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# ON SOME SOLUTIONS OF THE SYSTEM OF EQUATIONS OF STEADY VIBRATION IN THE PLANE THERMOELASTICITY THEORY WITH MICROTEMPERATURES

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**Abstract**. In the present paper the linear 2D theory of thermoelasticity with microtemperatures is considered. The representation of regular solution of the system of equations of steady vibrations in the considered theory is obtained. The fundamental and singular solutions for a governing system of equations of this theory are constructed. Finally, the single-layer, double-layer and volume potentials are presented.

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## Introduction

A thermodynamic theory for elastic materials with inner structure the particles of which, in addition to microdeformations, possess microtemperatures was proposed by Grot [1]. Iesan and Quintanilla [2] have formulated the boundary value problems (BVPs) and presented an uniqueness result and a solution of the Boussinesq-Somigliana-Galerkin type. The fundamental solutions of the equations of the 3D theory of thermoelasticity with microtemperatures were constructed by Svanadze [3]. The representations of the Galerkin type and general solutions of the system of equations in this theory were obtained by Scalia, Svanadze and Tracinà [4]. In [5], a wide class of external BVPs of steady vibrations is investigated and Sommerfeld-Kupradze type radiation conditions and the basic properties of thermoelastopotentials are established. Here the uniqueness and existence theorems of regular solutions of the external BVPs are proved using the potential method and the theory of singular integral equations. The fundamental solutions of the equations of the two-dimensional (2D) theory of thermoelasticity with microtemperatures were constracted by Basheleishvili, Bitsadze and Jaiani [6]. The 2D BVPs of statics of the theory of thermoelasticity with microtemperatures are formulated and the uniqueness and existence theorems are presented in [7]. The basic results and extensive review of the theory of elastic materials with microstructure are given in the literature [8].

For investigation, boundary-value problems of the theory of elasticity and thermoelasticity by potential method are necessary to construct fundamental solutions of respective systems of partial differential equations and to establish their basic properties. There are several known methods to construct a fundamental solution of systems of differential equations of the theory of elasticity and thermoelasticity [9-12].

In the present paper the linear 2D theory of thermoelasticity with microtemperatures is considered. The representation of regular solution of the system of equations of steady vibration of the theory of thermoelasticity with microtemperatures is obtained.The fundamental and singular solutions for a governing system of equations of this theory are constructed. Finally, the single-layer, double-layer and volume potentials are presented.

**Basic equations**

We consider an isotropic elastic material with microtemperatures. Let $D^+(D^-)$ be a bounded (respectively, an unbounded) domain of the Euclidean 2D space $E_2$ bounded by the contour $S$. $\overline{D^+} := D^+ \bigcup S$, $D^- := E_2 \backslash \overline{D^+}$. Let $\mathbf{x} := (x_1, x_2) \in E_2$, $\partial \mathbf{x} := \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right)$. In 2D space "rot" is defined as a scalar

$$rot\boldsymbol{\phi} := \frac{\partial \phi_2}{\partial x_1} - \frac{\partial \phi_1}{\partial x_2}$$

for a vector $\phi := (\phi_1, \phi_2)$ and as a vector

$$rot\psi := \left(\frac{\partial \psi}{\partial x_2}, -\frac{\partial \psi}{\partial x_1}\right)$$

for a scalar $\psi$.

The basic system of equations of steady vibrations in the linear 2D theory of thermoelasticity with microtemperatures has the following form [1],[2]

$$\mu\Delta\mathbf{u} + (\lambda+\mu)graddiv\mathbf{u} - \beta grad\theta + \varrho\omega^2\mathbf{u} = -\varrho\mathbf{N}, \tag{1}$$

$$k_6\Delta\mathbf{w} + (k_4+k_5)graddiv\mathbf{w} - k_3 grad\theta + k_8\mathbf{w} = \rho\mathbf{M}, \tag{2}$$

$$(k\Delta + a_0)\theta + \beta_0 div\mathbf{u} + k_1 div\mathbf{w} = -\rho s, \tag{3}$$

where $\mathbf{u} = (u_1, u_2)^T$ is the displacement vector, $\mathbf{w} = (w_1, w_2)^T$ is the microtemperature vector, $\theta$ is the temperature measured from the constant absolute temperature $T_0$ ($T_0 > 0$) by the natural state (i.e. by the state of the absence of loads), $\rho$ is the reference mass density ($\rho > 0$), $\mathbf{N} = (N_1, N_2)$ is the body force, $\mathbf{M} = (M_1, M_2)$ is first heat source moment vector, $s$ is the heat supply, $a_0 = i\omega a T_0$, $\beta_0 = i\omega\beta T_0$, $k_8 = i\omega b - k_2$, $b > 0$, $\lambda$, $\mu$, $\beta$, $k$, $k_j$, $(j = 1, ..., 6)$, are the constitutive coefficients, $\Delta$ is the 2D Laplace operator and $\omega$ is the oscillation frequency ($\omega > 0$). The superscript "$T$" denotes transposition.

We introduce the matrix differential operator

$$\mathbf{A}(\partial\mathbf{x}, \omega) :=\| A_{lj}(\partial\mathbf{x}, \omega) \|_{5\times5},$$

where

$$A_{\alpha\gamma} := \mu\delta_{\alpha\gamma}(\Delta + \rho\omega^2) + (\lambda+\mu)\frac{\partial^2}{\partial x_\alpha \partial x_\gamma},$$

$$A_{\alpha+2;\gamma+2} := \delta_{\alpha\gamma}(k_6\Delta + k_8) + (k_4+k_5)\frac{\partial^2}{\partial x_\alpha \partial x_\gamma},$$

$$A_{\alpha,\gamma+2} := A_{\alpha+2,\gamma} = 0, \quad A_{\alpha 5} := -\beta\frac{\partial}{\partial x_\alpha}, \quad A_{\alpha+2;5} := -k_3\frac{\partial}{\partial x_\alpha},$$
$$A_{5\gamma} := \beta_0\frac{\partial}{\partial x_\gamma}, \quad A_{5;\gamma+2} := k_1\frac{\partial}{\partial x_\gamma}, \quad A_{55} := k\Delta + a_0, \quad \alpha,\gamma = 1,2,$$

