ON SOME SOLUTIONS OF STATICS OF THE THEORY OF ELASTIC TRANSVERSALLY ISOTROPIC BINARY MIXTURES

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Abstract

The purpose of this paper is to consider three - dimensional version of statics of the theory of elastic transversally-isotropic binary mixtures, which is the simplest anisotropic one and for which we can do explicit computation. The basic equations are given in [1] (see Rushchitski [1] and references cited therein). The fundamental and other matrixes of singular solutions are constructed for the equation of statics of a transversally isotropic elastic binary mixtures. Using the fundamental matrix there is constructed simple and double layer potentials and there is studied its properties. Applying this potentials the basic BVPs are reduced to a system of integral equations.

Key words and phrases: Boundary value problems, transversally-isotropic elastic mixtures, fundamental solution

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1 Basic equations and BVPs

The basic homogeneous equations of statics of the elastic mixture theory are written in terms of displacement components as follows [1]

$$C(\partial x)U = \begin{pmatrix} C^{(1)}(\partial x) & C^{(3)}(\partial x) \\ C^{(3)}(\partial x) & C^{(2)}(\partial x) \end{pmatrix} U = 0, \tag{1}$$

where

$$\begin{aligned}
&C^{(j)}(\partial x) = \|C^{(j)}_{kp}(\partial x)\|_{3\times 3}, \quad j = 1, 2, 3,\\
&C^{(j)}_{11}(\partial x) = c^{(j)}_{11}\frac{\partial^2}{\partial x_1^2} + c^{(j)}_{66}\frac{\partial^2}{\partial x_2^2} + c^{(j)}_{44}\frac{\partial^2}{\partial x_3^2},\\
&C^{(j)}_{12}(\partial x) = (c^{(j)}_{11} - c^{(j)}_{66})\frac{\partial^2}{\partial x_1 \partial x_2},\\
&C^{(j)}_{k3}(\partial x) = (c^{(j)}_{13} + c^{(j)}_{44})\frac{\partial^2}{\partial x_k \partial x_3}, \quad k = 1, 2,\\
&C^{(j)}_{22}(\partial x) = c^{(j)}_{66}\frac{\partial^2}{\partial x_1^2} + c^{(j)}_{11}\frac{\partial^2}{\partial x_2^2} + c^{(j)}_{44}\frac{\partial^2}{\partial x_3^2},\\
&C^{(j)}_{33}(\partial x) = c^{(j)}_{44}(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}) + c^{(j)}_{33}\frac{\partial^2}{\partial x_3^2}.
\end{aligned}$$

here $U(x)^T = U(u', u'')$ -is six-dimensional displacement vector, $u'(u'_1, u'_2, u'_3)^T$ and $u"(u"_1, u"_2, u"_3)^T$ -are partial displacement vectors, $c^{(j)}_{pq}$-are constants, characterizing the physical properties of the mixture and satisfying certain inequalities caused by the positive definiteness of potential energy.Throughout this paper "T" denotes transposition. The generalized stress vector acting on the arc from the side of positive normal $n(n_1, n_2, n_3)$ is written as

$$P(\partial x, n)U = \begin{pmatrix} P^{(1)}(\partial x, n) & P^{(3)}(\partial x, n) \\ P^{(3)}(\partial x) & P^{(2)}(\partial x) \end{pmatrix} U, \qquad (2)$$

where

$$\begin{aligned}
P^{(j)}(\partial x, n) &= \|P^{(j)}_{kp}(\partial x, n)\|_{3x3}, \\
P^{(j)}_{11}(\partial x) &= c^{(j)}_{11} n_1 \frac{\partial}{\partial x_1} + c^{(j)}_{66} n_2 \frac{\partial}{\partial x_2} + c^{(j)}_{44} n_3 \frac{\partial}{\partial x_3}, \\
P^{(j)}_{12}(\partial x) &= \alpha^{(j)}_0 n_1 \frac{\partial}{\partial x_2} + n_2 \gamma^{(j)}_0) \frac{\partial}{\partial x_1}, \\
P^{(j)}_{13}(\partial x) &= n_1 \beta^{(j)}_0 \frac{\partial}{\partial x_3} + n_3 \delta^{(j)}_0 \frac{\partial}{\partial x_1}, P^{(j)}_{21}(\partial x) \\
&= \alpha^{(j)}_0 n_2 \frac{\partial}{\partial x_1} + n_1 \gamma^{(j)}_0) \frac{\partial}{\partial x_2}, \\
P^{(j)}_{22}(\partial x) &= c^{(j)}_{66} n_1 \frac{\partial}{\partial x_1} + c^{(j)}_{11} n_2 \frac{\partial}{\partial x_2} + c^{(j)}_{44} n_3 \frac{\partial}{\partial x_3}, \\
P^{(j)}_{23}(\partial x) &= n_2 \beta^{(j)}_0 \frac{\partial}{\partial x_3} + n_3 \delta^{(j)}_0 \frac{\partial}{\partial x_2}, \\
P^{(j)}_{31}(\partial x) &= n_3 \beta^{(j)}_0 \frac{\partial}{\partial x_1} + n_1 \delta^{(j)}_0 \frac{\partial}{\partial x_3},
\end{aligned}$$

