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THIRD AND FOURTH BOUNDARY VALUE PROBLEMS OF STATICS OF THE THEORY OF ELASTIC TRANSVERSALLY ISOTROPIC BINARY MIXTURES FOR A HALF PLANE

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Abstract

In the paper the basic two-dimensional boundary value problems (BVPs) of statics of elastic transversally isotropic binary mixtures are investigated for a half plane. Using the potential method and the theory of singular integral equations, Fredholm type equations are obtained for all the considered problems. By the aid of these equations, Poisson type formulas of explicit solution are constructed for a half plane.

Key words and phrases: Boundary value problems, transversally-isotropic elastic mixtures, explicit solution.

AMS subject classification: 74E30; 74G05.

Introduction

The equation of the theory of elastic mixtures and a lot of important results have been obtained concerning mathematical problems of three-dimensional models (see Rushchitski [1] and references cited therein). As to the corresponding two-dimensional problems, they are not deeply investigated so far. The purpose of this paper is to consider the two-dimensional version of statics of the theory of elastic transversally-isotropic binary mixtures, which is the simplest anisotropic one and for which we can do explicit computations (it is assumed that the second component of the three-dimensional partial displacement vectors are equal to zero and the other components depend only on the variable x_1 and x_3). The fundamental and some other matrices of singular solutions for the system of equations of statics of a transversally-isotropic elastic mixtures are constructed in [2]. Using these matrices, the potentials are composed and the solution of basic BVPs for half-plane are constructed in [2,3,4].

In this paper we will explicitly construct solutions to the III and IY BVPs for a half plane. Applying a special integral representation formula for the displacement vector the problems are reduced to a simple system of integral equations.By the aid of these equations, Poisson type formula of explicit solutions are constructed for a half plane.

Some previous results

Let D denote the upper half-plane $x_3 > 0$. The boundary S of D is x_1 axis.

We say that a body is subject to a plane deformation if the second components u_2' and u_2'' of the partial displacements vectors $u'(u_1', u_2', u_3')$ and $u''(u_1'', u_2'', u_3'')$ vanish and the other components are functions of the variables only x_1, x_3. Then the basic equations of statics of a transversally isotropic elastic mixtures in the case of plane deformation read as [1]

$$C(\partial x)U = \begin{pmatrix} C^{(1)}(\partial x) & C^{(3)}(\partial x) \\ C^{(3)}(\partial x) & C^{(2)}(\partial x) \end{pmatrix} U = 0, \tag{1}$$

where

$$C^{(j)}(\partial x) = \begin{pmatrix} c_{11}^{(j)}\dfrac{\partial^2}{\partial x_1^2} + c_{44}^{(j)}\dfrac{\partial^2}{\partial x_3^2} & \left(c_{13}^{(j)} + c_{44}^{(j)}\right)\dfrac{\partial^2}{\partial x_1 \partial x_3} \\ (c_{13}^{(j)} + c_{44}^{(j)})\dfrac{\partial^2}{\partial x_1 \partial x_3} & c_{44}^{(j)}\dfrac{\partial^2}{\partial x_1^2} + c_{33}^{(j)}\dfrac{\partial^2}{\partial x_3^2} \end{pmatrix}, \quad j = 1, 2, 3.$$

$U(x) = U(u', u'')$ -is four-dimensional displacement vector, $u'(u_1', u_3')$ and $u''(u_1'', u_3'')$ are partial displacement vectors, depending on the variables x_1, x_3. $c_{pq}^{(j)}$ are constants.

The stress vector is defined as follows [2]

$$T(\partial_x, n)U = \begin{pmatrix} T^{(1)}(\partial x, n) \; T^{(3)}(\partial x, n) \\ T^{(3)}(\partial x, n) \; T^{(2)}(\partial x, n) \end{pmatrix}, \tag{2}$$

$$T^{(j)}(\partial_x, n) = \begin{pmatrix} c_{44}^{(j)}\partial x_3, \; c_{44}^{(j)}\partial x_1, \\ c_{13}^{(j)}\partial x_1, \; c_{33}^{(j)}\partial x_3, \end{pmatrix}, \quad j = 1, 2, 3,$$

$$\partial_x = (\partial x_1, \partial x_3), \quad \partial x_k = \frac{\partial}{\partial x_k}, k = 1, 3.$$

where $(0, 0, 1)$ are components of outside normal vector.

Definition. A vector function U defined in the region D is called regular , if $u_k', u_k'' \in C^2(D) \cap C^1(\bar{D})$, and the following conditions at infinity $u_k' = O(1), u_k'' = O(1), \varrho\partial_k u' = O(1), \varrho\partial_k u'' = O(1), k = 1, 3$ to be fulfilled with $\varrho^2 = x_1^2 + x_3^2$.

