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A MODIFIED THEORY $M\tau SR$

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Abstract. Modifications of the logic τSR [1] are introduced allowing to define the notions of a definition, a proof, contracting proof, and a proof text. Namely the language of the logic τSR is extended by adding some metasymbols and auxiliary symbols. The main purpose of the modified theory is a computer realization of the mathematical research.

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1. Introduction

In [1] the notion of a contracted symbol is defined and main theorems on the properties of contracted forms and operations on them are proved. These results as well as the results stated in [2] imply that the types I-IV, II', and IV' of contracted symbols are rational in the following sense [2]:

> "On the one hand, the system is so general that we can define almost every contracted symbol used in the classical mathematical theories. On the other hand, the system has so rich properties that we have guarantee of the freedom for operating with the contracted forms".

Thus it is desirable to use the types I-IV, II, and IV of contracted symbols in formal and nonformal mathematical theories. Note that the types III, IV, and IV' of contracted symbols represent a generalization of the types I, II, and II' respectively. The types III, IV, and IV' of contracted symbols are very complicated and it is difficult to use them in automated reasoning.

An advantage of the logic τSR [1] is that its alphabet allows to use only the types I, II, and II' of a definition of contracted symbols for introducing all the operators introduced by the rational system.

We propose an artificial modification $M\tau SR$ of the logic τSR supporting an implementation of mathematical problems by the computer.

2. The Language of the Theory $M\tau SR$

The language of the theory $M\tau SR$ contains the following symbols.
1) The basic symbols of the τSR theory:

- logical connectives $\neg$ (of the weight 1), $\vee$, $\leftrightarrow$ (each of the weight 2);

- a logical operational symbol τ of the weight (1,1);
- a substantive special substitution operator S of the weight (1,2);
- a relational special substitution operator R of the weight (1,2) and with the logicality indicator 2;
- object letters $X_0, X_1, X_2, ...$;
- predicate symbols $=$, $\in$ (each of the weight 2);
- predicate letters $P_0^n, Q_0^n, P_1^n, Q_1^n, \dots$ $(n = 0, 1, 2, \dots)$;
- a functional symbol $\supset$ of the weight 2;
- functional letters $f_0^n, g_0^n, f_1^n, g_1^n, \dots$ $(n = 0, 1, 2, \dots)$;
- the left and right brackets [,].

2) the symbols defined by the types I, II, and II' of definitions;
3) metasymbols

- metaconstants for the natural numbers $0, 1, 2, \dots, 10, \dots$;
- metavariables for the object letters $x, y, z, x_1, y_1, z_1, \dots$;
- metavariables for the forms $\Phi, \Phi_1, \Phi_2, \dots$;
- metavariables for the formulae $A, B, A_1, B_1, \dots$;
- metavariables for the terms $T, U, T_1, U_1, \dots$;

4) auxiliary symbols $\vdash$, $\dashv$, [,] , $\Rightarrow$, *and*, *or*, $\cong$, /, $M\tau SR$, C, I, D, , . . . ,.

A sign combination is a finite sequence of the symbols of the $M\tau SR$ theory. If a sign combination does not contain auxiliary symbols, then we call it *a word*; if a word contains only the basic symbols of the τSR theory, then it is called *a basic word.* If a word contains at least one symbol from the group 2), then we call it *a short word.*

Words type τx is logical substantive operators with weight 1 of $M\tau SR$ theory, Sx and Rx type words are operators with weight 2 of the same theory, besides Sx is special substantial partial quantifier with binding indicator 2, Rx operator is logico-special relational partial quantifiers with binding and logical indicator 2.

Formulas and terms of $M\tau SR$ theory are defined in the following way.

1) The object letters, the metavariables for the object letters and for the terms, and the metaconstants for the natural numbers are atomic terms (forms).

2) The metavariables for the formulas are atomic formulas (forms).

3) The metavariables for the forms are atomic forms.

4) If σ is an n-ary logico-special operator, then $\sigma A_1 \dots A_n$ $(\sigma T_1 \dots T_n)$ is a form, namely either a formula or a term depending on whether the operator σ is relational or substantive.