$$\begin{aligned}
P^{(j)}_{32}(\partial x) &= n_3 \beta^{(j)}_0 \frac{\partial}{\partial x_2} + n_2 \delta^{(j)}_0 \frac{\partial}{\partial x_3}, \\
P^{(j)}_{33}(\partial x) &= c^{(j)}_{44} (n_1 \frac{\partial}{\partial x_1} + n_2 \frac{\partial}{\partial x_2}) + c^{(j)}_{33} n_3 \frac{\partial}{\partial x_3}, \alpha^{(j)}_0 + \gamma^{(j)}_0 \\
&= c^{(j)}_{11} - c^{(j)}_{66}, \delta^{(j)}_0 + \beta^{(j)}_0 = c^{(j)}_{13} + c^{(j)}_{44}.
\end{aligned}$$

when $\beta^{(j)}_0 = c^{(j)}_{13}, \gamma^{(j)}_0 = c^{(j)}_{66}, \alpha^{(j)}_0 = c^{(j)}_{11} - 2c^{(j)}_{66}, \delta^{(j)}_0 = c^{(j)}_{44}, j = 1, 2, 3,$ we have the stress vector.

Let $D^+(D^-)$ be a finite (an infinite) three- dimensional domain bounded by surface S.

Definition 1.The vector U defined in the region $D^+(D^-)$ is called regular,if it has integrable in $D^+(D^-)$ continuous second derivatives and U itself and its first derivatives are continuously extendable at every point of the boundary of $D^+(D^-)$. For the region D^- the conditions at infinite are added

$$U(x) = O(1), | x |^2 \frac{\partial U}{\partial x_k} = O(1).$$

Where O(1) denotes a bounded function and $\mid x \mid = x_1^2 + x_2^2 + x_3^2$.

For the equation (1),we pose the following BVPs.Find a regular vector function U, satisfying in $D^+(D^-)$ the equation (1), if on the boundary S one of the following conditions is given.

Problem 1. The displacement vector U is given in the form

$$U^{\pm} = f,$$

where $(.)^{\pm}$ denotes the limiting value from $D^{\pm}$, and f is a given vector.

Problem 2. The stress vector is given in the form

$$(TU)^{\pm} = f.$$

Matrix of fundamental solution.The fundamental matrix of the equation (1),which is denoted by $\Gamma(x-y)$,is the following

$$\Gamma(x-y) = \sum_{k=1}^{6} \|\Gamma_{pq}^{(k)}\|_{6x6}, \tag{3}$$

where $\Gamma_{pq}^{(k)} = \Gamma_{qp}^{(k)}$,

$$\begin{aligned}
&\Gamma_{pp}^{(k)} = \tfrac{A_{11}^{(k)}}{r_k} + A_{12}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_p^2}, \quad \Gamma_{12}^{(k)} = A_{12}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1 x_2},\\
&\Gamma_{p3}^{(k)} = A_{13}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_p x_3}, \quad p=1,2,\\
&\Gamma_{14}^{(k)} = \tfrac{A_{41}^{(k)}}{r_k} + A_{42}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1^2}, \quad \Gamma_{15}^{(k)} = A_{42}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1 x_2},\\
&\Gamma_{p6}^{(k)} = A_{16}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_p x_3}, \quad p=1,2,\\
&\Gamma_{24}^{(k)} = \Gamma_{15}^{(k)}, \quad \Gamma_{25}^{(k)} = \tfrac{A_{41}^{(k)}}{r_k} + A_{42}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_2^2}, \quad \Gamma_{33}^{(k)} = \tfrac{A_{33}^{(k)}}{r_k},\\
&\Gamma_{34}^{(k)} = A_{34}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1 x_3}, \quad \Gamma_{35}^{(k)} = A_{34}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_2 x_3},\\
&\Gamma_{36}^{(k)} = \tfrac{A_{36}^{(k)}}{r_k}, \quad \Gamma_{44}^{(k)} = \tfrac{A_{44}^{(k)}}{r_k} + A_{45}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1^2},\\
&\Gamma_{45}^{(k)} = A_{45}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1 x_2}, \quad \Gamma_{46}^{(k)} = A_{46}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_1 x_3},\\
&\Gamma_{55}^{(k)} = \tfrac{A_{44}^{(k)}}{r_k} + A_{45}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_2^2}, \quad \Gamma_{56}^{(k)} = A_{46}^{(k)}\tfrac{\partial^2\Phi_k}{\partial x_2 x_3}, \quad \Gamma_{66}^{(k)} = \tfrac{A_{6}^{(k)}}{r_k},\\
&A_{11}^{(k)} = \tfrac{(-1)^k b_0(c_{44}^{(2)} - c_{66}^{(2)} a_k)}{(a_1-a_2)r_0}, \quad k=1,2, \quad r_0 = c_{66}^{(1)}c_{66}^{(2)} - c_{66}^{(3)2} > 0,
\end{aligned}$$

