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STUDY OF STRESS-STRAIN STATE OF A TWO-LAYER ELLIPTIC CYLINDER

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Abstract. In the paper, elastic state of a two-layer elliptic ring is studied in the elliptic coordinates system. The layers composing the ring are made of steel and technical rubber and have different thickness and disposition.

Keywords and phrases: Elliptic cylinder, rigid contact, sliding contact, boundary element method, fictitious load.

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1. Introduction

We consider an elastic equilibrium of a two-layer elliptic ring in the elliptic coordinates system. The layers of the composed ring are made of steel and technical rubber and have different thickness and disposition. Besides, thickness of the composed ring and a load imposed on the ring are constant (under the thickness we mean $\xi_2-\xi_1$, where ξ_1 and ξ_2 are boundary lines of the composed ring). The corresponding stress-strain states are brought.

Such problems are considered in many works, e.g. [1] and [2]. In monograph [1], sufficient complete bibliography on considered problems is given. It should be noted that in our paper, unlike other works, different contact conditions and domain geometry are considered.

2. Problems setting

In the system of elliptic curvilinear orthogonal coordinates ξ, η $(0 \leq \xi < \infty$, $0 \leq \eta < 2\pi)$ [3] stress-strain state of the two-layer elastic body $\Omega = \Omega_1 + \Omega_2$, is considered, where $\Omega_1 = \{\xi_1 < \xi < \xi_{12},\ \ 0 < \eta < 2\pi\}$, $\Omega_2 = \{\xi_{12} < \xi < \xi_2,\ \ 0 < \eta < 2\pi\}$$(0 \leq \eta < 2\pi$ in the firstly given domain; $0 < \eta < \frac{\pi}{2}$ due to the load symmetry on the construction). If x, y are Cartesian coordinates, then $x = c\cosh\xi\cos\eta$, $\ y = c\sinh\xi\sin\eta$, and metric coefficients $h_\xi = h_\eta = h = \frac{c}{\sqrt{2}}\sqrt{\cosh(2\xi) - \cos(2\eta)}$, where c is a scale factor. In our case c=1. One layer of the considered body is of steel and the other of technical rubber. Young's module E_i and Poisson coefficient ν_i (i=1, 2) for steel are equal to $2\cdot 10^6$ kg/cm^2 and $0,3$; for technical rubber $2\cdot 10^2$ kg/cm^2 and $0,42$, respectively.

It should be noted that in the paper, plane deformed state in the system of cylindrical elliptic coordinates ξ, η, z is considered. Besides, it is easy to see that along the linear coordinate z stress-deformed state does not change.

Boundary value conditions have the form:

$$\text{when } \ \eta = 0 \ \text{ and } \eta = \frac{\pi}{2}: \quad v^{(1)} = 0, \quad \tau^{(1)}_{\xi\eta} = 0, \quad v^{(2)} = 0, \quad \tau^{(2)}_{\xi\eta} = 0, \tag{1}$$

$$\text{when } \ \xi = \xi_1: \quad \sigma^{(1)}_{\xi\xi} = p\sin^3\eta, \quad \tau^{(1)}_{\xi\eta} = 0, \tag{2}$$

$$\text{when } \xi = \xi_2 : \qquad \sigma_{\xi\xi}^{(2)} = 0, \quad \tau_{\xi\eta}^{(2)} = 0. \tag{3}$$

Contact conditions have the form:
when $\theta = \theta_{12}$:

$$\begin{aligned} &\text{a) } \sigma_{\xi\xi}^{(1)} = \sigma_{\xi\xi}^{(2)}, \quad \sigma_{\xi\eta}^{(1)} = \sigma_{\xi\eta}^{(2)}, \quad u^{(1)} = u^{(2)}, \; v^{(1)} = \; v^{(2)} \quad \text{(rigid contact)} \\ &\text{or b) } u^{(1)} = u^{(2)}, \quad \sigma_{\xi\xi}^{(1)} = \sigma_{\xi\xi}^{(2)}, \quad \sigma_{\xi\eta}^{(1)} = 0, \qquad \sigma_{\xi\eta}^{(2)} = 0 \quad \text{(sliding contact).} \end{aligned} \tag{4}$$

where ξ_{12} is a contact surface, $u^{(i)}$ and $v^{(i)}$ (i=1,2) components of a displacement vector along the ξ = *const* surface normal and tangent, respectively; $\sigma_{\xi\xi}^{(i)}$, $\sigma_{\eta\eta}^{(i)}$ are normal stresses, $\tau_{\xi\eta}^{(i)}$ - tangential stresses. Equilibrium equations have the form:

$$\begin{aligned} &\frac{\partial D^{(i)}}{\partial \theta} - \frac{\partial K^{(i)}}{\partial \alpha} = 0, \qquad && \frac{\partial u^{(i)}}{\partial \theta} + \frac{\partial v^{(i)}}{\partial \alpha} = \frac{\kappa_i - 1}{\kappa_i \mu_i} h^2 D^{(i)}, \\ &\frac{\partial D^{(i)}}{\partial \alpha} + \frac{\partial K^{(i)}}{\partial \theta} = 0, \qquad && \frac{\partial v^{(i)}}{\partial \theta} - \frac{\partial u^{(i)}}{\partial \alpha} = \frac{1}{\mu_i} h^2 K^{(i)}, \qquad i = 1, 2. \end{aligned} \tag{5}$$

where $\kappa_i = 4\,(1 - \nu_i)\,, \;\; \mu_i = \frac{E_i}{2(1-\nu_i)}$.

