ON THE ASYMPTOTIC METHOD OF SOLUTION OF ONE CLASS OF ASTROPHYSIC PROBLEMS

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Abstract

This work proposes an asymptotic method of solution for a system of nonlinear nonhomogeneous equations of one class of mixed problems with an unknown external boundary in the domain. The problem of a central explosion of a nonhomogeneous gas sphere (star) that is balanced in its own gravitating field is discussed as the first test problem. The problem of a central explosion of homogeneous gas sphere collapsing at zero pressure and folloved by a thermonuclear detonation is discussed here.

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To resolve a number of astrophysic problems one has to investigate the dynamics of the gas bodies that interact with a gravitating field. It is clear that the conceptions of astrophysic problems investigation can be based on the statement and solution of a number of gas motion dynamic problems. These problems are regarded as theoretic models that include important peculiarities of the motion and evolution of stars.

The methods, devices and considerations of modern theoretical gas dynamics and aerodynamics must be used for the construction and investigation of such models and the statement and solution of corresponding mechanic problems related to astrophysic ones.

Numerical modelling of problems of processes that take place in the nuclei of stars has been widely used for establishing the phenomena of supernovae flashes [1- 3].Main attention is paid to physical processes related to thermonuclear reactions and spreading of neutrinous radiation. Less attention is paid to the gas dynamics as a whole. It was considered for a long time that neutrinous, formed in electric seizure and radiated by the central nucleus of a star must transfer a radial component of its impulse to the external layers of the star, thus causing an supernovae explosion. However, we had to reject such mechanism of explosion after the discovery

(theoretically and experimentally) in weak interaction of neutral currents that lead to keeping of neutrinous in the star nucleus [4].

This work proposes an asymptotic method of solution for a system of nonlinear nonhomogeneous equations of one class of mixed problems with an unknown external boundary in the domain. The system of equations describes an adiabatic spherical and symmetrical motion of a gravitating gas, while a moving shock or detonation wave (a spherical surface where the solution undergoes the first kind of discontinuity) is the external boundary of the domain.

The problem of a central explosion of a nonhomogeneous gas sphere (star) that is balanced in its own gravitating field is discussed as the first test problem. The asymptotic method of a thin shock layer is used for the solution. Analytical formulae of the first two approximations of the solution are found . Gas dispersion in the vacuum after the shock wave body comes out on the surface is also described.

The problem of a central explosion ($t_0 < 0$ is the instance of explosion) of a homogeneous gas sphere collapsing at zero pressure and followed by a thermonuclear detonation is discussed here. The first two approximations for the motion law and the thermodynamic characteristics of the medium are calculated. The analysis of the solution shows that beginning from a certain instance a disseminating detonating wave begins to be brought to the centre.

1. Let us discuss the equations of the adiabatic spherical and symmetrical motion of a gas that are written in Lagrange's form [5]

$$\begin{cases} \dfrac{\partial^2 r}{\partial t^2} + 4\pi r^2 \dfrac{\partial p}{\partial m} + \dfrac{km}{r^2} = 0, \\ p = (\gamma - 1) f(m) \rho^\gamma, \\ \rho = \left[4\pi r^2 \dfrac{\partial r}{\partial m} \right]^{-1}. \end{cases} \tag{1}$$

Here m is the $r(m,t)$ radius sphere mass, k is the gravitation constant, γ is the adiabatic indicator, $f(m)$ is the function connected with the distribution of entropy by Lagrange's m coordinate. $r = r(m,t)$ is medium motion law, $p(m,t)$ is medium pressure, $\rho(m,t)$ is medium density.

The first equation of system (1) is the motion equation, the second equation is the adiabation equation, the third equation is the mass continuity equation. $r(m,t), p(m,t), \rho(m,t)$ functions are unknown.