$\delta_{\alpha\gamma}$ is the Kronecker delta. Then the system (1)-(3) can be rewritten as

$$\mathbf{A}(\partial\mathbf{x},\omega)\mathbf{U} = \mathbf{F}, \tag{4}$$

where

$$\mathbf{U} := (u_1,u_2,w_1,w_2,\theta)^T, \quad \mathbf{F} = (-\varrho\mathbf{N}, \varrho\mathbf{M}, -\varrho s).$$

When $\mathbf{F}=0$, we have homogeneous system of equations of steady vibrations in the 2D theory of thermoelasticity with microtemperatures

$$\mu\Delta\mathbf{u} + (\lambda+\mu)graddiv\mathbf{u} - \beta grad\theta + \varrho\omega^2\mathbf{u} = 0, \tag{5}$$

$$k_6\Delta\mathbf{w} + (k_4+k_5)graddiv\mathbf{w} - k_3 grad\theta + k_8\mathbf{w} = 0, \tag{6}$$

$$(k\Delta + a_0)\theta + \beta_0 div\mathbf{u} + k_1 div\mathbf{w} = 0. \tag{7}$$

The matrix $\widetilde{\mathbf{A}}(\partial\mathbf{x},\omega) :=\| \widetilde{A}_{lj}(\partial\mathbf{x},\omega) \|_{5\times5} := \mathbf{A}^T(-\partial\mathbf{x},\omega)$, will be called the associated operator to the differential operator $\mathbf{A}(\partial\mathbf{x},\omega)$. Thus, the homogeneous associated system to (4) has the following form

$$\mu\Delta\mathbf{u} + (\lambda+\mu)graddiv\mathbf{u} - \beta_0 grad\theta + \rho\omega^2 u = 0,$$

$$k_6\Delta\mathbf{w} + (k_4+k_5)graddiv\mathbf{w} - k_1 grad\theta + k_8\mathbf{w} = 0,$$

$$(k\Delta + a_0)\theta + k_3 div\mathbf{w} + \beta div\mathbf{u} = 0.$$

We assume that $\mu\mu_0 k k_6 k_7 \neq 0$, where $\mu_0 := \lambda+2\mu,\ k_7 := k_4+k_5+k_6$. Obviously, if the last condition is satisfied, then $\mathbf{A}(\partial\mathbf{x},\omega)$ is the elliptic differential operator.

**Representation of regular solutions**

**Definition**. A vector function $\mathbf{U}(\mathbf{u},\mathbf{w},\theta)$ is called regular in $D^-$(*or* $D^+$) if

$$1.\quad \mathbf{U}\in C^2(D^-)\cap C^1(\bar{D}^-) \quad or \quad (\mathbf{U}\in C^2(D^+)\cap C^1\bar{D}^+),$$

$$2.\quad \mathbf{u} = \sum_{j=1}^{4}\mathbf{u}^{(j)}(\mathbf{x}), \quad \mathbf{w} = \sum_{j=1,2,3,5}\mathbf{w}^{(j)}(\mathbf{x}), \quad \theta = \sum_{j=1}^{3}\theta^{(j)}(\mathbf{x}),$$

$$\begin{aligned} &3.\quad (\Delta+\lambda_j^2)\mathbf{u}^{(j)} = 0, \quad (\Delta+\lambda_l^2)\mathbf{w}^{(l)} = 0, \quad (\Delta+\lambda_m^2)\theta^{(m)} = 0,\\ &\mathbf{u}^{(j)} = (u_1^{(j)},u_2^{(j)}), \quad \mathbf{w}^{(l)} = (w_1^{(l)},w_2^{(l)}),\\ &j=1,2,3,4, \quad l=1,2,3,5, \quad m=1,2,3 \end{aligned} \tag{8}$$

and

$$\begin{aligned} &\left(\frac{\partial}{\partial|\mathbf{x}|} - i\lambda_j\right)u_l^{(j)} = e^{i\lambda_j|\mathbf{x}|}o(|\mathbf{x}|^{-\frac{1}{2}}), \quad j=1,2,3,4, \quad l=1,2,\\ &\left(\frac{\partial}{\partial|\mathbf{x}|} - i\lambda_l\right)w_k^{(l)} = e^{i\lambda_l|\mathbf{x}|}o(|\mathbf{x}|^{-\frac{1}{2}}), \quad l=1,2,3,5, \quad k=1,2, \end{aligned} \tag{9}$$

$$\left(\frac{\partial}{\partial|\mathbf{x}|}-i\lambda_m\right)\theta^{(m)}=e^{i\lambda_m|\mathbf{x}|}o(|\mathbf{x}|^{-\frac{1}{2}}),\quad m=1,2,3\quad for\quad |\mathbf{x}|=\sqrt{x_1^2+x_2^2}\quad >>1;$$

where $\lambda_j^2,\quad j=1,2,3$, are roots of equation $D(-\xi)=0$,

$$D(\Delta)=(\mu_0\Delta+\rho\omega^2)k_1k_3\Delta+(k_7\Delta+k_8)[\beta\beta_0\Delta+(\mu_0\Delta+\rho\omega^2)(k\Delta+a_0)]=$$

$$\mu_0kk_7(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)$$

and the constants $\lambda_4^2$ and $\lambda_5^2$ are determined by the formulas

$$\lambda_4^2:=\frac{\rho\omega^2}{\mu}>0,\quad \lambda_5^2:=\frac{k_8}{k_6}.$$

The quantities $\lambda_j^2,\quad j=1,2,3,5$ are complex numbers and are chosen so as to ensure positivity of their imaginary part, i.e. it is assumed that $Im\lambda_j^2>0$.

Equalities in (9) are Sommerfeld-Kupradze type radiation conditions in the linear theory of thermoelastisity with microtemperatures.

