AT THE ORIGINS OF MATHEMATICS IN GEORGIA

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Abstract. It is considered how trough centuries the mathematical knowledge was accumulated in Georgia which formed a basis for establishing in 20th century higher mathematical education in Georgian language and initiation research in mathematics.

Introduction

The historical development of Georgian nation was complicated and hard. During the long existence there were frequent internal wars and invasions from outside, which caused permanent struggle for survival and defending nationality. Despite such conditions Georgians were trying to develop national economy and culture, arts, literature and sciences. But very often the only way to perform this activity lay through cooperation with foreign countries more developed technically and culturally.

There are no written direct sources about creative work in and/or teaching of mathematics in Georgia in old times. But high level of architecture of a lot of old monuments (churches, castles, etc.) could not be reached without some mathematical knowledge (see [3]). Ancient Georgian manuscripts tell us that Georgians used their alphabet to elaborate an original system of numeration, they have had the own chronology. In early centuries there have been translations of books devoted to astronomy. Beginning from medieval times Georgians have translated or compiled mathematical textbooks, composed encyclopedic dictionaries. Mathematical knowledge is contained also in manuscripts involving a heritage of some Georgian philosophers.

1. Georgian Alphabetical Numeration. Georgian Chronology

Every nation's chronology is connected to some important date. Georgian chronology is connected with the foundation of the independent kingdom of Iberia and begins in 284 BC. It is the date of consolidation of "Parnavazianta" dynasty, under the ruling of *King Parnavaz* ($4^{\text{th}} - 3^{\text{th}}$ c. BC) Georgian Kingdom was formed.

According to chronological data of "Kartlis Tskhovreba" ("Life of Kartli", compendium of Georgian chronographic manuscripts; here Kartli means the whole Georgia, not only its central part) Parnavaz became the king in $303 - 302$ BC.

"Kartlis Tskhovreba" also tells us that Parnavaz spread the language of Kartli that is the Georgian language on the whole country and he created the Georgian writing. Some contemporary scientists (see, e.g., [7]) interpret this as introduction of the Georgian capital alphabet by Parnavaz as all-Georgian alphabet in 284 BC, while the alphabet itself was created much earlier (possibly in 5th century BC). Georgian

capital alphabet or "Mrgvlovani" is the oldest Georgian alphabet and is one of the oldest alphabets in the world.

The tradition of using an alphabet to designate numbers is known from ancient times, such as numerations of Babylonians and ancient Greeks. In Georgia alphabetical numeration was used from ancient times to the 19th century, though from the 10th century Indian figures were also used.

Georgian alphabetical numeration is based on Georgian alphabet in consecutive way. Some other numerations do not always follow the alphabet exactly and are disposed according to Greek alphabet. We can conclude that Georgian alphabetical numeration is only based on Georgian alphabet and was created independently. Contemporary researchers found to exist a developed system of cryptographic information put into the alphabet by its creator(s) via alphabetical numeration system (see [7] and [1]).

Adoption of chronology without creating calendar system was impossible. It is clear, that if Georgian chronology was introduced in 284 BC, the chronological calendar system was introduced at the same time.

In the ancient time Georgians used the moon calendar, according to which there are 354 days in a year. Later in the 5th century BC (some think 1st century) the moon calendar was changed by the sun calendar according to which a year consisted of 365 days. In the 1st century BC (or 1st century AD) a leap year system was introduced [7].

There were different chronological systems at the same time. In Christian Georgia there were three systems:

1. *Chronology "Dasabamitgan"* (from the very beginning). It is the period from the time when the God created the world, or "from Adam to Christ's birth".

2. *Chronology "Choronikoni"* "choronical era" and at the same time "March style year" at the time of 8th − 9th centuries. It was the chronology of 532 year-cycle. 532 year cycle was called influx.