5) If σ is an n-ary special substitution operator with the logicality indicator $(n_1, \dots, n_k)$ and $\phi_1, \dots, \phi_n$ is a sequence of the forms such that $\phi_{n_1}, \dots, \phi_{n_k}$ is the maximal subsequence of formulas from $\phi_1, \dots, \phi_n$, then $\sigma\phi_1, \dots, \phi_n$ is a form, namely either a formula or a term depending on whether the operator σ is relational or substantive.

6) Only such words as arise from repeated applications of 1) – 5) are forms (terms and formulas) of $M\tau SR$ theory.

Some sign combination are called special prescript sign combination. They will be introduced as required. We begin with the following special prescript sign combination.

1) $M\tau SR|A_1, \dots, A_n|$ is a special prescript sign combination of the theory $M\tau SR$; it is read "A theory obtained adding the axioms $A_1, \dots, A_n$ to $M\tau SR$".

2) $M\tau SR|A_1, \dots, A_n|[\mathbf{B_1}, \dots, \mathbf{B_n}]\mathbf{A}$ is a special prescript sign combination of $M\tau SR$ theory; it is read "If $B_1, \dots, B_n$ are theorems of $M\tau SR|A_1, \dots, A_n|$, then A is a theorem of the theory $M\tau SR|A_1, \dots, A_n|$". A special prescript sign combinations of this kind is called an inference rule of $M\tau SR|A_1, \dots, A_n|$, where the formulas $B_1, \dots, B_n$ are premises and and A is a conclusion.

3) $\phi \cong \phi_1$ is a special prescript sign combination of the theory $M\tau SR$; it is read "The forms ϕ and ϕ_1 are congruent".

4) If E_1 and E_2 are special prescript sign combinations of the theory $M\tau SR$, then $E_1 \Rightarrow E_2$,$E_1\, and\, E_2$, $E_1\, or\, E_2$ are special prescript sign combinations of the theory $M\tau SR$; they are read "If E_1, then E_2", "E_1 and E_2", "E_1 or E_2" respectively.

5) $C0, C1, C2, \dots$ are special prescript sign combination of the theory $M\tau SR$; they are used for enumerating the inference rules.

3. The Definition of a Proof in the Theory $M\tau SR$

To complete the description of the theory $M\tau SR$ we define the notions of *a proof* and *a proof text*. This is done in the following way.

1. First we specify some formulas of the theory $M\tau SR$ as its *explicit axioms*; the object, predicate, and functional letters occurring into the explicit axioms are called *bound constants*.

2. Then some special prescript sign combinations of the type 2) above are specified as *basic derivation rules* of the theory $M\tau SR$.

3. Finally, some rules are specified as *schemes* of the theory $M\tau SR$.

Any formula obtained by applying some schemes of the theory $M\tau SR$ is called *an implicit axiom* of the theory $M\tau SR$.

A proof of the theory $M\tau SR$ is defined recursively.

1. Any sequence of explicit and implicit axioms of the theory $M\tau SR$ is a proof of the theory $M\tau SR$.

2. A sequence $A_1, \dots, A_m$ of the formulas of the theory $M\tau SR$ is a proof of the theory $M\tau SR$ if at least one of the following conditions is fulfilled for every formula A in this sequence:

a) there is a proof of the theory $M\tau SR$ containing A;

b) in the sequence mentioned above there exist formulas $A_{i_1}, \dots, A_{i_k}$ $(1 \le i_1 < \dots < i_k \le m)$ preceding A such that $A_{i_1}, \dots, A_{i_k}$ are the premises of some derivation rule of the theory $M\tau SR$, while A is the conclusion of this derivation rule.

A theorem of the theory $M\tau SR$ is a formula occurring in some proof of the theory $M\tau SR$.

Sign combinations of the form $\vdash M\tau SR|A_1,\dots,A_m|$ (read "the beginning of the proof of the theory $M\tau SR|A_1,\dots,A_m|$)") and $\dashv M\tau SR|A_1,\dots,A_m|$ (read "the end of the proof of the theory $M\tau SR|A_1,\dots,A_m|$"), $m \geq 0$, are called *opposite*. In this case we call a sign combination $\vdash M\tau SR|A_1,\dots,A_m|$ and $\dashv M\tau SR|A_1,\dots,A_m|$ *an opening* and *a closing* sign combination respectively.