The integral equation of the energy of the gas layer situated between the $m = 0$ and $m = M(t)$ surfaces is as follows:

$$T+U+V=$$

$$=E+\int\limits_{t_0}^{t}\left[\overset{\bullet}{M}\left(\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2+\frac{p}{(\gamma-1)\rho}-\frac{kM}{R}+Q\right)-4\pi R^2\frac{\partial r}{\partial t}p\right]_1 d\tau, \quad (2)$$

$$T=\frac{1}{2}\int\limits_0^M\left(\frac{\partial r}{\partial t}\right)^2 dm,\ U=\frac{1}{\gamma_2-1}\int\limits_0^M\frac{pdm}{\rho},$$

$$V=-k\int\limits_0^M\frac{mdm}{r},\qquad \overset{\bullet}{M}\equiv\frac{dM\,(t)}{dt}.$$

Here T,U,V are the kinetic, inner and potential (gravitation) energies of the gas, Q is the energy excreted during the burning of a gas mass unit on the $m=M(t)$ surface, E is the explosion energy, $m=M(t)$ is the law of motion shock $(Q=0)$ or detonation $(Q\neq 0)$ wave with gas mass, $R=r(M(t),t)$ is the radius of a shock or detonation wave. 1,2 indices denote correspondingly the gas position in front of and behind the wave.

Boundary conditions on the $m=M(t)$ discontinuity are as follows

$$\left[\rho\left(\overset{\bullet}{R}-\frac{\partial r}{\partial t}\right)\right]_1^2=0,\quad \left[p+\rho\left(\frac{\partial r}{\partial t}-\overset{\bullet}{R}\right)^2\right]_1^2=0, \quad (3)$$

$$\left[\frac{1}{2}\left(\frac{\partial r}{\partial t}-\overset{\bullet}{R}\right)^2+\frac{\gamma}{\gamma-1}\frac{p}{\rho}\right]_1^2=Q,\qquad [\varphi]_1^2\equiv\varphi_2-\varphi_1.$$

If boundary conditions (3) are solved with respect to parameters of the gas behind the wave we get the following

$$\rho_2=\frac{\gamma_2+1}{\gamma_2-1}\rho_1\left[1+\frac{1}{\gamma_2-1}\left(\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}+1-g\right)\right]^{-1},\quad a_1^2=\frac{\gamma_1 p_1}{\rho_1},$$

$$p_2=\frac{1}{\gamma_2+1}\left[p_1+\rho_1\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2+\rho_1\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2 g\right], \quad (4)$$

$$\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_2=\frac{\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1}{\gamma_2+1}\left[\gamma_2+\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}-g\right],$$

$$g=\left[\left(1-\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right)^2+\frac{2(\gamma_2+1)\,(\gamma_1-\gamma_2)\,a_1^2}{\gamma_1(\gamma_1-1)\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}-\frac{2(\gamma_2^2-1)Q}{\left(\overset{\bullet}{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right]^{\frac{1}{2}}.$$

Besides, the continuity of Euler's and Lagrange's variables ought to be taken into account.

$$[r]_1^2=0,\qquad [m]_1^2=0. \tag{5}$$

In fact, we get a mixed problem for the system (1) of nonlinear, nonhomogeneous equations, where the $r(m,t),p(m,t),\rho(m,t)$ functions are unknown.

Initial conditions ($t=t_0$, phone) determine the initial state of a gas sphere and are the exact $r_1(m,t),p_1(m,t),\rho_1(m,t)$ solutions of the (1) system.

Thus, the mixed problem is considered in the domain Ω: $\Omega=\{t\in(t_0,t_*),$ $m\in(0,M(t))\}$, where t_0 is the moment of explosion, t_* is the moment of time when the wave comes out on the surface of the body, or the moment of collapse.

Boundary conditions on the external unknown boundary $m=M(t)$ are like (4),(5), and $r(m,t)=0$ when $m=0$.

2. For the most of the gases $\varepsilon=\frac{\gamma_2-1}{\gamma_2+1}$ is a small parameter. Besides, it is included in (1) as a system of equations, in the boundary conditions (4) and in the integral equation (2), whence the $R(t)$ law of wave motion is established.

Thus, the analysis of the system of equations and boundary conditions makes it clear that the solution can be sought for behind the wave with respect to the small parameter ε as a kind of several decompositions.

But the decomposition becomes irregular near the symmetry centre ($m=0$) [5-7]. For the solution regularization in this domain we use the method of consecutive approximation the essence of which is that the members of the series area ε maintained in the zero approximation $\rho_0(m,t)$ of the expression $\rho\,(m,t)$. Then the first approximation for the medium motion and wave laws is found from the continuity equation by means of the boundary condition $r(0,t)=0$ and the zero approximation. The first approximations of the $p\,(m,t)$ and $\rho(m,t)$ functions will be found in the rest of the system (1) of equations.

