencies of plastic-rigid slightly curved shells

During the investigation of the bearing capacity of composite shells it is assumed that the separate elements are linked by the longitudinal and cross connections (hinges), the lines of arrangement of which coincide with the lines of principal curvatures (Fig.3.1). At the points of the arrangement of hinges the first-order derivative of sagging undergoes first-kind discontinuity and the bending moments are equal to zero.

The design diagram indicated can occur to the complete casting of the welds between the composite elements in the process of installation. During determination of the bearing capacity it is used theoretical studies of M.Sh.Mikeladze on the technical theory of thin ideally-plastic shells [1].

The condition of fluidity of the material of shell has the following form [2, 3]

$$f(\sigma_{jj}) = \frac{\sigma_{11}^2}{\sigma_{S1}^2} - \frac{\sigma_{11}\sigma_{22}}{\sigma_{31}\sigma_{32}} + \frac{\sigma_{22}^2}{\sigma_{32}^2} + \frac{\sigma_{12}^2}{\tau_S^2} - 1 = 0.$$

The dependence between the speed of dissipation of the mechanical energy, in reference to the unit of the area of median surface of a shell, and the coefficient of plasticity takes the form [1]

$$D = 2h\lambda, \tag{1.1}$$

where $2h$ is the thickness of a shell:

$$\begin{aligned}\lambda = \Bigg[& \frac{1}{3}(\sigma_{S1}^2 \cdot \varepsilon_1^2 + \sigma_{S1}\sigma_{S2} \cdot \varepsilon_1\varepsilon_2 + \sigma_{S2}^2 \cdot \varepsilon_2^2) + \frac{\tau_S^2 W^2}{4} + \\ & + \frac{h^2}{36}(\sigma_{S1}^2 \cdot \dot{\varkappa}_1^2 + \sigma_{S1}\sigma_{S2} \cdot \dot{\varkappa}_1\dot{\varkappa}_2 + \sigma_{S2}^2 \cdot \dot{\varkappa}_2^2) + \frac{\tau_S^2 h^2}{48} + \dot{\chi}^2 \Bigg]^{1/2}.\end{aligned} \tag{1.2}$$

From the equality of the powers of works accomplished by external and internal forces

$$P \int_F \dot{Z} dF = \int_\Gamma D dF$$

is being determined the bearing capacity of a shell on top [2]

$$P = \frac{\int_F D dF}{\int_F \dot{Z} dF}, \tag{1.3}$$

where $\dot{Z}$ is the kinematically permissible field of velocities of displacements of the points of the middle surface of shells.

The parameters determining deformation speed of median surface of a shell will be:

$$\begin{aligned}
&\dot{\varepsilon}_1 = \frac{\partial \dot{U}}{\partial x} + \frac{\dot{W}}{R_1};\ \dot{\varepsilon}_2 = \frac{\partial \dot{V}}{\partial y} + \frac{\dot{W}}{R_2};\ \dot{\omega} = \frac{\partial \dot{U}}{\partial y} + \frac{\partial \dot{V}}{\partial x};\\
&\dot{\varkappa}_1 = -\frac{\partial^2 W}{\partial x^2} + \frac{1}{R_1}\frac{\partial \dot{U}}{\partial x};\ \dot{\varkappa}_2 = -\frac{\partial^2 W}{\partial y^2} + \frac{1}{R_2}\frac{\partial \dot{V}}{\partial y};\\
&\chi = \frac{1}{R_1}\frac{\partial \dot{U}}{\partial y} + \frac{1}{R_1}\frac{\partial \dot{V}}{\partial x} - 2\frac{\partial^2 W}{\partial x \partial y}.
\end{aligned} \tag{1.4}$$

Assume that the kinematically permissible field of velocities of tangential displacements equal to zero, then using formulas (1.1), (1.2), (1.3) and (1.4), the bearing capacity of a shell on top can be written as follows [2]:

$$P < P_{ar} + \frac{2h}{\sqrt{3}}\left(\frac{\sigma_{S1}}{R_1^2} + \frac{\sigma_{S1}\cdot\sigma_{S2}}{R_1\cdot R_2} + \frac{\sigma_{S1}^2}{R_2^2}\right)^{1/2} \tag{1.5}$$

where

$$P_{ar} = \frac{\frac{h^2}{\sqrt{3}}\int\limits_F \left\{\frac{1}{3}\left[\left(\frac{\partial^2 \dot{W}}{\partial x^2}\right)^2 \sigma_{S1}^2 + \sigma_{S1}\sigma_{S2}\frac{\partial^2 \dot{W}}{\partial x^2}\frac{\partial^2 \dot{W}}{\partial y^2} + \sigma_{S2}^2\left(\frac{\partial^2 \dot{W}}{\partial x^2}\right)^2 + \tau_S^2\left(\frac{\partial^2 \dot{W}}{\partial x \partial y}\right)^2\right]\right\}^{1/2} dF}{\int\limits_F W^2 dF} \tag{1.6}$$

is the bearing capacity of a plate, which has the same dimensions in the plan as a slightly curved shell being investigated [4].