Let

$$D_1,\dots,D_n \tag{1}$$

be a sequence of closing and opening sign combinations.

We say that sequence 1 is *normal* if the following conditions are fulfilled:

1) D_1 and D_n are opposite sign combinations, D_1 being $\vdash M\tau SR$ and D_n being $\dashv M\tau SR$;

2) For each right sign combination D' in the sequence (1) there exist left sign combinations D'' in (1) corresponding to D' and vice versa. Also, between D' and D'' there are only occurences of formulas and pairs of the corresponding sign combinations.

We say that a formula A in (1) is directly connected with a theory

$$M\tau SR|A_1,\dots,A_m|$$

in (1) if $M\tau SR|A_1,\dots,A_m|$ precedes A in (1) and between $M\tau SR|A_1,\dots,A_m|$ and A only formulas and pairs of the corresponding sign combinations may occur. It is obvious that for each formula A of a normal sequence there exists a unique right sign combination in this sequence with which A is connected.

Consider a subsequence

$$D_1,\dots,D_i \ (i = 2,...,m) \tag{2}$$

of the sequence (1), where D_i is a formula A. Remove from (2) first all the terms occurring between the pairs of the corresponding sign combinations and then all the terms which are not formulas. We say that the remainder of the sequence of (1) is connected with A. A normal sequence $D_1,\dots,D_n$ is called a conclusive text of the theory $M\tau SR$ if for each formula A in (1) connected with $M\tau SR|A_1,\dots,A_m|$ in (1) the following condition is fulfilled: a subsequence of the sequence (1) connected with A is a proof of the theory $M\tau SR|A_1,\dots,A_m|$.

The character combination $A\dots$ ($A.Cm$, AIm, $AIDm$) read as "A is a theorem (according to the conditions, Cm, Im, IDm respectively)". In these cases, we say that a formula A is given with a commentary. We say that a given conclusive text of the theory $M\tau SR$ is a conclusive text with a commentary if this text contains formulas with a commentary.

The inference rules of the theory $M\tau SR$ are the following:

ra. if A and $\neg A \vee B$, then B;

rb. if A and $B \leftrightarrow A$, then B;

rc. if $A \leftrightarrow B$, then $B \leftrightarrow A$;

rd. if A and $B \cong A$, then B;

re. if $M\tau SR \mid A \mid B$, then $M\tau SRA \rightarrow B$.

The axiom schemes of the theory $M\tau SR$ are the following:

I1. $\neg[A \vee A] \vee A$

I2. $\neg A \vee [A \vee B]$

I3. $[A \vee B] \rightarrow [B \vee A]$

I4. $[A \rightarrow B] \rightarrow [[A_1 \vee A] \rightarrow [A_1 \vee B]]$

I5. $[A \leftrightarrow B] \rightarrow [A \rightarrow B]$

I6. $RxTA \rightarrow \exists xA$

I7. $RxTA \leftrightarrow (T/x)A$

I8. $SxTU = (T/x)U$

Note that in the axiom shemas I7 and I8 the substitution (T/x) does not bind free variables in T.

I9. $\forall x[A \leftrightarrow B] \rightarrow [\tau xA = \tau xB]$

I10. $[\forall x[A \leftrightarrow B] \bigwedge [T = U]] \rightarrow [RxTA \leftrightarrow RxUB]$

I11. $[T = U] \bigwedge [T_1 = U_1]] \rightarrow [SxT_1T = SxU_1U]$

Finally, assume that Φ — Φ_1 is a definition D_m $(m = 1, 2, \dots)$, then $\Phi \leftrightarrow \Phi_1$ $(\Phi = \Phi_1$) is an axiom shema if Φ is a formula (respectively, if Φ is a term). This axiom shema is denoted by IDm.

Examples of the $Dm[k, j]$ definition (where m is an index of the definition, k denotes the type of the definition, and j denotes the level of the operator obtained by the definition) and the axiom shema IDm are following.

$D1[1, 1]$. $A \rightarrow B - - - \neg A \vee B$ (read "If A, then B");

I$D1$. $[A \rightarrow B] \leftrightarrow [\neg A \vee B]$.