**Remark.** The equalities (9) imply [5]

$$U_l(\mathbf{x})=e^{i\lambda_j|\mathbf{x}|}O(|\mathbf{x}|^{-\frac{1}{2}})\quad for\quad |\mathbf{x}|>>1,\quad l,j=1,..,5. \tag{10}$$

**Theorem 1.** *The regular solution $\boldsymbol{U}=(\boldsymbol{u},\boldsymbol{w},\theta)$ of the systems (5)-(7) admits in the domain of regularity a representation*

$$\boldsymbol{U}=(\overset{\mathbf{1}}{\mathbf{u}}+\overset{\mathbf{2}}{\mathbf{u}},\overset{\mathbf{1}}{\mathbf{w}}+\overset{\mathbf{2}}{\mathbf{w}},\theta),$$

*where $\overset{\mathbf{1}}{\mathbf{u}}$, $\overset{\mathbf{2}}{\mathbf{u}}$, $\overset{\mathbf{1}}{\mathbf{w}}$ and $\overset{\mathbf{2}}{\mathbf{w}}$ are the regular vectors, satisfying the conditions*

$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\overset{\mathbf{1}}{\mathbf{u}}=0,\quad rot\overset{\mathbf{1}}{\mathbf{u}}=0,$$

$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\overset{\mathbf{1}}{\mathbf{w}}=0,\quad rot\overset{\mathbf{1}}{\mathbf{w}}=0,$$

$$(\Delta+\lambda_4^2)\overset{\mathbf{2}}{\mathbf{u}}=0,\quad div\overset{\mathbf{2}}{\mathbf{u}}=0,\quad (\Delta+\lambda_5^2)\overset{\mathbf{2}}{\mathbf{w}}=0,\quad div\overset{\mathbf{2}}{\mathbf{w}}=0,$$

$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\theta=0.$$

**Proof.** Let $\mathbf{U}=(\mathbf{u},\mathbf{w},\theta)$ be a regular solution of the equations (5)-(7). Taking into account the identity

$$\Delta\mathbf{w}=graddiv\mathbf{w}-rotrot\mathbf{w}, \tag{11}$$

where

$$rotrot\mathbf{w}:=\left(\frac{\partial}{\partial x_2}\left(\frac{\partial w_2}{\partial x_1}-\frac{\partial w_1}{\partial x_2}\right),-\frac{\partial}{\partial x_1}\left(\frac{\partial w_2}{\partial x_1}-\frac{\partial w_1}{\partial x_2}\right)\right),$$

from (5),(6) we obtain

$$\mathbf{u}=-\frac{\mu_0}{\rho\omega^2}graddiv\mathbf{u}+\frac{\mu}{\rho\omega^2}rotrot\mathbf{u}+\frac{\beta}{\rho\omega^2}grad\theta,$$

$$\mathbf{w} = -\frac{k_7}{k_8} graddiv\mathbf{w} + \frac{k_6}{k_8} rotrot\mathbf{w} + \frac{k_3}{k_8} grad\theta,$$

Let

$$\overset{1}{\mathbf{u}} := -\frac{\mu_0}{\rho\omega^2} graddiv\mathbf{u} + \frac{\beta}{\rho\omega^2} grad\theta, \tag{12}$$

$$\overset{2}{\mathbf{u}} := \frac{\mu}{\rho\omega^2} rotrot\mathbf{u}, \tag{13}$$

$$\overset{1}{\mathbf{w}} := -\frac{k_7}{k_8} graddiv\mathbf{w} + \frac{k_3}{k_8} grad\theta, \tag{14}$$

$$\overset{2}{\mathbf{w}} := \frac{k_6}{k_8} rotrot\mathbf{w}. \tag{15}$$

Clearly

$$\mathbf{u} = \overset{1}{\mathbf{u}} + \overset{2}{\mathbf{u}}, \quad \mathbf{w} = \overset{1}{\mathbf{w}} + \overset{2}{\mathbf{w}} \quad rot\overset{1}{\mathbf{u}} = 0, \quad rot\overset{1}{\mathbf{w}} = 0, \quad div\overset{2}{\mathbf{u}} = 0, \quad div\overset{2}{\mathbf{w}} = 0. \tag{16}$$

Taking into account the identity $\Delta\overset{2}{\mathbf{u}} = -rotrot\overset{2}{\mathbf{u}}$, $\Delta\overset{2}{\mathbf{w}} = -rotrot\overset{2}{\mathbf{w}}$, from (13)-(15) we get

$$(\Delta + \lambda_4^2)\overset{2}{\mathbf{u}} = 0, \quad (\Delta + \lambda_5^2)\overset{2}{\mathbf{w}} = 0. \tag{17}$$

Applying the operator $div$ to equations (5), (6) we obtain

$$\begin{aligned} &(\mu_0\Delta + \rho\omega^2) div\boldsymbol{u} - \beta\Delta\theta = 0, \\ &(k_7\Delta + k_8) div\boldsymbol{w} - k_3\Delta\theta = 0, \\ &(k\Delta + a_0)\theta + k_1 div\boldsymbol{w} + \beta_0 div\boldsymbol{u} = 0, \end{aligned} \tag{18}$$

Rewrite system (18) as follows

$$D(\Delta)\Psi := \begin{pmatrix} \mu_0\Delta + \rho\omega^2 & 0 & -\beta\Delta \\ 0 & k_7\Delta + k_8 & -k_3\Delta \\ \beta_0 & k_1 & k\Delta + a_0 \end{pmatrix} \Psi = 0,$$

where $\Psi = (div\boldsymbol{u}, div\boldsymbol{w}, \theta)^T$. Clearly, $detD = \mu_0 k k_7(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)$,

$$\begin{aligned} &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2) div\boldsymbol{u} = 0, \\ &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2) div\boldsymbol{w} = 0, \\ &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)\theta = 0. \end{aligned} \tag{19}$$

Applying the operator $(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)$ to equations (12), (14) using the last relations we obtain

$$(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)\overset{1}{\mathbf{u}} = 0,$$

$$(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)\overset{1}{\mathbf{w}} = 0,$$

$$(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)\theta = 0.$$

The last formulas prove the theorem.