$$\begin{aligned}
&A_{11}^{(k)} = \tfrac{(-1)^k b_0(c_{44}^{(2)} - c_{66}^{(2)} a_k)}{r_0(a_1-a_2)}, A_{14}^{(k)} = -\tfrac{(-1)^k b_0(c_{44}^{(3)} - c_{66}^{(3)} a_k)}{r_0(a_1-a_2)}, k=1,2,\\
&A_{12}^{(k)} = \tfrac{A_{11}^{(k)}}{a_k}, A_{24}^{(k)} = \tfrac{A_{14}^{(k)}}{a_k}, A_{45}^{(k)} = \tfrac{A_{44}^{(k)}}{a_k},\\
&A_{44}^{(k)} = \tfrac{(-1)^k b_0(c_{44}^{(1)} - c_{66}^{(1)} a_k)}{r_0(a_1-a_2)}, k=1,2,\\
&A_{12}^{(k)} = \tfrac{\delta_k}{a_k}[-q_3 c_{44}^{(2)} + a_k t_{12} - a_k^2 t_{11} + c_{11}^{(2)} q_4 a_k^3],\\
&A_{42}^{(k)} = \tfrac{\delta_k}{a_k}[q_3 c_{44}^{(3)} + a_k t_{13} - a_k^2 t_{22} - c_{11}^{(3)} q_4 a_k^3],\\
&A_{45}^{(k)} = \tfrac{\delta_k}{a_k}[-q_3 c_{44}^{(1)} + a_k t_{23} - a_k^2 t_{33} + c_{11}^{(1)} q_4 a_k^3],\\
&A_{33}^{((k))} = \delta_k[q_4 c_{33}^{(2)} - a_k t_{42} + a_k^2 t_{44} - c_{44}^{(2)} q_1 a_k^3],
\end{aligned}$$

$$\begin{aligned}
A_{36}^{(k)} &= \delta_k[-q_4c_{33}^{(3)} - a_kt_{62} + a_k^2t_{66} + c_{44}^{(3)}q_1a_k^3],\\
A_{66}^{(k)} &= \delta_k[q_4c_{33}^{(1)} - a_kt_{52} + a_k^2t_{55} - c_{44}^{(1)}q_1a_k^3],\\
A_{13}^{(k)} &= \delta_k[v_{13} - v_{11}a_k + v_{12}a_k^2],\\
A_{16}^{(k)} &= \delta_k[w_{13} - w_{12}a_k + v_{11}a_k^2], k = 3,..,6, \sum_{k=1}^{6} A_{12}^{(k)} = 0,\\
A_{34}^{(k)} &= \delta_k[v_{23} - v_{21}a_k + v_{22}a_k^2],\\
A_{46}^{(k)} &= \delta_k[w_{34} - w_{14}a_k + w_{24}a_k^2], k = 3,..,6,
\end{aligned}$$

$$\begin{aligned}
\sum_{k=1}^{6} A_{13}^{(k)} &= 0, \sum_{k=1}^{6} A_{45}^{(k)} = 0, \sum_{k=1}^{6} A_{34}^{(k)} = 0, \sum_{k=1}^{6} A_{16}^{(k)} = 0, \sum_{k=1}^{6} A_{46}^{(k)} = 0,\\
\delta_k &= d_k(a_1 - a_k)(a_2 - a_k),\\
r_k &= \sqrt{a_k}[(x_1 - y_1)_1^2 + (x_2 - y_2)^2] + (x_3 - y_3)^2,\\
\Phi &= (x_3 - y_3)\ln(x_3 - y_3 + r_k) - r_k.
\end{aligned}$$