$D2[1, 1]$. $A \bigwedge B - - - \neg[\neg A \vee \neg B]$ (read "A and B";

I$D2$. $[A \bigwedge B \leftrightarrow \neg[\neg A \vee \neg B]$.

$D3[1, 1]$. $\exists x\, A - - - Rx\tau xAA$ (read "There exists x such that A"; $\exists x$ is a logical relational operator;

I$D3$. $\exists x\, A \leftrightarrow Rx\tau xAA$.

$D4[1, 2]$. $\forall x\, A - - - \neg \exists x\, \neg A$ (read "For all x, A"; $\forall x$ is a logical relational operator;

I$D4$. $\forall x\, A \leftrightarrow \neg \exists x \neg A$.

$D5[2', 3]$. $< \mathit{set} > xA\ - - - \tau y[\forall x\, [x \in y \leftrightarrow A]]$, where the variable y is different from x and does not occur in A (read "A is the set of objects possessing the property A"); $< \mathit{set} > x$ is a logical substantive operator;

I$D5$. $< \mathit{set} > xA = \tau y[\forall x\, [x \in y \leftrightarrow A]]$.

$D6[2', 1]$. $< \mathit{represent} > xUT - - - \exists y\, [U = SxyT]$, where the variable y is different from x and does not occurin the terms T and U (read "U can be represented as T with respect to the x"); $< \mathit{represent} > x$ is a special relational partial quantifier with the binding indicator (2);

I$D6$. $< \mathit{represent} > xUT \leftrightarrow \exists y\, [U = SxyT]$.

$D7[2', 3]$. $< \mathit{subset} > xTA - - - \tau y[\forall x\, [x \in y \leftrightarrow [x \in T \bigwedge A]]]$, where x and y are distinct variables, x does not occur in T and y does not occur in T and A (read "The set of all the elements of T with the property A"); $< \mathit{subset} > x$ is a logico-special substantial partial quantifier with the logical and binding indicator (2);

I$D7$. $< \mathit{subset} > xTA = \tau y[\forall x[x \in y \leftrightarrow [x \in T \bigwedge A]]]$.

$D8[2',4]$. $<$ *complement* $>$ UT $---$ $<$ *set* $>$ $x[x \in T \bigwedge x \in U]$, where the variable x does not occur in the terms T and U (read "The complements of the set U with respect to the set T"); $<$ *complement* $>$ is a special substantive operator;

I$D8$. $<$ *complement* $>$ $UT =$ $<$ *set* $>$ $x[x \in T \bigwedge x \in U]$.

$D9[1,1]$. $<$ *root* $>$ xTA $---$ $RxTA]$ (read "T is a solution of the formula A with respect to x"); $<$ *root* $>$ x is a logico-special relational partial quantifier with the logical and binding indicator (2);

I$D9[1,1]$. $<$ *root* $>$ $xTA \leftrightarrow RxTA]$.

According to the inference rules and axioms we prove analogies of deductive criteria given in the monography [3]. For examples, prove the criteria C1 – C15.

C1. $M\tau SR[A, \neg A \vee B]B$

$$\vdash M\tau SR, A\textbf{..},\ \neg A \vee B\textbf{..},\ B\textbf{.},\ ra, \dashv M\tau SR$$

C2. $M\tau SR[A, B \leftrightarrow A]B$

$$\vdash M\tau SRA\textbf{..}, B \leftrightarrow A, B\textbf{.}rb, \dashv M\tau SR$$

C3. $M\tau SR[A \leftrightarrow B]B \leftrightarrow A$

$$\vdash M\tau SR,\ A \leftrightarrow B\textbf{..},\ B \leftrightarrow A\textbf{.}\ rc, \dashv M\tau SR$$

C4. $M\tau SR[A, A \leftrightarrow B]B$

$$\vdash M\tau SR\ A\textbf{..},\ A \leftrightarrow B\textbf{..},\ \ B \leftrightarrow A\textbf{.}\ \ C3,\ B\textbf{.}\ C2\textbf{.}, \dashv M\tau SR$$

C5. $M\tau SR[A, A \to B]B$

$$\vdash M\tau SR,\ A\textbf{..},\ A \to B\textbf{..},\ [A \to B] \leftrightarrow [\neg A \vee B]\textbf{.}\ ID1,$$
$$\neg A \vee B\textbf{.}\ \ C4,\ B\textbf{.}\ C1, \dashv M\tau SR$$