**Theorem 2.** *The regular solution* $\boldsymbol{U} = (\boldsymbol{u}, \boldsymbol{w}, \theta) \in C^2(D)$ *of equation* $\boldsymbol{A}(\partial \boldsymbol{x})\,\boldsymbol{U} = 0$ *for* $\boldsymbol{x} \in D$, *is represented as the sum*

$$\boldsymbol{u} = \sum_{j=1}^{4} \boldsymbol{u}^{(j)}(\boldsymbol{x}), \quad \boldsymbol{w} = \sum_{j=1,2,3,5} \boldsymbol{w}^{(j)}(\boldsymbol{x}), \quad \theta = \sum_{j=1}^{3} \theta^{(j)}, \tag{20}$$

*where* $D$ *is a domain in* $E_2$ *and* $\boldsymbol{u}^{(j)}, \boldsymbol{w}^{(j)}$*and* $\theta^{(j)}$ *are regular functions satisfying the following conditions*

$$\begin{aligned} &(\Delta + \lambda_j^2)\boldsymbol{u}^{(j)} = 0, \quad (\Delta + \lambda_l^2)\boldsymbol{w}^{(l)} = 0, \quad (\Delta + \lambda_m^2)\theta^{(m)} = 0, \\ &j = 1,2,3,4, \quad l = 1,2,3,5, \quad m = 1,2,3. \end{aligned} \tag{21}$$

**Proof.** Applying the operator div to the equations (5) and (6) and taking into account the relations (18) and (19) we obtain

$$\begin{aligned} &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta + \lambda_4^2)\boldsymbol{u} = 0, \\ &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta + \lambda_5^2)\boldsymbol{w} = 0, \\ &(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)\theta = 0. \end{aligned} \tag{22}$$

We introduce the notations:

$$\begin{aligned} \mathbf{u}^{(j)} &= \left[\prod_{l=1;l\neq j}^{4} \frac{\Delta + \lambda_l^2}{\lambda_l^2 - \lambda_j^2}\right]\mathbf{u}, \quad j = 1,2,3,4, \\ \mathbf{w}^{(p)} &= \left[\prod_{l=1,2,3,5} \frac{\Delta + \lambda_l^2}{\lambda_l^2 - \lambda_p^2}\right]\mathbf{w}, \quad l \neq p, \quad p = 1,2,3,5, \\ \theta^{(q)} &= \left[\prod_{l=1}^{3} \frac{\Delta + \lambda_l^2}{\lambda_l^2 - \lambda_q^2}\right]\theta, \quad l \neq j, \quad j = 1,2,3. \end{aligned} \tag{23}$$

By virtue of (23), it follows that

$$\mathbf{u} = \sum_{j=1}^{4} \mathbf{u}^{(j)}, \quad \mathbf{w} = \sum_{j=1,2,3,5} \mathbf{w}^{(j)}, \quad \theta = \sum_{j=1}^{3} \theta^{(j)}, \tag{24}$$

$$(\Delta + \lambda_j^2)\mathbf{u}^{(j)} = 0, \quad (\Delta + \lambda_l^2)\mathbf{w}^{(l)} = 0, \quad (\Delta + \lambda_m^2)\theta^{(m)} = 0,$$

$$j = 1,2,3,4, \quad l = 1,2,3,5, \quad m = 1,2,3.$$

Thus, the regular in $D$ solution of equation $\boldsymbol{A}(\partial \mathbf{x}, \omega)\mathbf{U} = 0$ is represented as a sum of functions $\mathbf{u}^{(j)}$, $\mathbf{w}^{(j)}$, $\theta^{(j)}$, which satisfy Helmholtz' equations in $D$.

### Matrix of fundamental solutions

We introduce the matrix differential operator $\mathbf{B}(\partial \mathbf{x})$ consisting of cofactors of elements of the transposed matrix $\mathbf{A}^T$ divided on $\mu\mu_0 k k_6 k_7$

$$\mathbf{B}(\partial \mathbf{x}, \omega) :=\| B_{lj}(\partial \mathbf{x}, \omega) \|_{5\times 5},$$

where

$$B_{\alpha\gamma} := B_{11}^*\delta_{\alpha\gamma} - B_{12}^*\xi_\alpha\xi_\gamma, \quad B_{\alpha+2,\gamma+2} := B_{33}^*\delta_{\alpha\gamma} - B_{34}^*\xi_\alpha\xi_\gamma,$$

$$B_{1\gamma+2} := B_{13}^*\xi_1\xi_\gamma, \quad B_{2\gamma+2} := B_{13}^*\xi_2\xi_\gamma, \quad B_{\alpha 5} := B_{15}^*\xi_\alpha, \quad B_{5\alpha} := B_{51}^*\xi_\alpha,$$

$$B_{5\gamma+2} := B_{53}^*\xi_\gamma, \quad \xi_\alpha := \frac{\partial}{\partial x_\alpha}, \quad \alpha,\gamma = 1,2, \quad B_{55} := B_{55}^*,$$

$$B_{3\gamma} := B_{31}^*\xi_1\xi_\gamma, \quad B_{4\gamma} := B_{31}^*\xi_2\xi_\gamma, \quad B_{2+\gamma,5} := B_{35}^*\xi_\gamma,$$

$$B_{11}^* := \frac{1}{\mu}(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_5^2),$$

$$B_{12}^* := \frac{(\Delta+\lambda_5^2)}{kk_7\mu\mu_0}\left\{\beta\beta_0(k_7\Delta+k_8) + (\lambda+\mu)[(k\Delta+a_0)(k_7\Delta+k_8)+k_1k_3\Delta]\right\},$$

$$B_{13}^* := -\frac{\beta k_1}{\mu_0 kk_7}((\Delta+\lambda_4^2)(\Delta+\lambda_5^2), \quad B_{15}^* := \frac{\beta}{\mu_0 kk_7}(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)(k_7\Delta+k_8),$$

$$B_{51}^* := -\frac{\beta_0}{\mu_0 kk_7}(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)(k_7\Delta+k_8), \quad \mu_0 := \lambda+2\mu,$$

$$B_{53}^* := -\frac{hk_1}{\mu_0 kk_7}(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)(\mu_0\Delta+\rho\omega^2),$$

$$B_{55}^* := \frac{1}{\mu_0 kk_7}(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)(\mu_0\Delta+\rho\omega^2)(k_7\Delta+k_8),$$

$$B_{31}^* := -\frac{k_3\beta_0}{\mu_0 kk_7}((\Delta+\lambda_4^2)(\Delta+\lambda_5^2) \quad B_{35}^* := \frac{k_3}{\mu_0 kk_7}(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)(\mu_0\Delta+\rho\omega^2),$$

$$B_{33}^* := \frac{1}{k_6}(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2),$$

$$B_{34}^* := \frac{(\Delta+\lambda_4^2)}{\mu_0 kk_6k_7}\{k_1k_3(\mu_0\Delta+\rho\omega^2) + (k_4+k_5)[(\mu_0\Delta+\rho\omega^2)(k\Delta+a_0)+\beta\beta_0\Delta]\}.$$