For problems (1), (2), (3), (4), (5) numerical solutions are obtained by the boundary element method, in particular, by the fictitious load method [4].

We note, that except the boundary conditions considered by us, it is possible to be considered other boundary conditions in any quantity of layers.

3. Presentation of the solved problems

In this paper are brought corresponding figures of the obtained results, in particular, the following problems are considered.

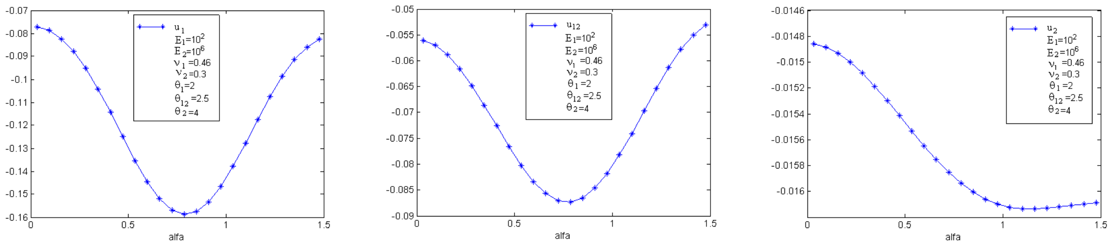
We find an elastic state of a two-layer elliptic cylinder when the domain Ω_1 is technical rubber, and Ω_2 is steel, in the same time a rigid contact is realized between the layers (4a), and conditions (1), (2), (3) are fulfilled at the boundaries. $p = 10$ kg/cm^2; $\xi_1 = 2$, $\xi_2 = 4$, a) $\xi_{12} = 2,5$; b) $\xi_{12} = 3$; c) $\xi_{12} = 3,5$; when contact surface $\xi_{12} = 2,5$, the problem is called Problem 1.1a. The Problems 1.1b and 1.1c are explained analogously.

Problems 1.2a, 1.2b and 1.2c are explained similarly to Problems 1.1a, 1.1b, 1.1c with the difference that the domain $\Omega_1$is steel, and Ω_2 is technical rubber.

Problems 2.1a, 2.1b and 2.1c are explained similarly to Problems 1.1a, 1.1b, 1.1c with the difference that there is realized a sliding contact (4b).

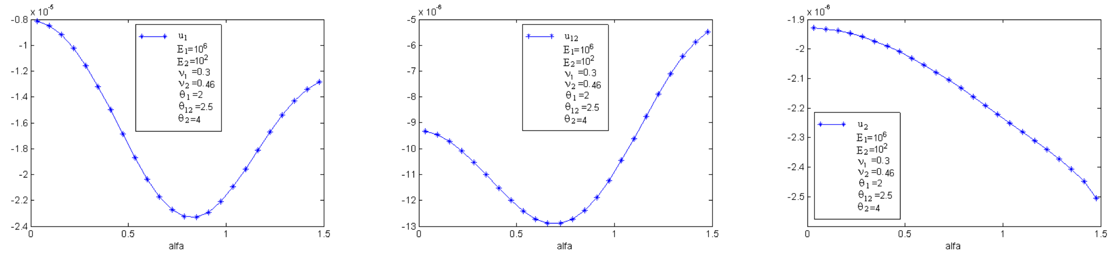
Problems 2.2a, 2.2b and 2.2c are explained analogously to 2.1a, 2.1b and 2.1c with the difference that the domain$\Omega_1$is steel, and Ω_2 is rubber.

In graphs below the displacements u obtained in some problems on the some surfaces of the considered body are presented.



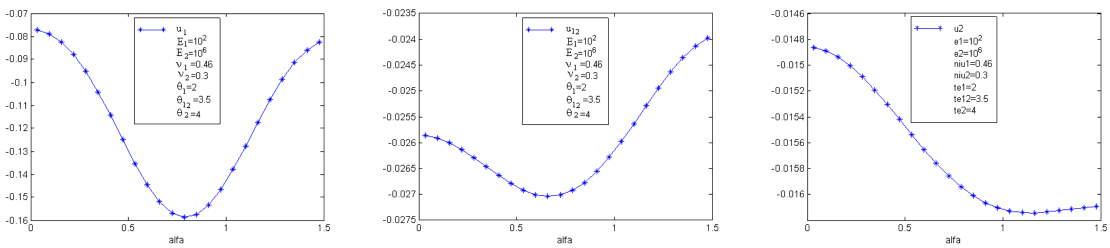
a) b) c)

Fig.1. Normal displacements u obtained in Problem 1.1a for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 2,5$ and c) on the boundary $\xi = \xi_2$.