For the equation (1), we pose the following basic (BVPs). Find a regular vector U satisfying the system of equations (1) in D, if on the boundary S one of the following boundary conditions are given:

Problem III. The components u_1',u_1'' of displacement vector U and the components $(Tu)_3'$,$(Tu)"_3$ of stress vector are given on S

$$\lim_{x \to t} u_1'(t) = f_1(t), x \in D, t \in S,$$

$$\lim_{x \to t} u"_1(t) = f_3(t), x \in D, t \in S,$$

$$\lim_{x \to t} [Tu]_3' = f_2(t), x \in D, t \in S,$$

$$\lim_{x \to t} [Tu]_3" = f_4(t), x \in D, t \in S,$$

$$f_1, f_3 \in C^{1,\alpha}(S), f_2, f_4 \in C^{0,\alpha}(S), f_1, f_3 = c_0 + \frac{a}{\mid t \mid^{1+\beta}}, \int\limits_{-\infty}^{+\infty} f_2(t)dt = 0, \int\limits_{-\infty}^{+\infty} f_4(t)dt = 0.$$

$$f_2, f_4 = \frac{a}{\mid t \mid^{1+\beta}},$$

for large $\mid t \mid$, $a, c = constants, \beta > 0, \alpha > 0$.

Problem IY. The components $u_3', u_3"$ of displacement vector U and the components $(Tu)_1', (Tu)"_1$ of stress vector are given on S

$$\lim_{x \to t} u_3'(t) = f_1(t), x \in D, t \in S,$$

$$\lim_{x \to t} u_3"(t) = f_2(t), x \in D, t \in S,$$

$$\lim_{x \to t} [Tu]_1' = f_3(t), x \in D, t \in S,$$

$$\lim_{x \to t} [Tu]_1'' = f_4(t), x \in D, t \in S,$$

$$f_2, f_4 \in C^{1,\alpha}(S), f_1, f_3 \in C^{0,\alpha}(S), f_1, f_3 = \frac{a}{\mid t \mid^{1+\beta}}, \int\limits_{-\infty}^{+\infty} f_1(t)dt = 0, \int\limits_{-\infty}^{+\infty} f_3(t)dt = 0.$$

$$f_2, f_4 = c_0 + \frac{a}{\mid t \mid^{1+\beta}},$$

for large $\mid t \mid$, $a, c = constants, \beta > 0, \alpha > 0$. where f is given vector on S.

Solution of the third BVP

A solution of the III boundary value problem for a half plane will be sought in the form

$$u(x) = \frac{1}{\pi} Im \sum_{k=1}^{4} R^{(k)T} m \int\limits_{-\infty}^{+\infty} \frac{g(t)dt}{z_k - t}, \tag{3}$$

where g is unknown real vector function, m is an arbitrary real constant matrix which will be defined below, $z_k = x_1 + \alpha_k x_3$, α_k are the roots of the characteristic equation [2], $R^{(k)T}$ denote transposition of matrix $R^{(k)}$

$$\begin{aligned}
R^{(k)} &= \| R^{(k)}_{pq} \|_{4,4}, \\
R^{(k)}_{1j} &= c^{(1)}_{44}(A^{(k)}_{1j}\alpha_k + A^{(k)}_{j2}) + c^{(3)}_{44}(A^{(k)}_{j3}\alpha_k + A^{(k)}_{j4}), R^{(k)}_{1j} = -\alpha_k R^{(k)}_{2j}, \\
R^{(k)}_{2j} &= (c^{(1)}_{33}A^{(k)}_{2j} + c^{(3)}_{33}A^{(k)}_{j4})\alpha_k + c^{(1)}_{13}A^{(k)}_{1j} + c^{(3)}_{13}A^{(k)}_{j3}, R^{(k)}_{(3j} = -\alpha_k R^{(k)}_{4j}, \\
R^{(k)}_{3j} &= c^{(3)}_{44}(A^{(k)}_{1j}\alpha_k + A^{(k)}_{j2}) + c^{(2)}_{44}(A^{(k)}_{j3}\alpha_k + A^{(k)}_{j4}), \\
R^{(k)}_{4j} &= (c^{(3)}_{33}A^{(k)}_{2j} + c^{(2)}_{33}A^{(k)}_{j4})\alpha_k + c^{(3)}_{13}A^{(k)}_{1j} + c^{(2)}_{13}A^{(k)}_{j3}, j = 1, 2, 3, 4.
\end{aligned}$$