Substituting the vector $\mathbf{U}(\mathbf{x}) = \mathbf{B}(\partial\mathbf{x},\omega)\boldsymbol{\Psi}$ into $\boldsymbol{A}(\partial\mathbf{x},\omega)\mathbf{U} = 0$, where $\boldsymbol{\Psi}$ is a five-component vector function, we get

$$B(\Delta) = (\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\boldsymbol{\Psi}.$$

Whence, applying the method developed in [6], after some calculations, the vector $\Psi$ can be represented as

$$\boldsymbol{\Psi} = \sum_{j=1}^{5} d_j H_0^{(1)}(\lambda_j r), \quad \sum_{j=1}^{5} d_j = 0, \quad \sum_{j=1}^{5} d_j(\lambda_m^2-\lambda_j^2) = 0, \quad m = 4,5, \tag{25}$$

$$\sum_{j=1}^{5} d_j(\lambda_4^2-\lambda_j^2)(\lambda_5^2-\lambda_j^2) = 0, \quad d_j = \prod_{m=1}^{5}\frac{1}{\lambda_j^2-\lambda_m^2}, \quad j\neq m, \quad j = 1,2,..,5,$$

where $H_0^{(1)}(\lambda_j r)$ are Hankel's functions of the first kind with the index equal to 0 and $r = |x-y|$.

Substituting (25) into $\mathbf{U}=\mathbf{B\Psi}$, we obtain the matrix of fundamental solution, which we denote by $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\omega)$

$$\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=\|\ \Gamma_{kj}(\mathbf{x}\text{-}\mathbf{y},\omega)\ \|_{5\times 5},$$

where

$$\Gamma_{\alpha\gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=\delta_{\alpha\gamma}\frac{H_0^{(1)}(\lambda_4 r)}{\mu}-\frac{\partial^2\Psi_{11}}{\partial x_\alpha\partial x_\gamma},\quad \Psi_{11}:=-\frac{H_0^{(1)}(\lambda_4 r)}{\mu\lambda_4^2}$$
$$+\sum_{m=1}^{3}\frac{l_m}{\lambda_m^2\mu_0 kk_7}[(k_8-k_7\lambda_m^2)(a_0-k\lambda_m^2)-k_1k_3\lambda_m^2]H_0^{(1)}(\lambda_m r)$$
$$\Gamma_{\alpha+2,\gamma+2}(\mathbf{x}\text{-}\mathbf{y},\omega):=\delta_{\alpha\gamma}\frac{H_0^{(1)}(\lambda_5 r)}{k_6}-\frac{\partial^2\Psi_{33}}{\partial x_\alpha\partial x_\gamma},\quad \Psi_{33}:=-\frac{H_0^{(1)}(\lambda_5 r)}{k_6\lambda_5^2}$$
$$+\sum_{m=1}^{3}\frac{l_m}{\lambda_m^2\mu_0 kk_7}[(a_0-k\lambda_m^2)(\rho\omega^2-\mu_0\lambda_m^2)-\beta\beta_0\lambda_m^2]H_0^{(1)}(\lambda_m r),$$
$$\Gamma_{55}(\mathbf{x}\text{-}\mathbf{y},\omega):=\frac{1}{kk_7\mu_0}\sum_{m=1}^{3}l_m((\rho\omega^2-\mu_0\lambda_m^2)(k_8-k_7\lambda_m^2)H_0^{(1)}(\lambda_m r),$$
$$\Gamma_{\alpha 5}(\mathbf{x}\text{-}\mathbf{y},\omega):=\beta\frac{\partial\psi_{15}}{\partial x_\alpha},\quad \Gamma_{2+\alpha,5}(\mathbf{x}\text{-}\mathbf{y},\omega):=k_3\frac{\partial\psi_{51}}{\partial x_\alpha},\quad \alpha,\gamma-1,2,$$
$$\Gamma_{5\gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=-\beta_0\frac{\partial\psi_{15}}{\partial x_\gamma},\quad \psi_{15}=\frac{1}{kk_7\mu_0}\sum_{m=1}^{3}l_m(k_8-k_7\lambda_m^2)H_0^{(1)}(\lambda_m r),$$
$$\Gamma_{5,2+\gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=-k_1\frac{\partial\psi_{51}}{\partial x_\gamma},\quad \psi_{51}=\frac{1}{kk_7\mu_0}\sum_{m=1}^{3}l_m(\rho\omega^2-\mu_0\lambda_m^2)H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{\alpha,2+\gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=-k_1\beta\frac{\partial^2\psi_{13}}{\partial x_\alpha\partial x_\gamma},\quad \psi_{13}:=\frac{1}{kk_7\mu_0}\sum_{m=1}^{3}l_m H_0^{(1)}(\lambda_m r),$$
$$\Gamma_{\alpha+2,\gamma}(\mathbf{x}\text{-}\mathbf{y},\omega):=-k_3\beta_0\frac{\partial^2\psi_{13}}{\partial x_\alpha\partial x_\gamma},\quad l_m=d_m(\lambda_4^2-\lambda_m^2)(\lambda_5^2-\lambda_m^2),\quad l=1,2,3,$$
$$\sum_{m=1}^{3}l_m=0,\quad \sum_{m=1}^{3}l_m\lambda_m^2=0,\quad \sum_{m=1}^{3}l_m\lambda_m^4=1.$$

We can easily prove the following

**Theorem 3.** *The elements of the matrix* $\mathbf{\Gamma}(\boldsymbol{x}\text{-}\boldsymbol{y},\omega)$ *has a logarithmic singularity as* $\boldsymbol{x}\to\boldsymbol{y}$ *and each column of the matrix* $\mathbf{\Gamma}(\boldsymbol{x}\text{-}\boldsymbol{y},\omega)$, *considered as a vector, is a solution of the system* $\boldsymbol{A}(\partial\boldsymbol{x},\omega)\boldsymbol{U}=0$ *at every point* $\boldsymbol{x}$ *if* $\boldsymbol{x}\neq\boldsymbol{y}$.