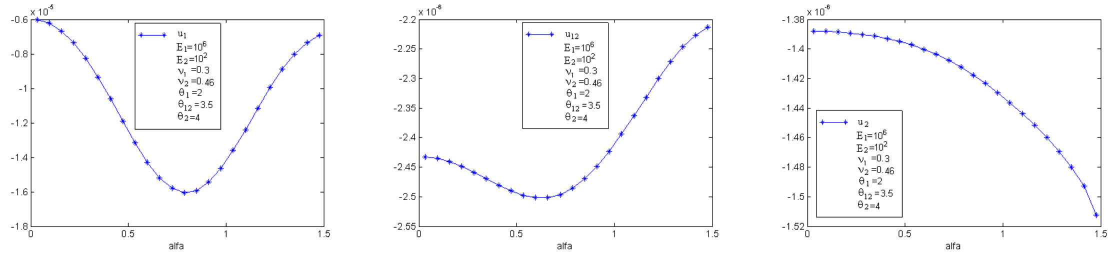
a) b) c)

Fig.2. Normal displacements u obtained in Problem 1.2a for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 2,5$ and c) on the boundary $\xi = \xi_2$.



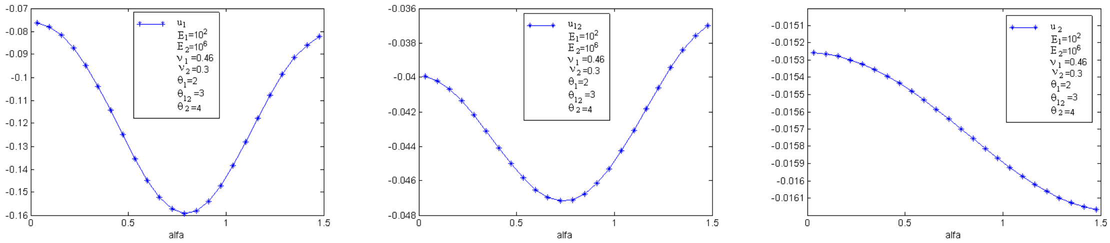
a) b) c)

Fig.3. Normal displacements u obtained in Problem 1.1c for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 3,5$ and c) on the boundary $\xi = \xi_2$.



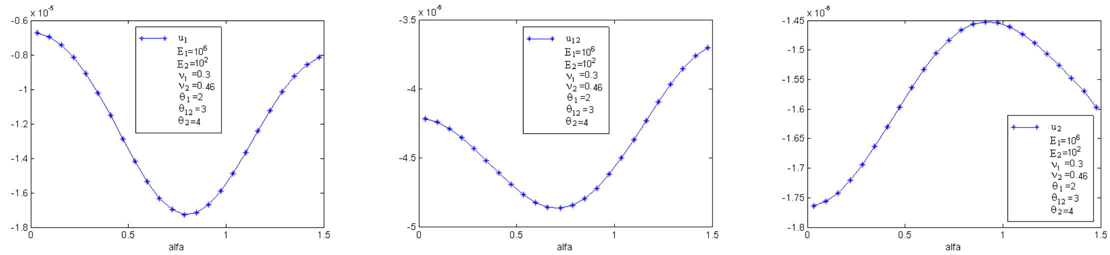
a) b) c)

Fig.4. Normal displacements u obtained in Problem 1.2c for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 3,5$ and c) on the boundary $\xi = \xi_2$.



a) b) c)

Fig.5. Normal displacements u obtained in Problem 2.1b for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 3$ and c) on the boundary $\xi = \xi_2$.



a) b) c)

Fig.6. Normal displacements u obtained in Problem 2.2b for two-layer elliptic cylinder: a) on the boundary $\xi = \xi_1$; b) on the contact surface $\xi = \xi_{12} = 3$ and c) on the boundary $\xi = \xi_2$.

R E F E R E N C E S

1. Koltunov M.F., Vasil'ev, Yu.N., Chernykh V.A. Elastisity and solidity of cylindrical bodies. (Russian) *"Vysshaya Shkola", Moscow,* 1970.

2. Khomasuridze N. Thermoelastic equilibrium of bodies in generalized cylindrical coordinates. *Georgian Math. J.,* **5**, 6 (1998), 521-544.

3. Bermant A.F. Mapping Linear Coordinates. Transformation. (Russian) *Green's Formulas, "Fizmatgiz" Publishers, Moscow,* 1958.

4. Crouch S.L., Starfield A.M. Boundary element methods in solid mechanics. *Publishers George Allen & Unwin, London-Boston-Sydney,* 1983.

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