The bearing capacity from below is determined by formula

$$P > \frac{2h}{\sqrt{3}}\left(\frac{\sigma_{S1}^2}{R_1^2} + \frac{\sigma_{S1}\cdot\sigma_{S2}}{R_1\cdot R_2} + \frac{\sigma_{S2}^2}{R_2^2}\right)^{1/2}. \tag{1.7}$$

For convenience in further computations of expression (1.5) let us introduce dimensionless coordinates ξ and, η which are connected with the basic variables by the following relationships:

$x = \xi l_1$ and $y = \eta l_2 = \eta k l_1$ $(l_2 = k l_1)$,

where l_1 and l_2 are the lengths of sides of a shell in the plan. Yield points σ_{S2} and τ_S are expressed by the following equalities:

$$\sigma_{S2} = t\sigma_{S1} \ \text{ and } \ \tau_S = \tau\sigma_{S1}\cdot\frac{1}{\sqrt{3}}.$$

Then expression (1.5) takes the form:

$$P < P_{ar} + \frac{2h\sigma_{S1}}{\sqrt{3}}\left(\frac{1}{R_1^2} + \frac{t}{R_1 R_2} + \frac{t^2}{R_2^2}\right)^{1/2} \tag{1.8}$$

where

$$P_{ar} = \frac{\frac{h}{\sqrt{3}}\frac{\sigma_{S1}}{l_1^2}\int\limits_0^1\int\limits_0^1\left[\left(\frac{\partial^2\dot{W}}{\partial\xi^2}\right)^2 + \frac{t}{K^2}\frac{\partial^2\dot{W}}{\partial\xi^2}\frac{\partial^2\dot{W}}{\partial\eta^2} + \frac{t^2}{K}\left(\frac{\partial^2\dot{W}}{\partial\eta^2}\right)^2 + \frac{1}{K^2}\left(\frac{\partial^2\dot{W}}{\partial\xi\partial\eta}\right)^2\right]^{1/2} d\xi d\eta}{\int\limits_0^1\int\limits_0^1 W\cdot d\xi d\eta}. \tag{1.9}$$

During the determination of upper boundary of the shell's carrying ability we will use mainly expression (1.9), to which is being added the constant expressed by the second term of (1.8).

2 The bearing capacity of slightly curved shell without the intermediate coupling

The bearing capacity of the shell without intermediate ties must be more than the carrying ability of the shell with ties (when other conditions are equal). The mentioned circumstance will allow to evaluate the accuracy of obtained results of the bearing capacity.

Let us determine the bearing capacity of the shell without intermediate ties (Fig.3.2).

For obtaining the kinematically permissible field of velocities of the saggings of the shell in the direction of axis "x" let us consider the fourth order polynomial

$$W(\xi) = a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3 + a_4\xi^4. \tag{2.1}$$

The conditions, which must be satisfied by expression (2.1) are:

$$\xi = 0;\ \begin{cases} W(\xi) = 0; \\ W''(\xi) = 0; \end{cases} \ \xi = \frac{1}{2};\ \frac{\partial W}{\partial \xi} = 0;\ \ \xi = 1;\ \begin{cases} W(\xi) = 0; \\ W'''(\xi) = 0. \end{cases} \tag{2.2}$$

Using conditions (2.2) we have:

$$W(\xi) = c(2\xi^3 - \xi^2 - \xi^4).$$

In the given case and afterwards we assume that $k = 1$ and $t = 1$, then for the direction of axis "y" the sagging is being obtained the same, i.e.

$$W(\eta) = c(2\eta^3 - \eta^2 - \eta^4).$$

The kinematically permissible field of velocities of saggings for the shell takes the form:

$$W = W(\xi)W(\eta) = c^2(2\xi^3 - \xi^2 - \xi^4)(2\eta^3 - \eta^2 - \eta^4). \tag{2.3}$$

The bearing capacity of the plate is determined according to (1.9) by the value of the integral of denominator, which is determined easily, it is equal to 0,000276 s^2.

The double integral of numerator is determined from Simpson's cubature formula [5] , and it is equal to 0,15 s^2.

Thus the upper boundary of carrying ability of the plate is equal

$$P_{ar} = 543,47 \cdot \frac{h^2 \sigma_{S1}}{sl^2} kg/cm^2$$

3 The bearing capacity of gently sloping composite shell

Let us examine a shell, whose elements are connected by two longitudinal and cross connections (Fig.3.3). For obtaining the kinematically permissible field of velocities of saggings of the shell we will use Mac Laurin's generalized formula [6,7].