According to the method developed in [5], we construct the matrix $\widetilde{\mathbf{\Gamma}}(\mathbf{x},\omega):=\mathbf{\Gamma}^T(-\mathbf{x},\omega)$ and the following basic properties of $\widetilde{\mathbf{\Gamma}}(\mathbf{x},\omega)$ may be easily verified:

**Theorem 4.** *Each column of the matrix* $\widetilde{\mathbf{\Gamma}}(\boldsymbol{x}\text{-}\boldsymbol{y},\omega)$, *considered as a vector, satisfies the associated system* $\widetilde{\boldsymbol{A}}(\partial\boldsymbol{x})\widetilde{\mathbf{\Gamma}}(\boldsymbol{x}\text{-}\boldsymbol{y},\omega)=0$, *at every point* $\boldsymbol{x}$ *if* $\boldsymbol{x}\neq\boldsymbol{y}$ *and the elements of the matrix* $\widetilde{\mathbf{\Gamma}}(\boldsymbol{x}\text{-}\boldsymbol{y},\omega)$ *have a logarithmic singularity as* $\boldsymbol{x}\to\boldsymbol{y}$.

**Matrix of singular solutions**

In solving BVPs of the theory of thermoelasticity with microtemperatures by the potential method, besides the matrix of fundamental solutions, some other matrices of singular solutions to equations (5)-(7) are of a great importance. Using the matrix of fundamental solutions, we construct the so-called singular matrices of solutions by means of elementary functions.

We introduce the special generalized stress vector $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$, which acts on the element of the arc with the unit normal $\mathbf{n} = (n_1, n_2)$, where

$$\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n}) :=\| \overset{\tau}{\mathrm{R}}_{lj} \|_{5\times5},$$

$$\overset{\tau}{\mathrm{R}}_{\alpha\gamma} := \delta_{\alpha\gamma}\mu\frac{\partial}{\partial\mathbf{n}} + (\lambda+\mu)n_\alpha\frac{\partial}{\partial x_\gamma} + \tau_1\mathcal{M}_{\alpha\gamma}, \quad \overset{\tau}{\mathrm{R}}_{\alpha,\gamma+2} \equiv \overset{\tau}{\mathrm{R}}_{\alpha+2,\gamma} \equiv \overset{\tau}{\mathrm{R}}_{\alpha+2,5}$$
$$\equiv \overset{\tau}{\mathrm{R}}_{5\gamma} \equiv 0, \quad \overset{\tau}{\mathrm{R}}_{\alpha5} := -\beta n_\alpha, \quad \overset{\tau}{\mathrm{R}}_{\alpha+2;\gamma+2} := \delta_{\alpha\gamma}k_6\frac{\partial}{\partial\mathbf{n}} + (k_4+k_5)n_\alpha\frac{\partial}{\partial x_\gamma} + \tau_2\mathcal{M}_{\alpha\gamma},$$
$$\overset{\tau}{\mathrm{R}}_{5,\gamma+2} := k_1 n_\gamma, \quad \overset{\tau}{\mathrm{R}}_{55} := k\frac{\partial}{\partial\mathbf{n}}, \quad \mathcal{M}_{\alpha\gamma} := n_\gamma\frac{\partial}{\partial x_\alpha} - n_\alpha\frac{\partial}{\partial x_\gamma}, \quad \alpha,\gamma = 1,2, \tag{26}$$

here $\boldsymbol{\tau} = (\tau_1, \tau_2)$, $\tau_\alpha$, $\alpha = 1,2$, are the arbitrary numbers. If $\tau_1 = \mu$, $\tau_2 = k_5$, we denote the obtained operator by $\mathbf{P}(\partial\mathbf{x},\mathbf{n})$. The operator, which we get from $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})$ for $\tau_1 = \dfrac{\mu(\lambda+\mu)}{\lambda+3\mu}$, $\tau_2 = \dfrac{k_6(k_4+k_5)}{k_4+k_5+2k_6}$, will be denoted by $\mathbf{N}(\partial\mathbf{x},\mathbf{n})$ and the vector $\mathbf{N}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$ will be called the pseudostress vector.

Applying the operator $\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})$ to the matrix $\boldsymbol{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\omega)$, we construct the so-called singular matrix of solutions

$$\overset{\boldsymbol{\tau}}{\mathbf{R}}(\partial\mathbf{x},\mathbf{n})\boldsymbol{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\omega) := \| \overset{\tau}{\mathrm{M}}_{lj}(\partial\mathbf{x}) \|_{5\times5},$$

where

$$\overset{\tau}{\mathrm{M}}_{\gamma\gamma}(\partial\mathbf{x}) := \frac{\partial H_0^{(1)}(\lambda_4 r)}{\partial n} + (-1)^\gamma(\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_1\partial x_2} + n_\gamma\rho\omega^2\frac{\partial\Psi_{11}}{\partial x_\gamma},$$
$$\overset{\tau}{\mathrm{M}}_{12}(\partial\mathbf{x}) := \frac{\tau_1}{\mu}\frac{\partial}{\partial s}H_0^{(1)}(\lambda_4 r) - (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_2^2} + \rho\omega^2 n_1\frac{\partial\Psi_{11}}{\partial x_2},$$
$$\overset{\tau}{\mathrm{M}}_{21}(\partial\mathbf{x}) := -\frac{\tau_1}{\mu}\frac{\partial}{\partial s}H_0^{(1)}(\lambda_4 r) + (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_1^2} + \rho\omega^2 n_2\frac{\partial\Psi_{11}}{\partial x_1},$$
$$\overset{\tau}{\mathrm{M}}_{1,\gamma+2}(\partial\mathbf{x}) := k_1\beta\left[n_1\varrho\omega^2\frac{\partial\psi_{13}}{\partial x_\gamma} - (\mu+\tau_1)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_\gamma\partial x_2}\right],$$
$$\overset{\tau}{\mathrm{M}}_{2,\gamma+2}(\partial\mathbf{x}) := k_1\beta\left[n_2\varrho\omega^2\frac{\partial\psi_{13}}{\partial x_\gamma} + (\mu+\tau_1)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_\gamma\partial x_1}\right],$$
$$\overset{\tau}{\mathrm{M}}_{15}(\partial\mathbf{x}) := \beta\left[(\tau_1+\mu)\frac{\partial}{\partial x_2}\frac{\partial}{\partial s} - \rho\omega^2 n_1\right]\psi_{15},$$
$$\overset{\tau}{\mathrm{M}}_{25}(\partial\mathbf{x}) := -\beta\left[(\tau_1+\mu)\frac{\partial}{\partial x_1}\frac{\partial}{\partial s} + \rho\omega^2 n_2\right]\psi_{15},$$
$$\overset{\tau}{\mathrm{M}}_{3\alpha}(\partial\mathbf{x}) := k_3\beta_0\left[\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial}{\partial x_\alpha}H_0^{(1)}(\lambda_m r) - (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\alpha}\right],$$