Georgian "Big Choronicon" (chronology) (in 780 AD ended the 12th influx) and "Georgian beginning" 5 604 BC are interconnected

$$532 \times 12 = 6384; \ 5604 + 780 = 6384.$$

"Choronical" chronology as well as Georgian chronology "from the very beginning" are based on 532 year cycle. The number 532 is very convenient in any calendar calculations as $532 = 19 \times 28$, where 19 is the moon cycle and 28 is the sun cycle.

3. *Christian chronology* was the same in the whole Christian world. Its beginning is the birth of Christ.

2. 4th − 12th Centuries

There are many monuments, mostly clerical in Georgia and abroad, which have survived. But not many are left of Georgian manuscripts [6].

There is an old dated (492 − 493 AD) example of Georgian capital script curved on Bolnisi temple stone. Even older are the scripts discovered by Italian archeologists in Palestine, in Iuda desert in the ruins of the Georgian monastery near Bethlehem. The

floor of the monastery is designed with fine mosaic and on this mosaic they discovered Georgian writings in which some Georgian names are mentioned.

Arabs appeared in the world in the $7^{\text{th}} - 8^{\text{th}}$ centuries. They ruined most of the towns and castles in Georgia. As history says "No building was left and no human and animal food". Frequent wars destroyed farming and diminished population and stopped the process of constructing and cultural development.

Only in the $9^{\text{th}} - 10^{\text{th}}$ centuries started the country's economical development. The country began to flourish, Georgian language became very important, it was the language of literature, of ruling the country, civil communication and the only language of the church. There are many manuscripts and monuments of that epoch.

In the 11th century and later monasteries were very important places locally and abroad as the main educational schools were in the monasteries. Original and translating work was developing. Local centers had contacts with the centers abroad. With the help of the monasteries abroad Georgia could observe cultural successes in the educated countries. In Georgian centers abroad famous Georgian scholars (Ioane Mtatsmindeli on Mount Athos and his son Ekvtime Mtatsmindeli, also their successor Giorgi Mtatsmindeli, Ephrem Mtsire who worked in Syria on Black Mount and others) used their spare time for writing as they acknowledged the necessity and importance of their work for Georgian national interests. They actively took part in the social, cultural and state development of their country's life [6].

In the $11^{\text{th}} - 12^{\text{th}}$ centuries big cultural centers were built and developed which greatly contributed to the development of culture, science and education. In the conditions of that time in Georgia and in the middle age European countries monasteries were important cultural and educational centers. Many cultural centers were outside of Georgia too, in Syria and Palestine, in Sabatsminda monastery on Black Mount, near Antioch. Georgian monastery on Mount Athos in Greece, at Mangan Academy in Constantinople, in Petritsoni (present Bachkovo in Bulgaria) and others. In Georgia Ikalto and Gelati where Georgian clerics were active scientists and translators.

These centers were not only clerical but cultural development centers too. In old academies mathematics was always taught as one of the main subjects.

Unfortunately we have just a few practical mathematical manuscripts kept in Georgian museums which inform us about the level of mathematical education in Georgia. One of them is *Monk Ioane's* "Treatise". This monk lived in Jerusalem and the "Treatise" was written in 907. The main source of Georgian ecclesiastical technical chronology is the astronomical manuscript (A-38; Georgian museum) which was written in 1008. From this treatise we see that Georgians, as well as Jews and Greeks had the same type of numbering consecutively by 1 to 7 the week days Sunday to Saturday. We call these numbers "Day numbers". In Georgian treatises "the seven of the year" meant the number of the first day of the year. The author of the 11th century treatise at first chose the first of January as the beginning of the year, but to create the rule for finding "28 year sun cycle sevens" or "the sevens" of any year he accepted the 1st of March as the beginning of the year.

It is also very important to consider the treatise of the 12th century Georgian writer *Ioane Shavteli* in connection with Georgian calendar(this manuscript was copied in 1233). According to this treatise we can conclude that Georgia was the first to

introduce 25 of March as the beginning of the year.