The described method makes it possible to solve quite a wide class of mixed problems of a system of equations (1). It is natural that the choice of decomposition depends on the initial state of the gas sphere (the exact solution before the wave) and on the energy of the explosion.

3. Let us discuss the problem of the central explosion at the $t = 0$ moment of a homogeneous gas sphere (star) balanced in its own gravitation field as the first test problem.

Thus, the exact solution of the system of equations (1) that corresponds to the homogeneous gas sphere balanced in its own gravitation field $\left(\left(\frac{\partial r}{\partial t}\right)_1 = 0\right)$ is taken as an initial condition (phone). The gravitation constant, the sphere surface density and the sphere radius are taken as main units of dimension

$$r = \left(\frac{m}{4\pi\alpha}\right)^{\alpha}, \quad p = \frac{2\pi\alpha}{1-\omega}\left(1 - r^{2(1-\omega)}\right), \quad \rho = r^{-\omega}, \quad \alpha \equiv \frac{1}{3-\omega}. \tag{6}$$

It arises from (6) that the pressure p is equal to zero on the sphere surface ($r = 1$), i.e. the $r = 1$sphere is a boundary between a star and the interstellar space, as the density of the interstellar gas $\rho \sim 10^{-24} gr/sm^3$.

Boundedness conditions of the body mass

$$M_* = 4\pi \int_0^1 \rho r^2 dr < \infty$$

and the initial energy

$$|V| = \left| -\int_0^{M_*} \frac{mdm}{r} \right| < \infty, \qquad U = \frac{1}{\gamma - 1} \int_0^{M_*} \frac{pdm}{\rho} < \infty,$$

give the following restriction for the index ω

$$\omega \in [0, 1) \cup \left(1, \frac{5}{2}\right).$$

We get the mixed problem where the system of equations is like (1), the energy integral equation - (2) ($t_0 = 0, \gamma_1 = \gamma_2 = \gamma,\ Q = 0$), the boundary conditions - (4) ($\gamma_1 = \gamma_2 = \gamma,\ Q = 0$), the initial conditions - (6).

Let us introduce a small parameter $\varepsilon = \frac{\gamma-1}{\gamma+1}$.

The analysis of the energy integral equation and the condition of the existence of a strong shock wave before the moment of the body coming out on the surface leads us to the condition $E = E_0/\varepsilon^2, E_0 = \underset{=}{O}(1)$.

Besides, the time of shock wave motion before the sphere comes out on the surface will be of $\sqrt{\varepsilon}$series. That's why for the sake of simplicity we can additionally protract the time $\tau = t/\sqrt{\varepsilon}$.

The analysis of the system of equations and the boundary conditions has shown us that the solution behind the shock wave can be sought for as the following decomposition:

$$r = R_0(\tau) + \varepsilon H(m,\tau) + \cdots,\ R(\tau) = R_0(\tau) + \varepsilon R_1(\tau) + \cdots, \qquad (7)$$

$$p = \frac{p_0(m,\tau)}{\varepsilon} + p_1(m,\tau) + \cdots, \qquad \rho = \frac{\rho_0(m,\tau)}{\varepsilon} + \rho_1(m,\tau) + \cdots.$$

Using the mentioned regularization method the problem solution in zero approximation will be written down as follows:

$$p_0(m,\tau) = R_0^{-\omega} R_0^{'2} + \frac{R_0^{''}}{4\pi R_0^2}(M_0 - m),\ \ M_0 = 4\pi\alpha R_0^{1/\alpha}, \qquad (8)$$

$$\rho_0(m,\tau) = p_0^{1/\gamma}\left[1 + \frac{a_1^2(m)}{R_0^{'2}\,(T_0(m))}\right]^{-1}\left[R_0^{'2}\,(T_0(m))\right]^{-1/\gamma}\left[R_0\,(T_0(m))\right]^{\frac{-\omega(\gamma-1)}{\gamma}},$$

$$R_0(\tau) = \left[\frac{3(5-\omega)^2 E_0 \tau^2}{4\pi}\right]^{\frac{1}{5-\omega}},\quad R_0^{'} \equiv \frac{dR_0(\tau)}{d\tau},$$

$$a_1^2(m) = \frac{2\pi\alpha}{1-\omega}\left(\frac{m}{4\pi\alpha}\right)^{\omega\alpha}\left[1 - \left(\frac{m}{4\pi\alpha}\right)^{2(1-\omega)\alpha}\right],$$

$$T_0(m) = \left[\frac{4\pi}{3(5-\omega)^2 E_0}\right]^{\frac{1}{2}}\left(\frac{m}{4\pi\alpha}\right)^{\frac{(5-\omega)}{2}\alpha}.$$

Here $T_0(m)$ is the moment of time where the shock wave passes the particle with Lagrange's m coordinate.