$$\overset{\tau}{\mathrm{M}}_{35}(\partial\mathbf{x}):=\frac{k_3}{k\mu_0}\sum_{m=1}^{3}l_m(\rho\omega^2-\mu_0\lambda_m^2)\left[-n_1\lambda_m^2+\frac{\tau_2+k_6}{k_7}\frac{\partial}{\partial x_2}\frac{\partial}{\partial s}\right]H_0^{(1)}(\lambda_m r),$$

$$\overset{\tau}{\mathrm{M}}_{4\alpha}(\partial\mathbf{x}):=k_3\beta_0\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3}l_m\lambda_m^2\frac{\partial}{\partial x_\alpha}H_0^{(1)}(\lambda_m r)+(\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\alpha}\right],$$

$$\overset{\tau}{\mathrm{M}}_{45}(\partial\mathbf{x}):=-\frac{k_3}{k\mu_0}\sum_{m=1}^{3}l_m(\rho\omega^2-\mu_0\lambda_m^2)\left[n_2\lambda_m^2+\frac{\tau_2+k_6}{k_7}\frac{\partial}{\partial x_1}\frac{\partial}{\partial s}\right]H_0^{(1)}(\lambda_m r),$$

$$\overset{\tau}{\mathrm{M}}_{5\gamma}(\partial\mathbf{x}):=-\frac{\beta_0}{k_7\mu_0}\sum_{m=1}^{3}l_m\left[\frac{k_1k_3}{k}+k_8-k_7\lambda_m^2\right]\frac{\partial^2H_0^{(1)}(\lambda_m r)}{\partial n\partial x_\gamma},$$

$$\overset{\tau}{\mathrm{M}}_{5,\gamma+2}(\partial\mathbf{x}):=\left[\frac{n_\gamma}{k_6}H_0^{(1)}(\lambda_5 r)-\frac{\partial^2(\Psi_{33}+k\psi_{51})}{\partial x_\gamma\partial n}\right]k_1$$

$$\overset{\tau}{\mathrm{M}}_{55}(\partial\mathbf{x}):=\frac{1}{\mu_0k_7}\sum_{m=1}^{3}l_m(\rho\omega^2-\mu_0\lambda_m^2)\left[\frac{k_1k_3}{k}+k_8-k_7\lambda_m^2\right]\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial n},$$

$$\overset{\tau}{\mathrm{M}}_{2+\gamma,2+\gamma}(\partial\mathbf{x}):=\frac{\partial H_0^{(1)}(\lambda_5 r)}{\partial n}+(-1)^\gamma(\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_1\partial x_2}-n_\gamma\frac{\partial}{\partial x_\gamma}[k_1k_3\psi_{51}-k_8\Psi_{33}]$$

$$\overset{\tau}{\mathrm{M}}_{43}(\partial\mathbf{x}):=-\frac{\tau_2}{k_6}\frac{\partial H_0^{(1)}(\lambda_5 r)}{\partial s}+(\tau_2+k_6)\frac{\partial^2}{\partial x_1^2}\frac{\partial\Psi_{33}}{\partial s}-n_2\frac{\partial}{\partial x_1}[k_1k_3\psi_{51}-k_8\Psi_{33}],$$

$$\overset{\tau}{\mathrm{M}}_{34}(\partial\mathbf{x}):=\frac{\tau_2}{k_6}\frac{\partial H_0^{(1)}(\lambda_5 r)}{\partial s}-(\tau_2+k_6)\frac{\partial^2}{\partial x_2^2}\frac{\partial\Psi_{33}}{\partial s}-n_1\frac{\partial}{\partial x_2}[k_1k_3\psi_{51}-k_8\Psi_{33}]. \tag{27}$$

We prove the following theorem.

**Theorem 5.** *Every column of the matrix* $\left[\overset{\tau}{\mathbf{R}}(\partial\mathbf{y},\mathbf{n})\mathbf{\Gamma}(\mathbf{y}\text{-}\mathbf{x},\omega)\right]^T$, *considered as a vector, is a solution of the system* $\widetilde{\mathbf{A}}(\partial\mathbf{x},\omega)=0$ *at any point* $\mathbf{x}$ if $\mathbf{x}\neq\mathbf{y}$.

Let

$$\tilde{\boldsymbol{R}}^{\boldsymbol{\tau}}(\partial\mathbf{x},\mathbf{n}):=\begin{pmatrix}\overset{\tau}{\mathrm{R}}_{11} & \overset{\tau}{\mathrm{R}}_{12} & 0 & 0 & -\beta_0n_1\\ \overset{\tau}{\mathrm{R}}_{21} & R_{22} & 0 & 0 & -\beta_0n_2\\ 0 & 0 & \overset{\tau}{\mathrm{R}}_{33} & \overset{\tau}{\mathrm{R}}_{34} & 0\\ 0 & 0 & \overset{\tau}{\mathrm{R}}_{43} & \overset{\tau}{\mathrm{R}}_{44} & 0\\ 0 & 0 & k_3n_1 & k_3n_2 & \overset{\tau}{\mathrm{R}}_{55}\end{pmatrix},$$

where $\overset{\tau}{\mathrm{R}}_{\alpha\gamma},\ \overset{\tau}{\mathrm{R}}_{\alpha+2,\gamma+2},\ \overset{\tau}{\mathrm{R}}_{55},\ \alpha,\gamma=1,2$, are given by (26), then

$$\tilde{\boldsymbol{R}}^{\boldsymbol{\tau}}(\partial\mathbf{x},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y},\omega)=\|\widetilde{\mathrm{M}}^{\tau}_{lj}(\partial\mathbf{x},)\|_{5\times5},$$