C6. $M\tau SR\ A \to [A \vee B]$

$$\vdash M\tau SR,\ \neg A \vee [A \vee B]\textbf{.}\ I2,\ [A \to [A \vee B]] \leftrightarrow [\neg A \vee [A \vee B]]\textbf{.}\ ID1,$$
$$A \to [A \vee B]\textbf{.}\ C2, \dashv M\tau SR$$

C7. $M\tau SR\ [A \to B, B \to A_1]\ A \to A_1$

$$\vdash M\tau SR,\ B \to A_1\textbf{..}\ \ [B \to A_1] \to [[\neg A \vee B] \to [\neg A \vee A_1]]\textbf{.}\ I4,$$
$$[\neg A \vee B] \to [\neg A \vee A_1]\textbf{.}\ C5,\ A \to B\textbf{..},\ [A \to B] \leftrightarrow [\neg A \vee B]\textbf{.}\ ID1,\ \ \neg A \vee B\textbf{.}\ C4,$$
$$\neg A \vee A_1\textbf{.}\ \ C5,\ [A \to A_1] \leftrightarrow [\neg A \vee A_1]\textbf{.}\ ID1,\ A \to A_1\textbf{.}\ C2, \dashv M\tau SR$$

C8. $M\tau SR\ A \to A$.

$$\vdash M\tau SR, A \to [A \vee A].\ C6,\ \neg[A \vee A] \vee A.\ I1,\ [[A \vee A] \to A] \leftrightarrow [\neg[A \vee A] \vee A]. ID1,$$
$$[A \vee A] \to A.\ C2,\ A \to A.\ C7, \dashv M\tau SR$$

C9. $M\tau SR\ A \to [B \vee A]$

$$\vdash M\tau SR, A \to [A \vee B].\ C6,\ [A \vee B] \to [B \vee A].\ I3,\ A \to [B \vee A].\ C7, \dashv M\tau SR$$

C10. $M\tau SR\ [B] A \to B$

$$\vdash M\tau SR,\ B \to [\neg A \vee B].\ C9,\ B..,\ \neg A \vee B.\ C5,$$
$$A \to B \leftrightarrow [\neg A \vee B].\ ID1, A \to B.\ C2,\ \dashv M\tau SR$$

C11. $M\tau SR\ A \vee \neg A$

$$\vdash M\tau SR, A \to A.\ C8,\ [A \to A] \leftrightarrow [\neg A \vee A].\ ID1,\ \neg A \vee A.\ C2,$$
$$[\neg A \vee A] \to [A \vee \neg A].\ I3,\ A \vee \neg A.\ C5, \dashv M\tau SR$$

C12. $M\tau SR\ A \to \neg\neg A$

$$\vdash M\tau SR, \neg A \vee \neg\neg A.\ C11, [A \to \neg\neg A] \leftrightarrow [\neg A \vee \neg\neg A].\ ID1,$$
$$A \to \neg\neg A.\ C2, \dashv M\tau SR$$

C13. $M\tau SR\ [A \to B] \to [\neg B \to \neg A]$

$$\vdash M\tau SR,\ B \to \neg\neg B.\ C11,\ [B \to \neg\neg B] \to [[\neg A \vee B] \to [\neg A \vee \neg\neg B]].\ I4,$$
$$[\neg A \vee B] \to [\neg A \vee \neg\neg B].\quad C5,\ [\neg A \vee \neg\neg B] \to [\neg\neg B \vee \neg A].\ I3,$$
$$[\neg A \vee B] \to [\neg\neg B \vee \neg A].\quad C7,\ [\neg A \vee B] \to [\neg\neg B \vee \neg A].\ C7,$$
$$[A \to B] \leftrightarrow [\neg A \vee B].\ ID1,\ [A \to B] \to [\neg A \vee B].\quad I5,$$
$$[A \to B] \to [\neg\neg B \vee \neg A].\quad C7,\ [\neg B \to \neg A] \leftrightarrow [\neg\neg B \vee \neg A].\ ID1,$$
$$[\neg\neg B \vee \neg A] \leftrightarrow [\neg B \to \neg A].\ rc,\ [\neg\neg B \vee \neg A] \to [\neg B \to \neg A].\quad I5,$$
$$[A \to B] \to [\neg B \to \neg A].\ C7, \dashv M\tau SR$$

