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NECESSARY CONDITIONS OF OPTIMALITY FOR QUASILINEAR SYSTEMS WITH INCOMMENSURABLE DELAYS AND MIXED RESTRICTIONS

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Abstract. Initially the necessary conditions of optimality for nonlinear control systems with incommensurable delays in phase and control variables were given in [1]. In the present paper the necessary conditions of optimality for systems, linear with control parameters, with the mixed restrictions and incommensurable delays in phase and control variables are given. Unlike [2], the approach used by us, allows problem research when the mapping describing restrictions of a task has infinite codimension, that is typical for systems with the mixed restrictions. As against [3], necessary conditions of optimality in the case of continuous initial function are obtained.

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We consider the following problem

$$\int_{t_0}^{t_1} f^0\left(x\left(t\right), x\left(t-\theta\right), u\left(t\right), u\left(t-\tau\right)\right) dt \to inf \tag{1}$$

under the restrictions

$$\dot{x}(t) = f\left(x\left(t\right), x\left(t-\theta\right), u\left(t\right), u\left(t-\tau\right)\right), \tag{2}$$

$$g\left(x\left(t\right), u\left(t\right)\right) \leq 0, t \in [t_0, t_1], \tag{3}$$

$$\chi\left(u\left(t\right)\right) \leq 0, t \in [t_0-\tau, t_0], \tag{4}$$

$$x\left(t_1\right) = x_1, \tag{5}$$

$$x\left(t\right) = \varphi\left(t\right), t \in [t_0-\theta, t_0], \tag{6}$$

where vector function φ is continuous on $[t_0-\theta, t_0]$, $x_1 \in R^n$ is fixed, $t_0, t_1, \theta > 0, \tau > 0$ are fixed numbers, the scalar function $f^0(x, y, u, v)$ and the vector functions $f(x, y, u, v) \in R^n, g(x, y, u, v) \in R^m (x \in R^n, y \in R^n, u \in R^r, v \in R^r)$ are linear with respect to arguments u and v and continuously differentiable with respect to other their arguments, $\chi(u) \in R^s$ is linear. The conditions (2) - (3) and (4) are fulfilled accordingly for almost all $t \in [t_0, t_1]$ and almost all $t \in [t_0-\tau, t_0]$ and the restrictions (3) fulfilled the conditions of generality (see [4]).

Let vector function $\varphi(t)$ be fixed, the vector function $x(t)$ be absolutely continuous, the vector function $u(t)$ be integrable on $[t_0-\tau, t_1]$, i.e., $x(t) \in W^n_{1,1}[t_0, t_1]$, $u(t) \in L^r_1[t_0-\tau, t_1]$, the functions f^0, f, g satisfying condition of "convexity" (see [2]), the restrictions (4) fulfil the conditions of generality, then using the Lagrange Principle of taking restrictions from [5], we obtain the following theorem:

Theorem. *Let* $(x(t), u(t))$ *is a solution of the problem (1) - (6). Then there exist multipliers* $\psi_0 \geq 0$, $\psi(t) \in W^n_{1,1}[t_0, t_1]$,$\nu(t) \in L^s_\infty[t_0 - \tau, t_0]$ *and* $\mu(t) \in L^m_\infty[t_0, t_1]$, *such that, the following conditions are fulfilled*

$$\mu_j(t) \geq 0, \mu_j(t)\, g^j(x(t), u(t)) = 0, j = \overline{1,m}, \quad t \in [t_0, t_1], \tag{7}$$

$$\nu_k(t) \geq 0, \nu_k(t)\, \chi^k(u(t)) = 0, k = \overline{1,s}, t \in [t_0 - \tau, t_0], \tag{8}$$

$$H[t+\tau, u(t+\tau), u(t)] = \min_{u \in \{u | \chi(u) \leq 0\}} H[t+\tau, u(t+\tau), u], \quad t \in [t_0 - \tau, t_0], \tag{9}$$

$$H[t, u(t), u(t-\tau)] + H[t+\tau, u(t+\tau), u(t)] = \min_{u \in \{u | g(x(t),u) \leq 0\}}$$

$$H[t, u, u(t-\tau)] + H[t+\tau, u(t+\tau), u], \quad t \in [t_0, t_1 - \tau], \tag{10}$$

$$H[t, u(t), u(t-\tau)] = \min_{u \in \{u | g(x(t),u) \leq 0\}} H[t, u, u(t-\tau)], \quad t \in [t_1 - \tau, t_1], \tag{11}$$

$$\frac{d\psi}{dt} = \frac{\partial \Re(x(t), x(t-\theta), u(t), u(t-\tau), \psi_0, \psi(t), \mu(t))}{\partial x}$$

$$+\frac{\partial \Re(x(t+\theta), x(t), u(t+\theta), u(t+\theta-\tau), \psi_0, \psi(t+\theta), \mu(t+\theta))}{\partial y}, t \in [t_0, t_1 - \theta], \tag{12}$$

$$\frac{d\psi}{dt} = \frac{\partial \Re(x(t), x(t-\theta), u(t), u(t-\tau), \psi_0, \psi(t), \mu(t))}{\partial x}, \quad t \in [t_1 - \theta, t_1], \tag{13}$$

and

$$(\psi_0, \psi(\cdot)) \neq (0, 0),$$

where

$$H[t, u(t), u(t-\tau)] \equiv H(x(t), x(t-\theta), u(t), u(t-\tau), \psi_0, \psi(t))$$

$$\equiv \psi_0 f^0(x(t), x(t-\theta), u(t), u(t-\tau)) - \sum_{i=1}^{n} \psi_i(t)\, f^i(x(t), x(t-\theta), u(t), u(t-\tau)),$$

$$\Re(x(t), x(t-\theta), u(t), u(t-\tau), \psi_0, \psi(t), \mu(t))$$

$$\equiv H(x(t), x(t-\theta), u(t), u(t-\tau), \psi_0, \psi(t)) + \mu(t)\, g(x(t), u(t))$$

and the restrictions (7)-(13) are fulfilled for almost all t.

R E F E R E N C E S

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