In the next approximation we shall get the following from the continuity equation:

$$\frac{4\pi}{3}r^3 = \frac{4\pi}{3}R^3 - \int_m^M \frac{\varepsilon dm}{\rho_0(m,\tau)}. \qquad (9)$$

To establish the first approximation $R_1(\tau)$ of the shock wave motion law we shall use the boundary condition in the centre: $r = 0$ when $m = 0$.

Detailed calculations in correlation (9) with the use of (8) will give us gas and shock wave motion laws:

$$r = R_0\xi^{2\varepsilon\alpha}\left(1-\frac{2\varepsilon}{3}\ln 2\,(1+\xi)^{3\alpha}\right)+\frac{R_0\pi\varepsilon\alpha}{3E_0\,(1-\omega)}\left[\left(\frac{1}{4\pi\alpha}\right)^{\omega\alpha}M_0^{3\alpha}G\,(\xi,\omega)-\right.$$

$$\left.-\left(\frac{1}{4\pi\alpha}\right)^{1-\alpha}M_0^{2-\alpha}F\,(\xi,\omega)\right]+\underset{=}{O}\left(\varepsilon^2\right), \tag{10}$$

$$R=\left(\frac{M}{4\pi\alpha}\right)^{\alpha}=R_0\left\{1+\frac{\varepsilon}{3}\left[-2(6-\omega)\alpha\ln 2+\frac{\pi\alpha}{E_0\,(1-\omega)}\times\right.\right.$$

$$\left.\left.\times\left(\left(\frac{1}{4\pi\alpha}\right)^{\omega\alpha}M_0^{3\alpha}G\,(1,\omega)-\left(\frac{1}{4\pi\alpha}\right)^{1-\alpha}M_0^{2-\alpha}F\,(1,\omega)\right)\right]\right\}+\underset{=}{O}\left(\varepsilon^2\right),$$

$$\xi\equiv\frac{m}{M_0},\quad G\,(\xi,\omega)\equiv\int\limits_0^{\xi}\frac{y^{\omega\alpha}}{1+y}dy,\ \ F\,(\xi,\omega)\equiv\int\limits_0^{\xi}\frac{y^{1-\alpha}}{1+y}dy.$$

It is noteworthy that G,F integrals for the rational ω are axpressed in elementary functions (integrals from differential binomials).

Using the law of motion (10) of the gas found behind the shock wave we shall calculate $p_1(m,\tau)$ da $\rho_1(m,\tau)$ from the motion and adiabation equations. The construction of the first approximation is completely determined by

$$p_1(m,\tau)=R_0^{-\omega}R_0'^2\left(\frac{2R_1'}{R_0'}-\frac{\omega R_1}{R_0}-1\right)+\frac{R_0''R_0^{1-\omega}}{3}\left(-2(6-\omega)\alpha\ln 2+\Lambda\right)+I,$$

$$\Lambda\equiv\frac{\pi\alpha}{E_0\,(1-\omega)}\left(\beta^{\omega\alpha}M_0^{3\alpha}G\,(1,\omega)-\beta^{1-\alpha}M_0^{2-\alpha}F\,(1,\omega)\right),\quad \beta\equiv\frac{1}{4\pi\alpha},$$

$$I\equiv\frac{1}{4\pi R_0^2}\int\limits_m^{M_0}\left(\frac{\partial^2 H}{\partial\tau^2}-\frac{2HR_0''}{R_0}+\frac{m}{R_0^2}\right)dm,$$

$$\rho_1=\rho_0\left[\frac{p_1}{p_0}+\frac{R_0'^2(T_0)}{R_0'^2(T_0)+a_1^2(m)}\left(1-\frac{2R_1'(T_0)+2R_0''(T_0)T_1(m)}{R_0'(T_0)}\right)\right],$$