The coefficients $d_k,\ v_{ij}, ..., t_{ij}$ are given in [2]. $a_1, a_2, ..., a_6$ are the positive roots of the characteristic equation

$$(r_0a^2 - c_0a + q_4)(b_0a^4 - b_1a^3 + b_2a^2 - b_3a + b_4) = 0,$$

where

$$\begin{aligned}
&c_0 = c_{66}^{(1)}c_{44}^{(2)} + c_{66}^{(2)}c_{44}^{(1)} - 2c_{66}^{(3)}c_{44}^{(3)} > 0,\\
&b_k > 0, b_1b_2 - b_0b_3 > 0; \quad k = 1, 2, 3,\\
&b_1 = \delta_{13}\delta_{11} - k_{11}k_{44} + q_1\delta_{22}, \quad \delta_{13} = (c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2})\\
&\delta_{11} = c_{11}^{(1)}c_{44}^{(2} + c_{11}^{(2)}c_{44}^{(1)} - 2c_{11}^{(3)}c_{44}^{(3)},\\
&b_2 = \delta_{13}^2 + 2\delta_{13}k_{44} + q_1q_3 + \delta_{11}\delta_{22} - k_2,\\
&b_3 = q_3\delta_{11} + \delta_{22}\delta_{13} - k_{33}k_{44},\\
&\delta_{22} = c_{33}^{(1)}c_{44}^{(2)} + c_{33}^{(2)}c_{44}^{(1)} - 2c_{33}^{(3)}c_{44}^{(3)},\\
&\delta_{33} = c_{11}^{(1)}c_{33}^{(2)} + c_{11}^{(2)}c_{33}^{(1)} - 2c_{11}^{(3)}c_{33}^{(3)},\\
&k_2 = \alpha_{13}^{(2)2}c_{11}^{(1)}c_{33}^{(1)} + \alpha_{13}^{(1)2}c_{11}^{(2)}c_{33}^{(2)} +\\
&\alpha_{13}^{(3)2}(\delta_{33} + 4c_{33}^{(3)}c_{11}^{(3)}) - 2\alpha_{13}^{(1)}\alpha_{13}^{(3)}(c_{11}^{(2)}c_{33}^{(3)} + c_{11}^{(3}c_{33}^{(2)})\\
&-2\alpha_{13}^{(2)}\alpha_{13}^{(3)}(c_{11}^{(3)}c_{33}^{(1)} + c_{11}^{(1)}c_{33}^{(3)}) +\\
&2\alpha_{13}^{(1)}\alpha_{13}^{(2)}c_{11}^{(3)}c_{33}^{(3)}, k_{11} = c_{11}^{(2)}c_{13}^{(1)} + c_{11}^{(1)}c_{13}^{(2)} - 2c_{11}^{(3)}c_{13}^{(3)},\\
&k_{44} = c_{44}^{(2)}c_{13}^{(1)} + c_{44}^{(1}c_{13}^{(2)} -\\
&2c_{44}^{(3)}c_{13}^{(3)} + 2q_4, k_{33} = c_{33}^{(2)}c_{13}^{(1)} + c_{33}^{(1}c_{13}^{(2)} - 2c_{33}^{(3)}c_{13}^{(3)},
\end{aligned}$$

It is evident that $\Gamma(x)$ is a symmetric matrix. It can be shown, that every columns and every rows of the matrix $\Gamma(x)$ are solutions to the equation (1) with respect to x, for any $x \neq y$.

The following theorems are valid:

Theorem 1.All elements of the matrix $\Gamma(x)$ have the singularity

$$|\Gamma_{pq}^{(k)}(x)| \leq \frac{const}{|x|}$$

. **Proof**.Consider the term

$$\Gamma_{12}^{(k)}(x)$$

and applying the conditions $\sum\limits_{k=1}^{6} A_{12}^{(k)} = 0, \quad \Delta_k \Phi_k = 0, \quad k = 1, 2.., 6,$ we get

$$|\Gamma_{12}^{(k)}(x)| = |\sum_{k=1}^{6} A_{12}^{(k)} \frac{\partial^2 \Phi_k}{\partial x_1 x_2}|$$
$$= |\sum_{k=1}^{5} A_{12}^{(k)} (a_k - a_6)(\frac{1}{r_k + r_6} - \frac{x_1^2(a_6 r_k + a_k r_6)}{r_k r_6 (r_6 + r_k)^2})| \leq \frac{const}{|x|},$$

Analogously we obtain $|\Gamma(x)_{pq}| \leq \dfrac{const}{|x|}$.