The expression of sagging for the case in question takes the form:

$$\begin{aligned}
&W(\xi) = C\left(-\frac{\xi^2}{18} + \frac{\xi^3}{12} - \frac{\xi^4}{24}\right) \\
&\text{for } \ 0 < \xi < \frac{1}{3}, \\
&W(\xi) = C\left[-\frac{\xi^2}{18} + \frac{\xi^3}{12} - \frac{\xi^4}{24} + \frac{1}{72}\left(\xi - \frac{1}{3}\right)\right] \\
&\text{for } \ \frac{1}{3} < \xi < \frac{2}{3}, \\
&W(\xi) = C\left[-\frac{\xi^2}{18} + \frac{\xi^3}{12} - \frac{\xi^4}{24} + \frac{1}{72}\left(\xi - \frac{1}{3}\right) + \frac{1}{72}\left(\xi - \frac{2}{3}\right)\right] \\
&\text{for } \ \frac{2}{3} < \xi < 1,
\end{aligned}$$

where

$$C = \frac{ql^4}{EJ}.$$

The same form has a sagging in the direction of "y" axis.

The kinematically permissible field of velocities for the case under con-

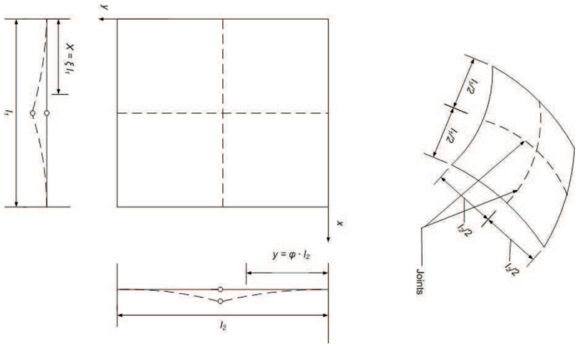

Figure 3.1: The design model of composite shell with longitudinal and transversal hinges.

sideration will takes the form:

$$W = W(\xi)W(\eta) = C^2\left(-\frac{\xi^2}{18}+\frac{\xi^3}{12}-\frac{\xi^4}{24}\right)\cdot\left(-\frac{\eta^2}{18}+\frac{\eta^3}{12}-\frac{\eta^4}{24}\right)$$
for $\xi < 0, \ \ \eta < \frac{1}{3}$;
$$W = W(\xi)W(\eta) = C^2\left(-\frac{\xi^2}{18}+\frac{\xi^3}{12}-\frac{\xi^4}{24}+\frac{1}{72}\left(\xi-\frac{1}{3}\right)\right)\times$$
$$\times\left(-\tfrac{\eta^2}{18}+\frac{\eta^3}{12}-\frac{\eta^4}{24}+\frac{1}{72}\left(\eta-\frac{1}{3}\right)\right)$$
for $\frac{1}{3} < \xi, \ \ \eta < \frac{2}{3}$.

The values of the integrals of numerator and numerator in formula (1.9) are respectively equal to:

$$J_r = 20,402; \; J_3 = 0,309.$$

The bearing ability

$$P_m = 66,03\frac{h^2\sigma_{s1}}{3l^2}kg/cm^2.$$

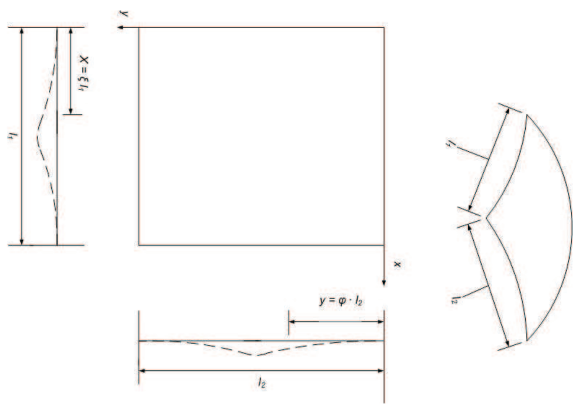

Figure 3.2: The design model for monolithic shell.

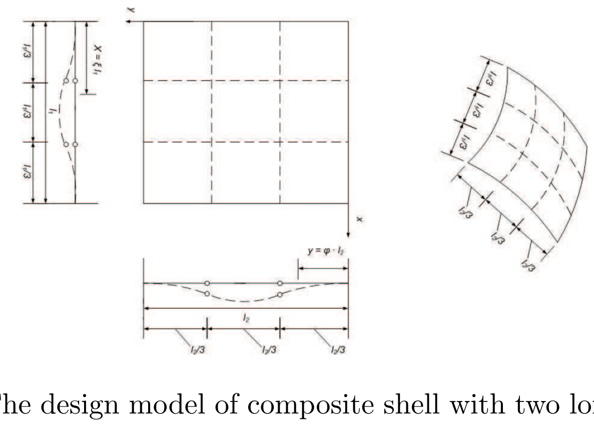

Figure 3.3: The design model of composite shell with two longitudinal and two transversal hinges.

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