$$H(m,\tau)=\frac{R_0}{\varepsilon}\left(\xi^{2\varepsilon\alpha}-1\right)-\frac{2}{3}R_0\xi^{2\varepsilon\alpha}\ln 2(1+\xi)^{3\alpha}+$$

$$+\frac{R_0\pi\alpha}{3E_0(1-\omega)}\left[\beta^{\omega\alpha}M_0^{3\alpha}G\left(\xi,\omega\right)-\beta^{1-\alpha}M_0^{2-\alpha}F\left(\xi,\omega\right)\right],$$

$$R_1=\frac{R_0}{3}\left(-2(6-\omega)\alpha\ln 2+\Lambda\right),$$

$$T_1(m)=\frac{5-\omega}{6}T_o(m)\left[2(6-\omega)\alpha\ln 2-\right.$$

$$\left.-\frac{\pi\alpha}{E_0\left(1-\omega\right)}\left(\beta^{\omega\alpha}m^{3\alpha}G\left(1,\omega\right)-\beta^{1-\alpha}m^{2-\alpha}F\left(1,\omega\right)\right)\right].$$

4. The shock wave comes out on the body surface at the t_* moment of time that is established from the condition $R(t_*)=1$ and calculated from (7), (8), (10)

$$t_*=\frac{\sqrt{\varepsilon}}{5-\omega}(\frac{4\pi}{3E_0})^{1/2}$$

that will cause the decomposition of the free discontinuity followed by gas expansion in the vacuum.

Let us make it clear that the discussed small parameter asymptotic method can also be used for the description of the process of the basic gas mass adiabatic dilation in the vacuum.

The gas motion equations are like (1) where function $f(m)$ is determined from the solution before the appearance of the shock wave (adiabatic dilation) on the sphere surface:

$$f(m)=6\varepsilon\alpha E\beta^{2\varepsilon\omega\alpha}m^{-1+2\varepsilon\omega\alpha}\left[1+\frac{\pi m\left((\beta m)^{\omega\alpha}-(\beta m)^{1-\alpha}\right)}{6E_0(1-\omega)}\right].\qquad(11)$$

We get the following from the motion equation in zero approximation and the boundary condition $p(M_*,t)=0$

$$r=R(t),p=\frac{\ddot{R}}{4\pi R^2}(M_*-m)+\frac{1}{8\pi R^4}(M_*^2-m^2),\qquad(12)$$

where $M_*=4\pi\alpha$ is the body mass.

The function $R(t)$ is determined from the energy integral equation. Besides, it is clear that solution (8) is used as initial conditions when $t=t_*$

$$\int\limits_1^{R(t)}\Phi^{-1/2}(y)dy=t-t_*,\qquad(13)$$

$$\Phi(y) = \frac{2E_*}{M_*} + \frac{M_*}{y} + \frac{2E_*}{M_*}\left(\frac{6\varepsilon\alpha E}{E_*} - 1 - \frac{M_*^2}{2E_*}\right) y^{-6\varepsilon},$$

$$E_* = E + \frac{16\pi^2\alpha}{3(\gamma-1)(5-2\omega)} - \frac{16\pi^2\alpha}{5-2\omega}.$$

It arises from (2), (12), (13) that the following correlations are true for the kinetic, inner and potential (gravitation) energies of the gas sphere dilatable in the vacuum

$$\frac{T}{E_*} = 1 - \left(1 - \frac{6\varepsilon\alpha E}{E_*}\right)\frac{1}{R^{6\varepsilon}} + \frac{M_*^2}{2E_*}\left(\frac{1}{R} - \frac{1}{R^{6\varepsilon}}\right), \tag{14}$$

$$\frac{U}{E_*} = \left(1 - \frac{6\varepsilon\alpha E}{E_*} + \frac{M_*^2}{2E_*}\right)\frac{1}{R^{6\varepsilon}}, \qquad \frac{V}{E_*} = -\frac{M_*^2}{2E_* R}.$$

The analysis of (14) makes it clear that only the inner energy of the body is important at the initial stage of dilation $R \geq 1$(at the distances of the radius series of a star). But at $R_{cr} \approx 2^{1/(6\varepsilon)}$ distances of the kinetic and inner energies of gas sphere are already comparative, and when $R \geq R_{**} \approx \varepsilon^{-1/(.6\varepsilon)}$ it is the kinetic energy that basically contributes to the energy equation. During the whole stage of dilation the gravitating energy, or it is of ε^2 series.