$A_{pq}^{(k)}$ are given in [2]. From (3) we obtain

$$T(\partial x, n)U = \frac{1}{\pi} Im \sum_{k=1}^{4} \int_{-\infty}^{+\infty} L^{(k)} m \frac{\partial}{\partial x_1} \frac{g(t)dt}{z_k - t}, \qquad (4)$$
$$(x_1, x_3) \in D,$$

where

$$L^{(k)} = \|L_{pq}^{(k)}\|_{4,4},$$
$$L_{11}^{(k)} = \alpha_k^2 L_{22}^{(k)},\ L_{12}^{(k)} = \alpha_k L_{22}^{(k)},\ L_{13}^{(k)} = \alpha_k^2 L_{24}^{(k)},\ L_{14}^{(k)} = -\alpha_k L_{42}^{(k)},$$
$$L_{23}^{(k)} = -\alpha_k L_{24}^{(k)},\ L_{34}^{(k)} = -\alpha_k L_{44}^{(k)},\ L_{33}^{(k)} = \alpha_k^2 L_{44}^{(k)},$$
$$L_{22}^{(k)} = -\Delta q_4 d_k [a_{44} + \alpha_k^2 (b_{11} + 2a_{34}) + a_{33}\alpha_k^4],$$
$$L_{24}^{(k)} = -\Delta q_4 d_k [a_{24} + \alpha_k^2 (-b_{33} + a_{14} + a_{23}) + a_{13}\alpha_k^4],$$
$$L_{44}^{(k)} = -\Delta q_4 d_k [a_{22} + \alpha_k^2 (b_{22} + 2a_{12}) + a_{11}\alpha_k^4],$$
$$\Delta = (c_{13}^{(1)} c_{13}^{(2)} - c_{13}^{(3)2})^2 - q_1 q_3 + \Delta(c_{11}^{(1)} a_{11} + c_{11}^{(2)} a_{33} + 2c_{11}^{(3)} a_{13}) > 0,$$
$$q_1 = c_{11}^{(1)} c_{11}^{(2)} - c_{11}^{(3)2}, q_3 = c_{33}^{(1)} c_{33}^{(2)} - c_{33}^{(3)2}, q_4 = c_{44}^{(1)} c_{44}^{(2)} - c_{44}^{(3)2},$$
$$d_1^{-1} = 2\alpha_1(\alpha_1^2 - \alpha_2^2)(\alpha_1^2 - \alpha_3^2)(\alpha_1^2 - \alpha_4^2),$$
$$d_2^{-1} = 2\alpha_2(\alpha_2^2 - \alpha_1^2)(\alpha_2^2 - \alpha_3^2)(\alpha_2^2 - \alpha_4^2),$$
$$d_3^{-1} = 2\alpha_3(\alpha_3^2 - \alpha_1^2)(\alpha_3^2 - \alpha_2^2)(\alpha_3^2 - \alpha_4^2),$$
$$d_4^{-1} = 2\alpha_4(\alpha_4^2 - \alpha_2^2)(\alpha_1^2 - \alpha_3^2)(\alpha_4^2 - \alpha_1^2),$$

$a_{11}, ..., a_{44}$ are the real constant values which characterizing mechanical properties of the elastic mixture in question and satisfy following conditions.