For calendar calculations to begin the year with 25 March was more convenient as the 25 of March and the 1 of January always fall on the same day of the week when it is a leap year, which makes easy to go from this calendar to Julius calendar which starts on the first of January. Such transition could happen by the rules of the treatise of the 11th and 12th century [10], reading which needs a good acquaintance with Diophantine arithmetic.

It is especially important to mention manuscript (A-65; Georgian museum), illustrated astronomical and astrological treatise. It contains many astronomical facts and corresponds to the astronomical scientific level of that time, which is close to the achievements of astronomers and mathematicians of Arabian countries. The manuscript is dated from 1186 and is masterly fulfilled technically.

We have to mention that in the 1008 treatise and earlier inscriptions on the 10th century monastery there are numbers represented by a system close to positional one and the used numerals are similar to Arabian (actually Indian) numerals.

At the end of the 11th century (1089) the king of Georgia became David IV, whom the historians named as *"Agmashenebeli"* (the man who reconstructed and built the country).

In the period of his ruling David united Georgia and made it a strong kingdom. This period was the era of economical, political and cultural renaissance for Georgia. The king was highly educated, he knew many languages and knew and respected many religions, paid great attention and supported Georgian culture and educational development. To carry out reforms he needed new thinkers and creative people. He began with founding Georgian cultural and educational centers. He built Gelati church (1106) and founded a monastery which had great national aims. The monastery not only had to be the religious center but it also had to prepare people for civil national work. An academy was founded at the monastery, where he gathered the most educated Georgians of that time. He invited many Georgian scholars and scientists from abroad, from Byzantine Empire and other Christian countries. The academy had the authority of science and culture so it became like a "small Athos". Among the thinkers there was a Georgian philosopher, the author and translator of many works, *Ioane Petritsi* (appr. 1055 - 1130).

I. Petritsi was born in South Georgia, in Samtskhe. He was educated in Constantinople, Mangan Academy. For some time he lived in Petritsoni monastery, then he returned to Georgia and was the head of Gelati Academy. (At that time in the academy seven main subjects of science were taught: grammar, philosophy, rhetoric, arithmetic, geometry, music and astronomy.) He formed a philosophical school and raised some disciples. We have his works in philosophy and other important works in original and translations. One of the most important translation is the work of a philosopher and researcher, mathematician Proclus Diadochos (who wrote an influential commentary on the first book of Euclid's "Elements of Geometry"). The book consists of three parts. In the introduction Petritsi speaks on the problems which he faces while translating Proclus' treatise. The second, the most extensive part is the review of Proclus' treatise which is written in the form of comments on Proclus' book "The Basis of Theology". The conclusion is a brief review of all the themes given in

the introduction and in the comments of Proclus' treatise [8], [2].

I. Petritsi attached wide comments to this work, thus the translation acquired a new original form. For example Proclus' first code of regulations deals with the meaning "one" and covers a half of the page. And in the translation of Petritsi this review contains 14 pages.

In his work Petritsi writes about physics and mechanics in the form of Aristotle's naturphilosophy. For the first time we meet many terms and notions of mathematics and mechanics used in contemporary Georgian scientific literature. We'll discuss the notion "force" which we meet the first time in Georgian scientific literature.

In Petritsi's opinion this notion comes from Aristotle, as the property of substance. Then it goes beyond physics limits and is defined as the force of life, spiritual force (strength), inspiration force, history force and so on. So many acpects of force are connected with the appearance of the dynamical picture of the universe. The main defining point of "force" Petritsi considers "action". Petritsi writes: "Every action of the body has its proper force". Under this action we can consider any change of the subject as philosophical understanding it is spiritual change and the movement of the subject, change of the position as mechanical movement. According to Petritsi the force acts not only in the space but in the superior, highest substance as well. That is why there are perfect and non-perfect forces. Action is the property of the perfect force. The non-perfect force doesn't possess an action property. For the non-perfect force action is the possibility. He mainly speaks on spiritual force (strength) and sometimes on the physical force. The ideas about spiritual force definitely spread on physical forces. In this work we meet such important issues as the law of action − counteraction: "Each action causes equal counteractive force". This general law of philosophy is one of the main laws of nature and society and involves spiritual as well as physical forces. He found the existence of internal and external forces and the force is considered as the quantity which has a direction: which moves upward, downward, to the angle. Petritsi mentions that the force may be decomposed into parts.