We get the following from (1) and (11)

$$\frac{4\pi}{3}\frac{\partial r^3}{\partial m} = \frac{1}{\rho} = \left[\frac{12E_0\alpha}{p}\rho^{2\varepsilon\omega\alpha} m^{-1+2\varepsilon\omega\alpha}\left(1 + \frac{\pi m\left((\beta m)^{\omega\alpha} - (\beta m)^{1-\alpha}\right)}{6E_0(1-\omega)}\right)\right]^{1-2\varepsilon}. \tag{15}$$

The boundary condition $r = 0$ when taking into account $m = 0$ and (12) correlations, the integration of (15) gives the following

$$r = (3\alpha)^{1/3} R(t)\left[\frac{1}{3\alpha}\left(\frac{m}{M_*}\right)^{6\varepsilon\alpha} + 1 - \left(1 - \frac{m}{M_*}\right)^{2\varepsilon}\right]^{1/3}. \tag{16}$$

Besides, the law of dilatable gas boundary motion is determined by the formula

$$R_*(t) = R(t)\left[\alpha(6-\omega)\right]^{1/3}.$$

Including the solution found for the medium motion law (13), (16) into the system (1), and using (11) we can find the next approximations for density and pressure distribution.

The analysis of the obtained medium motion law (16) makes it clear that almost the whole mass of the dilating gas will be gathered in the thin layer with the centre $r \approx R(t)$ and with $\varepsilon \ln \varepsilon$ series relative to thickness. When $R \geq R_{**}$ it arises from (13) that

$$R(t) = \left(\frac{2E}{M_*}\right)^{1/2} t \left(1 - \frac{1}{2} t^{-6\varepsilon} + \bar{\bar{O}}\left(t^{-6\varepsilon}\right)\right),$$

$$t \geq t_{**} = \left(\frac{2\pi\alpha}{E_0}\right)^{1/2} \varepsilon^{\frac{6\varepsilon-1}{6\varepsilon}}.$$

When the initial density distribution is like (6) the solution obtained shows us that the thin shock layer is not destroyed until the shock wave comes out the body surface (the basic mass of the gas accumulated in the small domain behind the wave). Afterwards, when the gas sphere dilates in the vacuum the basic mass of the gas is also accumulated in the thin layer that is already removed from the boundary of the body.

Thus, the analysis of the obtained solution makes it clear that when the explosion energy is quite big the initially nonhomogeneous self-gravitating gas sphere (star) will be completely dilated in the vacuum (interstellar medium) without central gravitation remainder after the shock wave comes out on its surface. Besides, the boundary of the body moves $[\alpha(6-\omega)]^{1/3}$ times faster than the basic gas mass. The permanent speed gas dilation in the vacuum will be taking place for quite a long time $t \geq t_{**}$.

5. Let us discuss the problem of a central explosion ($t_0 < 0$ is the moment of explosion) followed by a thermonuclear detonation of a homogeneous gas sphere collapsing at zero pressure.

The exact solution of the equations (1) corresponding to the homogeneous parabolic compression (collapse) of dust (gas pressure $p = 0$) is taken as initial data. Besides, the gravitation constant, the moment of explosion and the energy t_0, E are taken as basic dimension units

$$r = \left[\frac{9m(1-\tau)^2}{2}\right]^{1/3}, \quad \rho = \frac{1}{6\pi(1-\tau)^2}, \quad p = 0, \tau = 1 - \frac{t}{t_0}. \tag{17}$$

We obtain a mixed problem in the domain Ω

$$\Omega = \{\tau \in (0,1),\ m \in (0, M(\tau))\},$$

where the system of equations is like (1), the energy integral equation is like (2), the boundary conditions are like (4), the initial conditions are like (17).

Let us introduce a small parameter $\varepsilon = \frac{\gamma_2 - 1}{\gamma_2 + 1}$.