Let us consider the matrix $P^*(y-x)$, which is obtained from $P(\partial x, n)\Gamma(x-y)$ by transposition of the columns and rows and the variables x and y. We can easily prove that every column of the matrix $P^*(y-x)$ is a solution of the system (1) with respect to the point x, if $x \neq y$.

$$P(\partial x, n)\Gamma(x-y) = \sum_{k=1}^{6} \|M_{pq}^{(k)}\|_{6\times 6}, \tag{4}$$

where

$$M_{11}^{(k)} = R_{11}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} + R_{12}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_1 x_3} - q_{12}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{12}^{(k)} = -q_{11}^{(k)} \frac{\partial}{\partial s_3} \frac{1}{r_k} - q_{12}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2^2} + R_{12}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{13}^{(k)} = R_{13}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k} - q_{13}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{14}^{(k)} = R_{14}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} + R_{24}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_1 x_3} - q_{14}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{15}^{(k)} = -R_{15}^{(k)} \frac{\partial}{\partial s_3} \frac{1}{r_k} - q_{14}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2^2} + R_{24}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{16}^{(k)} = -q_{16}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2 x_3} + R_{16}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k},$$

$$M_{21}^{(k)} = q_{11}^{(k)} \frac{\partial_k}{\partial s_3} \frac{1}{r_k} + q_{12}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1^2} - R_{12}^{(k)} \frac{\partial}{\partial s_1} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$
$$M_{22}^{(k)} = R_{11}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} - R_{12}^{(k)} \frac{\partial}{\partial s_1} \frac{\partial^2 \Phi_k}{\partial x_2 x_3} + q_{12}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{23}^{(k)} = -R_{13}^{(k)} \frac{\partial}{\partial s_1} \frac{1}{r_k} + q_{13}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$
$$M_{24}^{(k)} = R_{15}^{(k)} \frac{\partial}{\partial s_3} \frac{1}{r_k} + q_{14}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1^2} - R_{24}^{(k)} \frac{\partial}{\partial s_1} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$
$$M_{25}^{(k)} = R_{14}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} - R_{24}^{(k)} \frac{\partial}{\partial s_1} \frac{\partial^2 \Phi_k}{\partial x_2 x_3} + q_{14}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{26}^{(k)} = -R_{16}^{(k)} \frac{\partial}{\partial s_1} \frac{1}{r_k} + q_{16}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$

$$M_{31}^{(k)} = -R_{31}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k} + R_{32}^{(k)} \frac{\partial_k}{\partial n} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$
$$M_{32}^{(k)} = R_{31}^{(k)} \frac{\partial}{\partial s_1} \frac{1}{r_k} + R_{32}^{(k)} \frac{\partial_k}{\partial n} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{33}^{(k)} = R_{33}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k}, \quad M_{36}^{(k)} = R_{36}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k},$$
$$M_{34}^{(k)} = -R_{34}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k} + R_{35}^{(k)} \frac{\partial_k}{\partial n} \frac{\partial^2 \Phi_k}{\partial x_1 x_3},$$
$$M_{35}^{(k)} = R_{34}^{(k)} \frac{\partial}{\partial s_1} \frac{1}{r_k} + R_{35}^{(k)} \frac{\partial_k}{\partial n} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$

$$M_{41}^{(k)} = R_{41}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} + R_{42}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_1 x_3} - q_{41}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{42}^{(k)} = -\mu_{42}^{(k)} \frac{\partial}{\partial s_3} \frac{1}{r_k} - q_{41}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2^2} + R_{42}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{43}^{(k)} = R_{43}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k} - q_{43}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{44}^{(k)} = R_{44}^{(k)} \frac{\partial_k}{\partial n} \frac{1}{r_k} + R_{45}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_1 x_3} - q_{44}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_1 x_2},$$
$$M_{45}^{(k)} = -\mu_{45}^{(k)} \frac{\partial}{\partial s_3} \frac{1}{r_k} - q_{44}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2^2} + R_{45}^{(k)} \frac{\partial}{\partial s_2} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$
$$M_{46}^{(k)} = R_{46}^{(k)} \frac{\partial}{\partial s_2} \frac{1}{r_k} - q_{46}^{(k)} \frac{\partial}{\partial s_3} \frac{\partial^2 \Phi_k}{\partial x_2 x_3},$$