In Petritsi's mechanical concept physical force is connected to movement (dynamic force) and deformation (static force).

Petritsi translated from Greek into Georgian a book by the $4^{\text{th}}-5^{\text{th}}$ centuries clerical writer Nemesios Emesian "For Human Nature" built on the works of Plato, Aristotle and other Greek philosophers. This book became the textbook for Gelati Akademy listeners.

The book was first published in Greek in 1802 in Halle, and its first Russian translation was published in 1901. Georgian translation was made seven centuries earlier in the $11^{\text{th}} - 12^{\text{th}}$ centuries.

Georgian philosophers respected Nemesios' book, they regarded the book as some kind of encyclopedia, in which ancient Greek philosophers' wisdom about "human" is collected. In a sense it is the measure of intellectual culture of Georgian thinkers in the $12^{\text{th}} - 13^{\text{th}}$ centuries.

Apart from other works Petritsi created astronomical and meteorological works in Gelati academy.

Arsen Ikaltoeli (from Ikalto) Vache's son, $11^{\text{th}}-12^{\text{th}}$ centuries, was a Georgian clergy man, writer, the representative of Georgian philosophical theology. He received educa-

tion in Mangan Academy (Constantinople). He started translating. In the 80s of the 11th century he worked in Syria, on Black Mount, under the guidance of Ephrem Mtsire. After David Agmashenebeli's invitation in about 1114 he returned to Georgia and continued working in Gelati Academy. Later, with the support of David Agmashenebeli he founded the academy of Ikalto at the monastery and he was the first rector of the academy. At last he settled and continued working in Shiomgvime monastery. Arsen Ikaltoeli translated a lot. He also composed a physics textbook which he taught at Gelati and Ikalto Academies.

In Gelati Academy Scientists observed astronomical events and they created original moon and sun calendars. They used for these calendars their own method of calculation.

At the end of the 12th century "The Man in the Panther's Skin" was written by *Shota Rustaveli*. The book is important not only for the literary qualities but also for its encyclopedic knowledge shown there.

11th − 12th centuries are the centuries of flourishing of the whole Georgia. The country was strong economically and it had military strength, which formed conditions for scientific development. This period is called "The Golden age of Georgian Nation".

3. 13th − 18th Centuries

There is absolutely different picture during the next few centuries. A hard time comes to the history of Georgian nation.

At the end of the 17th century there were better conditions to restore economics and culture. It was the period of rise. People tried to get education.

We have mathematical and technical writings created in the first half of the 18th century. They show that Georgians tried to be acquainted with the achievements of Asian and European Mathematicians.

Prince Alexander Bagrationi-Imeretian (the son of King Archil; 1674 − 1711) the author of the first technical textbook was born in Tbilisi. He knew several foreign languages. In 1697 he went abroad from Russia, where his father lived in emigration under patronage of Russian Emperor, to study military science. He got military education in Koenigsberg and Hague and used his knowledge to create artillery in Russia. He is the founder of Russian artillery and the first General-Feldzeuchmeister.

Prince Alexander created a new type of textbook in artillery. This manuscript is damaged, lacks the beginning, so it is unknown what title the author gave to the book. Now its name is "Artillery Study". This book is written in good Georgian and is illustrated. There are many drawings and schemes. The book is written to teach the basics of artillery. The book teaches how to use artillery equipment and war technique. There are treated some questions of mathematics, mechanics and chemistry. The book describes the technology of making artillery projectiles, their caliber and use, to aim the target, processes in the cannon while shooting (heating, overheating) and others. The book was written in 1705 − 1708.

There are the later versions of this book. In one of them it is a part of a collection which includes textbook of arithmetic and a description of missile composition [9], [4].

