DECOMPOSITION OF LAGUERRE POLYNOMIALS WITH RESPECT TO THE CYCLIC GROUP OF ORDER n

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Abstract

Let n be an arbitrary positive integer. We decompose the Laguerre polynomials $L_m^{(\alpha)}$ as the sum of n polynomials $L_m^{(\alpha,n,k)}; m \in \mathbb{N}; \mathrm{k} = 0, 1, ..., \mathrm{n} - 1$; defined by

$$L_m^{(\alpha,n,k)}(z) = \frac{1}{n} \sum_{l=0}^{n-1} \exp\left(-\frac{2i\pi kl}{n}\right) L_m^{(\alpha)}\left(z \exp\left(\frac{2i\pi l}{n}\right)\right); \quad z \in C.$$

In this paper, we establish the close relation between these components and the Brafman polynomials. The use of a technique described in an earlier work [2] leads us firstly to derive, from the basic identities and relations for $L_m^{(\alpha)}$, other analogous for $L_m^{(\alpha,n,k)}$ that turn out to be two integral representations, an operational representation, some generating functions defined by means of the generalized hyperbolic functions of order n and the hyper-Bessel functions, some finite sums including multiplication and addition formulas, a $(2n+1)$-term recurrence relation and a differential equation of order $2n$. Secondly, to express some identities of $L_m^{(\alpha)}$ as functions of the polynomials $L_m^{(\alpha),n,k}$. Some particular properties of $L_m^{(\alpha,n,0)}$, the first component, will be pointed out.