Here

$$\widetilde{\mathrm{M}}^{\tau}_{\alpha\gamma}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{\alpha\gamma}(\partial\mathbf{x}), \quad \widetilde{\mathrm{M}}^{\tau}_{\alpha+2,\gamma+2}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{\alpha+2,\gamma+2}(\partial\mathbf{x}), \quad \widetilde{\mathrm{M}}^{\tau}_{55}(\partial\mathbf{x}) := \overset{\tau}{\mathrm{M}}_{55}(\partial\mathbf{x}),$$

$$\widetilde{\mathrm{M}}^{\tau}_{1,\gamma+2}(\partial\mathbf{x}) := k_3\beta_0\left[n_1\rho\omega^2\frac{\partial\psi_{13}}{\partial x_\gamma} - (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\gamma}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{2,\gamma+2}(\partial\mathbf{x}) := k_3\beta_0\left[n_2\rho\omega^2\frac{\partial\psi_{13}}{\partial x_\gamma} + (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\gamma}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{15}(\partial\mathbf{x}) := \beta_0\left[-n_1\rho\omega^2\psi_{15} + (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial\psi_{15}}{\partial x_2}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{25}(\partial\mathbf{x}) := -\beta_0\left[n_2\rho\omega^2\psi_{15} + (\tau_1+\mu)\frac{\partial}{\partial s}\frac{\partial\psi_{15}}{\partial x_1}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{3\gamma}(\partial\mathbf{x}) := k_1\beta\left[\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial x_\gamma} - (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\gamma}\right]$$

$$\widetilde{\mathrm{M}}^{\tau}_{4\gamma}(\partial\mathbf{x}) := k_1\beta\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial x_\gamma} + (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\gamma}\right]$$

$$\widetilde{\mathrm{M}}^{\tau}_{35}(\partial\mathbf{x}) := -k_1\left[\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2(\rho\omega^2-\mu_0\lambda_m^2)H_0^{(1)}(\lambda_m r) - (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial\psi_{51}}{\partial x_2}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{45}(\partial\mathbf{x}) := -k_1\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2(\rho\omega^2-\mu_0\lambda_m^2)H_0^{(1)}(\lambda_m r) + (\tau_2+k_6)\frac{\partial}{\partial s}\frac{\partial\psi_{51}}{\partial x_1}\right],$$

$$\widetilde{\mathrm{M}}^{\tau}_{5\gamma}(\partial\mathbf{x}) := -\frac{\beta}{k_7\mu_0}\sum_{m=1}^{3} l_m\left[k_8 - k_7\lambda_m^2 + \frac{k_1k_3}{k}\right]\frac{\partial^2 H_0^{(1)}(\lambda_m r)}{\partial x_\gamma\partial n},$$

$$\widetilde{\mathrm{M}}^{\tau}_{5,\gamma+2}(\partial\mathbf{x}) := k_3\left[\frac{n_\gamma}{k_6}H_0^{(1)}(\lambda_5 r) - \frac{\partial^2(\psi_{33}+k\psi_{51})}{\partial x_\gamma\partial n}\right].$$

Let $[\widetilde{\mathbf{P}}(\partial\mathbf{y},\mathbf{n})\widetilde{\mathbf{\Gamma}}(\mathbf{y}\text{-}\mathbf{x}),\omega]^T$, be the matrix which we get from $\widetilde{\mathbf{P}}(\partial\mathbf{x},\mathbf{n})\widetilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y},\omega)$ by transposition of the columns and rows and the variables $\mathbf{x}$ and $\mathbf{y}$. The superscript "$T$" denotes transposition.

We prove the following theorem.

**Theorem 6.** *Every column of the matrix* $\left[\tilde{\boldsymbol{R}}^{\boldsymbol{\tau}}(\partial\mathbf{y},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{y}\text{-}\mathbf{x},\omega)\right]^T$, *considered as a vector, is a solution of the system* $\mathbf{A}(\partial\mathbf{x},\omega)\mathbf{U}=0$ at *any point* $\mathbf{x}$ *if* $\mathbf{x}\neq\mathbf{y}$.

Let $\mathbf{g}$ and $\boldsymbol{\phi}$ be continuous (or Hölder continuous) vectors and $S$ be a closed Lyapunov curve.

We introduce the potential of a single-layer

$$\mathbf{Z}^{(1)}(\mathbf{x},\mathbf{g}) = \int\limits_S \mathbf{\Gamma}(\mathbf{x}-\mathbf{y},\omega)\mathbf{g}(\mathbf{y})ds,$$

the potential of a double-layer

$$\mathbf{Z}^{(2)}(\mathbf{x},\mathbf{g}) = \int\limits_S [\tilde{\boldsymbol{R}}^{\boldsymbol{\tau}}(\partial\mathbf{y},\mathbf{n})\mathbf{\Gamma}^T(\mathbf{y}\text{-}\mathbf{x},\omega)]^T\mathbf{g}(\mathbf{y})ds$$

and the potential of volume

$$\mathbf{Z}^{(3)}(\mathbf{x},\boldsymbol{\phi})=\int\limits_{D^{\pm}}\boldsymbol{\Gamma}(\mathbf{x}-\mathbf{y},\omega)\boldsymbol{\phi}(\mathbf{y})ds,$$

where $\boldsymbol{\Gamma}$ is the fundamental matrix, $\mathbf{g}$ and $\boldsymbol{\phi}$ are five-component vectors.

The following theorem is valid:

**Theorem 7.** *The vectors* $\boldsymbol{Z}^{(j)}$ $(j=1,2,)$ *are the solutions of the system*

$$\boldsymbol{A}(\partial\boldsymbol{x},\omega)\,\boldsymbol{U}=0$$

*in both the domains* $D^{+}$ *and* $D^{-}$ *and the elements of the matrix* $\left[\tilde{\boldsymbol{R}}^{\boldsymbol{\tau}}(\partial\boldsymbol{y},\boldsymbol{n})\boldsymbol{\Gamma}^{\boldsymbol{T}}(\boldsymbol{y}\text{-}\boldsymbol{x},\omega)\right]^{T}$, *contain a singular part, which is integrable in the sense of the Cauchy principal value. The vector* $\boldsymbol{Z}^{(3)}(\boldsymbol{x},\boldsymbol{\phi})$ *is a solution of the system* $\boldsymbol{A}(\partial\boldsymbol{x},\omega)\boldsymbol{Z}^{(3)}=\boldsymbol{\phi}$.

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