Creating Georgian book is the deed of *King Vakhtang the VI* (1675−1737), whose ac-

tivity created the whole era in the Georgian cultural and educational history. Vakhtang VI gathered a scientific group of Georgian society: Sulkhan-Saba Orbeliani and Zosime Orbeliani, Prince Vakhushti Bagrationi, Mamuka Baratashvili and others. Their first aim was to create textbooks, teach foreign languages. They began to translate the literature of different scientific branches. They believed that their work would help to educate young generation. Great is Vakhtang VI's merit in the development of astronomical and mathematical education. He translated (from Persian) the textbook by well known 15th century Samarkandian astronomer Ulugh Beg (Ulugbek). It is a treatise of cosmography and geodesy "Astrology" ("Zidg") part of which was printed in the printing-house in 1721 by name "The Book of Knowledge of Creation". From this book we know that Georgians took the basis of trigonometry and many issues of mathematics from Arabs, not from Europeans. This work was translated in Georgia in $1713 - 1718$, much earlier than in Europe (it was translated into French by Sedilo in 1853). In the book there are tables of the locations of the sun, the moon and other planets and catalogues of the stars. Vakhtang VI turned this translation into scientific collection where he also added the works of ancient Greeks and Western Europian scientists.

More extensive is the handwritten copy in which together with "Zidg" comments, he added "choroniconi" (chronology), so he made a good textbook in Astronomy.

King Vakhtang made a short course of astronomy and cosmography, in which he explains different astronomical notions and terms. He created a short textbook of chemistry, medicine, optics, he created a chemistry textbook in which he described chemical processes and the equipment used in them, how to use them and some vessels used. We can guess from the book that Vakhtang was well-informed of the traditions of Georgian metallurgy, goldsmith's work and other practical knowledge. His works concern physics issues too. He writes on some rules of optics which are scientifically important even today.

Vakhtang VI also translated the book by Nasir al Tusi "Astrolabe Textbook", where he describes the equipment for astronomical and geodetic observations [9].

Under the guidance of Vakhtang VI the existed basis of the Georgian mathematical and technical terminology was developed which advanced Georgian mathematical and technical thinking.

Sulkhan-Saba Orbeliani $(1658-1725)$ a famous writer, poet, lexicographer, encyclopedist, clergyman, diplomat and the tutor of Vakhtang VI. His works influenced much the education of new generations.

In $1685 - 1716$ he created "Georgian Dictionary" ("Lexiconi Kartuli"). The dictionary is mainly explanatory. There are definitions of many scientific notions, and we can see that the facts were sought from ancient Georgian sources, from collected manuscripts and then he completed it from achievements of foreign science, thus the book obtained encyclopedical contents.

Among many issues in the "Dictionary" there are many terms and notions of mathematics, astronomy and mechanics, which are widely used today. On the first pages of the dictionary there is a table of Georgian alphabetical numeration and Arabian-Indian, or contemporary numeration table. There are definitions from different European and Arab sources concerning arithmetic, geometry, astronomy, and mechanics. Sulkhan-

Saba gives an information about Georgian measures (in Georgian metrology there are up to 25 length and area units, and more than 50 units of mass and weight capacity, grain volume and money). The mathematical contents of the dictionary is well described in [3].

Among mathematical textbooks we have to mention the book by a translator and author Michael Davitashvili "A Book of Arithmetic and Geometry".

It is also important to mention the son-in-law of Vakhtang VI *Dimitri Tsitsishvili* $(1723 - 1777)$, who was well educated and knew languages. He entered St. Petersburg Academy of Sciences in 1742. He knew Latin, German and French languages. He learned the world history, physics, arithmetic and geometry. He translated textbook of arithmetic from German (1739) and other textbooks. He tried to establish Latin terminology in Georgian mathematical language. D. Tsitsishvili wrote the first original textbook of practical geodesy in Russian which was printed in 1757.

In 1766 astronomer and geographer J. Dalil translated and published in Paris maps of Georgia created by Vakhushti Bagrationi.