$$
\begin{aligned}
M_{51}^{(k)} &= \mu_{42}^{(k)}\frac{\partial}{\partial s_3}\frac{1}{r_k}+q_{41}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1^2}-R_{41}^{(k)}\frac{\partial}{\partial s_1}\frac{\partial^2\Phi_k}{\partial x_1 x_3},\\
M_{52}^{(k)} &= \mu_{41}^{(k)}\frac{\partial_k}{\partial n}\frac{1}{r_k}-R_{41}^{(k)}\frac{\partial}{\partial s_1}\frac{\partial^2\Phi_k}{\partial x_2 x_3}+q_{41}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1 x_2},\\
M_{53}^{(k)} &= -R_{43}^{(k)}\frac{\partial}{\partial s_1}\frac{1}{r_k}+q_{43}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1 x_3},\\
M_{54}^{(k)} &= \mu_{45}^{(k)}\frac{\partial}{\partial s_3}\frac{1}{r_k}+q_{44}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1^2}-R_{44}^{(k)}\frac{\partial}{\partial s_1}\frac{\partial^2\Phi_k}{\partial x_1 x_3},\\
M_{55}^{(k)} &= \mu_{44}^{(k)}\frac{\partial_k}{\partial n}\frac{1}{r_k}-R_{44}^{(k)}\frac{\partial}{\partial s_1}\frac{\partial^2\Phi_k}{\partial x_2 x_3}+q_{44}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1 x_2},\\
M_{56}^{(k)} &= -R_{46}^{(k)}\frac{\partial}{\partial s_1}\frac{1}{r_k}+q_{46}^{(k)}\frac{\partial}{\partial s_3}\frac{\partial^2\Phi_k}{\partial x_1 x_3},
\end{aligned}
$$

$$
\begin{aligned}
M_{61}^{(k)} &= -R_{61}^{(k)}\frac{\partial}{\partial s_2}\frac{1}{r_k}+R_{62}^{(k)}\frac{\partial_k}{\partial n}\frac{\partial^2\Phi_k}{\partial x_1 x_3},\\
M_{62}^{(k)} &= R_{61}^{(k)}\frac{\partial}{\partial s_1}\frac{1}{r_k}+R_{62}^{(k)}\frac{\partial_k}{\partial n}\frac{\partial^2\Phi_k}{\partial x_2 x_3},\\
M_{63}^{(k)} &= R_{63}^{(k)}\frac{\partial k}{\partial n}\frac{1}{r_k},\quad M_{66}^{(k)} = R_{66}^{(k)}\frac{\partial_k}{\partial n}\frac{1}{r_k},\\
M_{64}^{(k)} &= -R_{64}^{(k)}\frac{\partial}{\partial s_2}\frac{1}{r_k}+R_{65}^{(k)}\frac{\partial_k}{\partial n}\frac{\partial^2\Phi_k}{\partial x_1 x_3},\\
M_{65}^{(k)} &= R_{64}^{(k)}\frac{\partial}{\partial s_1}\frac{1}{r_k}+R_{65}^{(k)}\frac{\partial_k}{\partial n}\frac{\partial^2\Phi_k}{\partial x_2 x_3},
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial_k}{\partial n} &= n_1\frac{\partial}{\partial y_1}+n_2\frac{\partial}{\partial y_2}+n_3 a_k\frac{\partial}{\partial y_3},\\
\frac{\partial}{\partial s_1} &= n_2\frac{\partial}{\partial y_3}-n_3\frac{\partial}{\partial y_2},\\
\frac{\partial}{\partial s_2} &= n_3\frac{\partial}{\partial y_1}-n_1\frac{\partial}{\partial y_3},\\
\frac{\partial}{\partial s_3} &= n_1\frac{\partial}{\partial y_2}-n_2\frac{\partial}{\partial y_1}.
\end{aligned}
$$

The coefficients $R_{pq}^{(k)},...,\mu_{pq}^{(k)}$ are constants and they are expressed by