The analysis of the system of equations and the boundary conditions makes it clear that the solution can be sought behind the detonating wave as the following decomposition

$$r = R_0(\tau) + \varepsilon H(m,\tau) + \cdots,\ R(\tau) = R_0(\tau) + \varepsilon R_1(\tau) + \cdots, \tag{18}$$

$$p = p_0(m,\tau) + \varepsilon p_1(m,\tau) + \cdots, \qquad \rho = \frac{\rho_0(m,\tau)}{\varepsilon} + \rho_1(m,\tau) + \cdots.$$

Including the decomposition (18) into the system of equations (1), in the integral equation (2) and the boundary conditions (4), we shall obtain the zero approximation of the problem solution using the regularization method.

$$p_0(m,\tau) = \frac{1}{6\pi(1-\tau)^2}\left(R_0^{/}(\tau) + \frac{2R_0(\tau)}{3(1-\tau)}\right)^2 + \frac{R_0^{//}(\tau)}{4\pi R_0^2(\tau)}\left(M_0 - m\right) +$$

$$+\frac{1}{8\pi R_0^4(\tau)}\left(M_0^2 - m^2\right), \tag{19}$$

$$\rho_0(m,\tau) = p_0^{1/\gamma_2}\left\{\left[6\pi(1-\tau)^2\right]^{\frac{1-\gamma_2}{\gamma_2}}\left(R_0^{/}(\tau) + \frac{2R_0(\tau)}{3(1-\tau)}\right)^{\frac{-2}{\gamma_2}} \times\right.$$

$$\left.\times\left[1 + \frac{2Q}{\left(R_0^{/}(\tau) + \frac{2R_0(\tau)}{3(1-\tau)}\right)^2}\right]^{-1}\right\}\Big|_{\tau=T_0(m)},$$

$$R_0(\tau) = \left[\frac{9M_0(1-\tau)^2}{2}\right]^{1/3},$$

where function $M_0(\tau)$ is the exact solution of Cauchy's special problem

$$3(1-\tau)^2 yy^{''} + (1-\tau)^2 y'^2 - 4(1-\tau)yy' - y^2 = 0, \quad \tau \in (0,1),$$

$$y = y(\tau),\ y(0) = 0,\ \lim_{\tau\to 0_+}\frac{y'(\tau)}{y(\tau)} = +\infty$$

and is as follows:

$$M_0(\tau) = (1-\tau)^{-\frac{\sqrt{17}+1}{8}}\left[1 - (1-\tau)^{\frac{\sqrt{17}}{3}}\right]^{\frac{3}{4}}. \tag{20}$$

Including the (20) into the formula (19), we shall obtain the zero approximation of the radius of the detonation wave and of the medium motion law.

$$R_0(\tau) = \left(\frac{9}{2}\right)^{\frac{1}{3}} (1-\tau)^{\frac{15-\sqrt{17}}{24}} \left[1-(1-\tau)^{\frac{\sqrt{17}}{3}}\right]^{\frac{1}{4}}. \tag{21}$$

In formula (19) $\tau = T_0(m)$ is the moment of time when the detonating wave passes the particle with Lagrange's m coordinate and is determined by the equation

$$m = (1-\tau)^{-\frac{\sqrt{17}+1}{8}} \left[1-(1-\tau)^{\frac{\sqrt{17}}{3}}\right]^{\frac{3}{4}}.$$

We shall obtain the following from the continuity equation in the next approximation

$$\frac{4\pi r^3}{3} = \frac{4\pi R^3}{3} - \int\limits_m^M \frac{\varepsilon dm}{\rho_0(m,\tau)}. \tag{22}$$

We shall use the boundary condition in the centre: $r = 0$ when $m = 0$, to establish the first approximation $R_1(\tau)$ of the detonation wave motion law. We shall obtain the following:

$$R(\tau) = \left[\frac{3}{4\pi} \int\limits_0^M \frac{\varepsilon dm}{\rho_0(m,\tau)}\right]^{\frac{1}{3}}, \tag{23}$$

where

$$M(\tau) = \frac{2R^3}{9(1-\tau)^2}.$$

Thus, taking into account (22), (23) the medium motion law is determined by the formula:

$$r(m,\tau) = \left[\frac{3}{4\pi} \int\limits_0^m \frac{\varepsilon dm}{\rho_0(m,\tau)}\right]^{\frac{1}{3}}. \tag{24}$$

Using the motion law of the gas found behind the detonating wave (24) we shall calculate $p_1\,(m,\tau)$ and $\rho_1\,(m,\tau)$ from the system (1) of equations.