C14. $M\tau SR\ [A \to B][B \to A_1] \to [A \to A_1]$

$$\vdash M\tau SR,\ A \to B..,\ [A \to B] \to [\neg B \to \neg A].\ C13,$$
$$\neg B \to \neg A.\ C5, [\neg B \to \neg A] \to [[A_1 \vee \neg B] \to [A_1 \vee \neg A]].\ I4,$$
$$[A_1 \vee \neg B] \to [A_1 \vee \neg A].\ C5, [A_1 \vee \neg A] \leftrightarrow [\neg A \vee A_1].\ I3,$$
$$[[A_1 \vee \neg A] \leftrightarrow [\neg A \vee A_1]] \to [[A_1 \vee \neg A] \to [\neg A \vee A_1].\ I5,$$
$$[A_1 \vee \neg A] \to [\neg A \vee A_1].\ C5,\ [A_1 \vee \neg B] \to [\neg A \vee A_1].\ C7,$$
$$[\neg B \vee A_1] \leftrightarrow [A_1 \vee \neg B].\ I3,$$
$$[[\neg B \vee A_1] \leftrightarrow [A_1 \vee \neg B]] \to [[\neg B \vee A_1] \to [A_1 \vee \neg B].I5,$$
$$[\neg B \vee A_1] \to [A_1 \vee \neg B].\ C5,$$
$$[\neg B \vee A_1] \to [\neg A \vee A_1].\ C7,$$
$$[B \to A_1] \leftrightarrow [\neg B \vee A_1].\quad ID1,$$
$$[[B \to A_1] \leftrightarrow [\neg B \vee A_1]] \to [[B \to A_1] \to [\neg B \vee A_1]].\ I5,$$
$$[B \to A_1] \to [\neg B \vee A_1].\ C5,$$
$$[B \to A_1] \to [\neg A \vee A_1].\ C7,$$
$$[A \to A_1] \leftrightarrow [\neg A \vee A_1].\ ID1,$$
$$[\neg A \vee A_1] \leftrightarrow [A \to A_1].\quad I3,$$
$$[[\neg A \vee A_1] \leftrightarrow [A \to A_1]] \to [[\neg A \vee A_1] \to [A \to A_1]].\ I5,$$
$$[\neg A \vee A_1] \to [A \to A_1].\ C5,\ [B \to A_1] \to [A \to A_1].\ C7, \dashv M\tau SR$$

C15. $[M\tau SR|\neg A|B \textit{ and } M\tau SR|\neg A|\neg B] \Rightarrow M\tau SR\ A$

$$\vdash M\tau SR, \vdash M\tau SR|\neg A|, B.., \neg B.., \neg B \to [\neg B \vee A].\ C6, \neg B \vee A.\ C5,$$
$$[B \to A] \leftrightarrow [\neg B \vee A].\quad ID1, [\neg B \vee A] \leftrightarrow [B \to A].\ C3,$$
$$[[\neg B \vee A] \leftrightarrow [B \to A]] \to [[\neg B \vee A] \leftrightarrow [B \to A]].\ I5, [\neg B \vee A] \leftrightarrow [B \to A].\ C5,$$
$$B \to A.\quad C5, A.\ C5,$$
$$\dashv M\tau SR|\neg A|, \neg A \to A.\ re, [\neg A \to A] \leftrightarrow [\neg\neg A \vee A].\ ID1,$$
$$\neg\neg A \vee A.\ C4, [\neg\neg A \vee A] \to [[A \vee \neg\neg A] \to [A \vee A].\ I4,$$
$$[A \vee \neg\neg A] \to [A \vee A].\ C5, [\neg\neg A \vee A] \to [A \vee \neg\neg A].\ I3,$$
$$A \vee \neg\neg A.\ C5, e\neg[A \vee A] \vee A.\ I1,$$
$$[[A \vee A] \to A] \leftrightarrow [\neg[A \vee A] \vee A].\ ID1, [A \vee A] \to A.\quad C2, A.\ C5, \dashv M\tau SR$$

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