Hook's constants

$$R_{11}^{(k)}=c_{66}^{(1)}A_{11}^{(k)}+c_{66}^{(3)}A_{41}^{(k)},$$
$$R_{12}^{(k)}=\delta_0^{(1)}A_{13}^{(k)}+\delta_0^{(3)}A_{61}^{(k)}+c_{44}^{(1)}A_{12}^{(k)}+c_{44}^{(3)}A_{42}^{(k)},$$
$$q_{12}^{(k)}=(\gamma_0^{(1)}+c_{66}^{(1)})A_{12}^{(k)}+(\gamma_0^{(3)}+c_{66}^{(3)})A_{42}^{(k)},$$
$$q_{11}^{(k)}=\gamma_0^{(1)}A_{11}^{(k)}+\gamma_0^{(3)}A_{41}^{(k)},$$
$$R_{13}^{(k)}=\delta_0^{(1)}A_{33}^{(k)}+\delta_0^{(3)}A_{63}^{(k)}+c_{44}^{(1)}A_{13}^{(k)}+c_{44}^{(3)}A_{43}^{(k)},$$
$$q_{13}^{(k)}=(\gamma_0^{(1)}+c_{66}^{(1)})A_{13}^{(k)}+$$
$$(\gamma_0^{(3)}+c_{66}^{(3)})A_{43}^{(k)},R_{14}^{(k)}=c_{66}^{(1)}A_{41}^{(k)}+c_{66}^{(3)}A_{44}^{(k)},$$
$$R_{24}^{(k)}=\delta_0^{(1)}A_{43}^{(k)}+$$
$$\delta_0^{(3)}A_{46}^{(k)}+c_{44}^{(1)}A_{42}^{(k)}+c_{44}^{(3)}A_{45}^{(k)},$$
$$q_{14}^{(k)}=(\gamma_0^{(1)}+c_{66}^{(1)})A_{42}^{(k)}+(\gamma_0^{(3)}+c_{66}^{(3)})A_{45}^{(k)},$$
$$R_{15}^{(k)}=\gamma_0^{(1)}A_{41}^{(k)}+\gamma_0^{(3)}A_{44}^{(k)},$$
$$q_{16}^{(k)}=(\gamma_0^{(1)}+c_{66}^{(1)})A_{61}^{(k)}+(\gamma_0^{(3)}+c_{66}^{(3)})A_{46}^{(k)},$$
$$R_{16}^{(k)}=\delta_0^{(1)}A_{63}^{(k)}+\delta_0^{(3)}A_{66}^{(k)}+c_{44}^{(1)}A_{61}^{(k)}+c_{44}^{(3)}A_{46}^{(k)},$$

$$R_{31}^{(k)}=\delta_0^{(1)}A_{11}^{(k)}+\delta_0^{(3)}A_{41}^{(k)},$$
$$R_{32}^{(k)}=\delta_0^{(1)}A_{12}^{(k)}+\delta_0^{(3)}A_{42}^{(k)}+c_{44}^{(1)}A_{13}^{(k)}+c_{44}^{(3)}A_{61}^{(k)},$$
$$R_{33}^{(k)}=\delta_0^{(1)}A_{13}^{(k)}+\delta_0^{(3)}A_{43}^{(k)}+c_{44}^{(3)}A_{63}^{(k)}+c_{44}^{(1)}A_{33}^{(k)},$$
$$R_{34}^{(k)}=\delta_0^{(1)}A_{41}^{(k)}+\delta_0^{(3)}A_{44}^{(k)}$$
$$R_{35}^{(k)}=\delta_0^{(1)}A_{42}^{(k)}+\delta_0^{(3)}A_{45}^{(k)}+c_{44}^{(1)}A_{43}^{(k)}+c_{44}^{(3)}A_{46}^{(k)},$$
$$R_{36}^{(k)}=\delta_0^{(1)}A_{61}^{(k)}+\delta_0^{(3)}A_{46}^{(k)}+$$
$$c_{44}^{(1)}A_{63}^{(k)}+c_{44}^{(3)}A_{66}^{(k)},R_{41}^{(k)}=c_{66}^{(3)}A_{11}^{(k)}+c_{66}^{(2)}A_{41}^{(k)},$$
$$R_{42}^{(k)}=\delta_0^{(3)}A_{13}^{(k)}+\delta_0^{(2)}A_{61}^{(k)}+$$
$$c_{44}^{(3)}A_{12}^{(k)}+c_{44}^{(2)}A_{42}^{(k)},$$
$$q_{41}^{(k)}=(\gamma_0^{(3)}+c_{66}^{(3)})A_{12}^{(k)}+(\gamma_0^{(2)}+c_{66}^{(2)})A_{42}^{(k)},$$
$$q_{44}^{(k)}=(\gamma_0^{(3)}+c_{66}^{(3)})A_{42}^{(k)}+(\gamma_0^{(2)}+c_{66}^{(2)})A_{45}^{(k)},$$
$$\mu_{42}^{(k)}=\gamma_0^{(3)}A_{11}^{(k)}+\gamma_0^{(2)}+A_{41}^{(k)},$$
$$q_{43}^{(k)}=(\gamma_0^{(3)}+c_{66}^{(3)})A_{13}^{(k)}+(\gamma_0^{(2)}+c_{66}^{(2)})A_{43}^{(k)},$$
$$R_{43}^{(k)}=\delta_0^{(3)}A_{33}^{(k)}+\delta_0^{(2)}A_{63}^{(k)}+c_{44}^{(3)}A_{13}^{(k)}+c_{44}^{(2)}A_{43}^{(k)},$$
$$R_{44}^{(k)}=c_{66}^{(3)}A_{41}^{(k)}+c_{66}^{(2)}A_{44}^{(k)},$$