The head of Georgian church, writer and scientist, philosopher *Anton I* (his name was *Teimuraz Bagrationi*, $1720 - 1788$) helped to develop schools, libraries, theatres, art, poetry and science. He established rich libraries.

He changed and improved the educational system, private schools were turned into state schools. In Tbilisi (1755) and later in Telavi (1782) he founded seminaries. He himself created textbooks. Among the textbooks of different subjects translated by him is the translation of physics' textbook by K. Wolf. Anton I used the book himself to teach physics in the seminary (King Erekle II attended his lectures). He worked on terminology of physics. The translation has 246 comments, which show that Anton I was very well informed in natural sciences.

It is worth to mention the manuscript of Georgian museum H-2200 from the first part of the 18$^{\text{th}}$ century which is a part of some lost algebra textbook. The author is unknown. The survived part of the handwriting includes only logarithms. There is definition of logarithm, and the rules of logarithms of product and quotient, detailed explanation of logarithms of decimal system and using their tables. It is not an original mathematical idea, but it is interesting that the author is trying to change Latin and Greek terms with Georgian terms.

In the Georgian museum there is also a manuscript of an unknown author H-252, which is written in capitals of the clerical version of Georgian alphabet. The author says that mathematics is divided into two parts: pure mathematics and applied mathematics. The author includes to pure mathematics: arithmetic, algebra, geometry and trigonometry and to applied mathematics or "mixed" mathematics mechanics, optics, acoustics, music, astronomy, geography, architecture, artillery. He explains that one part of mathematics is called "pure mathematics" because it can define quantity of subjects and not other properties (color, substance and others). The mathematics which teaches us how to measure subjects using pure mathematics is called applied

mathematics. The manuscript includes a part of geometry, too.

4. 19th Century – the Beginning of 20th Century

The level of mathematical and technical thinking grows intensively in Georgia from the 19th century.

In 1800 Mathematics textbook by *G. Tarkhnishvili* (manuscript S-1531) was written. The work consists of two parts "Arithmetic" and "Using Arithmetic in Trade", which considers issues of commercial arithmetic. In this textbook, except mathematics, there are many problems taken from life.

Ioane Batonishvili (King George XII's son; 1768 - 1830) wrote his work (H-2180, in K. Kekelidze Institute of Manuscripts, Tbilisi, Georgia), the most extensive one among all Georgian mathematical manuscripts (it has 740 pages), which covers arithmetic, algebra, geometry, trigonometry and analytical geometry.

Dealing with the arithmetic issues, the author follows the standards of his time, but definitions are not always quite precise. In geometry he doesn't keep the whole style. Some theorems are given without proof, the author gives drawings instead, the material of trigonometry and analytical geometry is very brief.

The whole work shows that mathematics is not the author's specialty and he may not know the level of mathematics of that time.

Ioane Batonishvili in $1813 - 1828$ wrote encyclopedia of the science of that time "Kalmasoba" or "Khumarstsavla" (studying with jokes) which represents information of all branches of natural and humanitarian sciences. In the work a great part is mathematics: there are characterized arithmetic, geometry, trigonometry. He poses many interesting questions and gives answers to them.

Batonishvili's brother *David Bagrationi*, in 1818 made a textbook of "Short Physics". He knew European languages. That helped him to consider the achievements of physics of that time. We can feel the influence of French materialists.

At the end of the 19th century and at the beginning of the 20th century many Mathematical textbooks were translated into Georgian and many original perfect courses were composed in arithmetic and algebra, geometry and trigonometry, collections of problems and exercises and methodical literature. All of them were almost as perfect methodically as other books of that time, but the terminology is far from the perfect one.

Beginning from the first decade of the 20th century, many talented Georgian young people were acquainted with the achievements in mathematical sciences at European and Russian Universities. This along with the foundation of Tbilisi University in 1918 gave a launch of the well-known Georgian mathematical school (see, e.g., [5]).

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