$$p_1(m,\tau)=$$

$$=\frac{2\left(R_0^{/}+\frac{2\overline{R}_0}{3(1-\tau)}\right)\left(R_1^{/}+\frac{2R_1}{3(1-\tau)}\right)-\left(R_0^{/}+\frac{2R_0}{3(1-\tau)}\right)^2\left(1+\frac{2Q}{\left(R_0^{/}+\frac{2R_0}{3(1-\tau)}\right)^2}\right)}{6\pi(1-\tau)^2}+$$

$$+\int_m^{M_0}\left(\frac{1}{4\pi R_0^2}\frac{\partial^2 H}{\partial\tau^2}+\frac{2H}{R_0}\frac{\partial p_0}{\partial m}-\frac{mH}{2\pi R_0^5}\right)dm,$$

$$\rho_1(m,\tau)=$$

$$=\frac{p_0(m,\tau)}{2Q+\left(R_0^{/}+\frac{2R_0}{3(1-\tau)}\right)^2}\left[\frac{2Q}{2Q+\left(R_0^{/}+\frac{2R_0}{3(1-\tau)}\right)^2}\left(\frac{6R_1^{'}(1-\tau)+4R_1}{3R_0^{'}(1-\tau)+2R_0}-1\right)+\right.$$

$$\left.+\frac{p_1(m,\tau)}{p_0(m,\tau)}-2\frac{R_1^{/}+\frac{2R_1}{3(1-\tau)}}{R_0^{/}+\frac{2R_0}{3(1-\tau)}}+1+\frac{2Q}{\left(R_0^{/}+\frac{2R_0}{3(1-\tau)}\right)^2}\right].$$

The following asymptotics are easily obtained from (20), (21) $R_0(\tau)\approx\left(\frac{9}{2}\right)^{\frac{1}{3}}\left(\frac{17}{9}\right)^{\frac{1}{8}}\tau^{\frac{1}{4}}$, when $\tau\to 0_+$ $M_0(\tau)\approx\left(\frac{17}{9}\right)^{\frac{3}{8}}\ \tau^{\frac{3}{4}}$, when $\tau\to 0_+$ $R_0(\tau)\approx\left(\frac{9}{2}\right)^{\frac{1}{3}}(\,1-\tau\,)^{\frac{15-\sqrt{17}}{24}}$, when $\tau\to 1_-$ $M_0(\tau)\approx(1-\tau)^{-\frac{\sqrt{17}+1}{8}}$, when $\tau\to 1_-$

The exact solution $R_0\,(\tau)$ (21) of the detonating wave radius zero approximation makes it clear that from the moment of time τ_{cr}

$$\tau_{cr}=1-\left(\frac{15-\sqrt{17}}{15+\sqrt{17}}\right)^{\frac{3}{\sqrt{17}}}$$

the initially divergent detonation wave begins to drift to the centre and when $\tau=1$ a collapse will take place.

References

1. Colgate S.A., White R.H. *The hydrodynamic behaviour of supernovae explosions*. Astrophys. J., 143 (1966), no 3, 626-681.

2. Nadyozhin D.K. *The collapse of iron-oxygen stars: physical and mathematical formulation of the problem and computational method*. Astrophys. and Space Science, 49 (1977), no 2, 399-425.

3. Domogatsky G.V., Eramzhyan R.A., Nadyozhin D.K. *Production of the light elements due to neutrinos emitted by collapsing stellar cores*. Astrophys. and Space Science, 58 (1978), no 2, 273-299.

4. Bete G.A. *Theory of Supernovae. Neuclear Astrophysics.* M.Mir, 1986, 418-445.

5. Golubyatnikov A.N., Chilachava T.I. *On the dispersion of the detonating explosive wave in the gravitating sphere with following dispersion in the vacuum*. Journal of the AS of the USSR, Fluids and Gas Mechanics, 1986, no 4, 187-191.

6. Golubyatnikov A.N., Chilachava T.I. *On the central explosion of the rotating gravitating body*. Papers of the AS of the USSR, 273 (1983), no 4, 825-829.

7. Chilachava T.I. *On the central explosion in the nonhomogeneous sphere balanced in its own gravitation field*. Journal of the AS of the USSR, Fluids and Gas Mechanics, 1988, no 3, 179-184.