$$\begin{aligned}
R_{45}^{(k)} &= \delta_0^{(3)} A_{43}^{(k)} + \delta_0^{(2)} A_{46}^{(k)} + c_{44}^{(3)} A_{42}^{(k)} + c_{44}^{(2)} A_{45}^{(k)},\\
\mu_{45}^{(k)} &= \gamma_0^{(3)} A_{41}^{(k)} + \gamma_0^{(2)} A_{44}^{(k)},\\
R_{46}^{(k)} &= \delta_0^{(3)} A_{63}^{(k)} + \delta_0^{(2)} A_{66}^{(k)} + c_{44}^{(3)} A_{61}^{(k)} + c_{44}^{(2)} A_{46}^{(k)},\\
q_{46}^{(k)} &= (\gamma_0^{(3)} + c_{66}^{(3)}) A_{61}^{(k)} + (\gamma_0^{(2)} + c_{66}^{(2)}) A_{46}^{(k)},\\
\mu_{41}^{(k)} &= c_{66}^{(3)} A_{11}^{(k)} + c_{66}^{(2)} A_{41}^{(k)},\\
\mu_{44}^{(k)} &= c_{66}^{(3)} A_{41}^{(k)} + c_{66}^{(2)} A_{44}^{(k)},\\
R_{62}^{(k)} &= \delta_0^{(3)} A_{12}^{(k)} + \delta_0^{(2)} A_{42}^{(k)} + c_{44}^{(3)} A_{13}^{(k)} + c_{44}^{(2)} A_{61}^{(k)},\\
R_{61}^{(k)} &= \delta_0^{(3)} A_{11}^{(k)} + \delta_0^{(2)} A_{41}^{(k)},\\
R_{63}^{(k)} &= \delta_0^{(3)} A_{13}^{(k)} + \delta_0^{(2)} A_{43}^{(k)} + c_{44}^{(3)} A_{33}^{(k)} + c_{44}^{(2)} A_{63}^{(k)},\\
R_{64}^{(k)} &= \delta_0^{(3)} A_{41}^{(k)} + \delta_0^{(2)} A_{44}^{(k)},\\
R_{65}^{(k)} &= \delta_0^{(3)} A_{42}^{(k)} + \delta_0^{(2)} A_{45}^{(k)} + c_{44}^{(3)} A_{43}^{(k)} + c_{44}^{(2)} A_{46}^{(k)},\\
R_{66}^{(k)} &= \delta_0^{(3)} A_{61}^{(k)} + \delta_0^{(2)} A_{46}^{(k)} + c_{44}^{(3)} A_{63}^{(k)} + c_{44}^{(2)} A_{66}^{(k)},
\end{aligned}$$

Let us introduce the following definitions.

Definition 2. The vector function defined by the equality

$$\varphi(x) = \frac{1}{2\pi} \iint_S \Gamma(x-y) g(y) ds,$$

where $\Gamma(x-y)$ is given by (3) and $g(y)$ is continuous vector function, S is a closed surface, is called a simple layer potential.

Definition 3.The vector function defined by the equality

$$\psi(x) = \frac{1}{2\pi} \iint_S P^*(y-x) g(y) ds,$$

where $P^*(y-x)$ is given by (4) and $g(y)$ is continuous vector function, S is a closed surface, is called a double layer potential.

It is evident that all potentials are solutions of the system (1) both in the domains D^+ and D^-. In the case when the point x tends to any point of the boundary z (from inside or from outside), the potentials possess some discontinuity or continuity properties. The direction of the outer (with respect to D^+ normal is assumed to be positive).

Theorem 2.If $S \in L_1(\alpha)$ and $g(y) \in C^{0,\beta}(S)$,$0 < \beta < \alpha \leq 1$,then the function $\varphi(x) \in C^{0,\beta}(S)$ and

$$[P(\partial x, n)u]^{\pm} = \mp g(t) + \frac{1}{2\pi} \iint_S P(\partial x, n) \Gamma(x-y) g(y) ds, \qquad (5)$$

Theorem 3.If $S \in L_1(\alpha)$ and $g(y) \in C^{1,\beta}(S)$,$0 < \beta < \alpha \leq 1$,then the function $\psi(t) \in C^{1,\beta}(S)$ and

$$[\psi(t)]^{\pm} = \mp g(t) + \frac{1}{2\pi} \iint_S P^*(t-y)g(y)ds, \tag{6}$$

where the symbol $[.]^{\pm}$ denote limits on S from $D^{\pm}$.

The integrals in (5)-(6) are singular and they are integrable in the sense of the principal value.

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