$$a_{11}\Delta = c_{11}^{(2)} q_3 - c_{33}^{(1)} c_{13}^{(2)2} + 2c_{33}^{(3)} c_{13}^{(2)} c_{13}^{(3)} - c_{33}^{(2)} c_{13}^{(3)2} > 0,$$
$$a_{13}\Delta = -c_{11}^{(3)} q_3 + c_{33}^{(2)} c_{13}^{(1)} c_{13}^{(3)} + c_{33}^{(1)} c_{13}^{(2)} c_{13}^{(3)} - c_{33}^{(3)} (c_{13}^{(1)} c_{13}^{(2)} + c_{13}^{(3)2}),$$
$$a_{22}\Delta = c_{33}^{(2)} q_1 - c_{11}^{(1)} c_{13}^{(2)2} + 2c_{11}^{(3)} c_{13}^{(2)} c_{13}^{(3)} - c_{11}^{(2)} c_{13}^{(3)2} > 0,$$
$$a_{33}\Delta = c_{11}^{(3)} q_3 - c_{33}^{(2)} c_{13}^{(1)2} + 2c_{33}^{(3)} c_{13}^{(1)} c_{13}^{(3)} - c_{33}^{(1)} c_{13}^{(3)2}) > 0,$$
$$a_{44}\Delta = c_{33}^{(1)} q_1 - c_{11}^{(2)} c_{13}^{(1)2} + 2c_{11}^{(3)} c_{13}^{(1)} c_{13}^{(3)} - c_{11}^{(1)} c_{13}^{(3)2} > 0,$$
$$a_{12}\Delta = c_{13}^{(2)} (c_{13}^{(1)} c_{13}^{(2)} - c_{13}^{(3)2}) - c_{11}^{(2)} c_{13}^{(1)} c_{33}^{(2)} - c_{33}^{(3)} c_{13}^{(2)} c_{11}^{(3)} + c_{13}^{(3)} (c_{33}^{(3)} c_{11}^{(2)} + c_{11}^{(3)} c_{33}^{(2)}),$$
$$a_{14}\Delta = -c_{13}^{(3)} (c_{13}^{(1)} c_{13}^{(2)} - c_{13}^{(3)2}) + c_{11}^{(2)} c_{13}^{(1)} c_{33}^{(3)} + c_{33}^{(1)} c_{13}^{(2)} c_{11}^{(3)}$$
$$-c_{13}^{(3)} (c_{33}^{(1)} c_{11}^{(2)} + c_{11}^{(3)} c_{33}^{(3)}),\ a_{23}\Delta = -c_{13}^{(3)} (c_{13}^{(1)} c_{13}^{(2)} - c_{13}^{(3)2}) + c_{11}^{(3)} c_{13}^{(1)} c_{33}^{(2)}$$
$$+c_{33}^{(3)} c_{13}^{(2)} c_{11}^{(1)} - c_{13}^{(3)} (c_{33}^{(2)} c_{11}^{(1)} + c_{11}^{(3)} c_{33}^{(3)}),\ a_{34}\Delta = c_{13}^{(1)} (c_{13}^{(1)} c_{13}^{(2)} - c_{13}^{(3)2})$$
$$-c_{11}^{(1)} c_{13}^{(2)} c_{33}^{(1)} - c_{33}^{(3)} c_{13}^{(1)} c_{11}^{(3)} + c_{13}^{(3)} (c_{33}^{(3)} c_{11}^{(1)} + c_{11}^{(3)} c_{33}^{(1)}),$$

Taking into account the following relation [5]

$$(\int_{-\infty}^{+\infty}\frac{g(t)dt}{t-z_k})^+ = \pi g(x_1)i + \int_{-\infty}^{+\infty}\frac{g(t)dt}{t-x_1},$$
$$\frac{g(t)}{(z_k-t)^2} = \frac{\partial}{\partial t}\frac{g(t)}{z_k-t} - \frac{g'(t)}{z_k-t}.$$

(4) can be rewritten in the form

$$T(\partial x, n)U = \frac{1}{\pi}Im\sum_{k=1}^{4}\int_{-\infty}^{+\infty}L^{(k)}m\frac{g'(t)dt}{z_k-t}, \quad (x_1,x_3)\in D, \tag{5}$$

for determining the unknown density $g(t)$, from (3) and (5) we obtain the following integral equation

$$ImDm(\pi g(x_1)i + \int_{-\infty}^{+\infty}\frac{g(t)dt}{t-x_1}) = C - F(x_1), x_1 \in S. \tag{6}$$

C is an arbitrary real vector, D is the following matrix

$$D = \begin{pmatrix} b_4 & R_{21} & 0 & R_{41} \\ 0 & L_{22} & 0 & L_{24} \\ 0 & R_{23} & b_4 & R_{43} \\ 0 & L_{24} & 0 & L_{44} \end{pmatrix},$$

$$R_{pq} = \sum_{k=1}^{4} R_{pq}^{(k)}, C = (0, c_2, 0, c_4)^T . F = (f_1, F_2, f_3, F_4)^T,$$
$$F_2 = \int_0^{x_1} f_2(t)dt, F_4 = \int_0^{x_1} f_4(t)dt,$$

Direct calculations give

$$detD = b_4^2(A_1C_1 - B_1^2)\Delta q_4\Delta_3 > 0, b_4 = q_3q_4,$$
$$\Delta_3 = b_4(m_1m_3 - 2\sqrt{\alpha_1\alpha_2\alpha_3\alpha_4}) + \Delta q_4\frac{1}{a_{11}}[(a_{11}a_{14} - a_{13}a_{24})^2 + \frac{a_{11}a_{33}q_1 + a_{14}^2q_3}{\Delta}].$$
$$C_1 = \sum_{k=1}^{4} d_k = iC, B_1 = \sum_{k=1}^{4} d_k\alpha_k^2 = -iB, A_1 = \sum_{k=1}^{4} d_k\alpha_k^4 = -iA,$$
$$D_1 = \sum_{k=1}^{4} d_k\alpha_k^6 = -iD,$$
$$m_1 = \sum_{k=1}^{4}\sqrt{a_k}, m_3 = (\sqrt{a_1a_2a_3} + \sqrt{a_1a_2a_4} + \sqrt{a_1a_3a_4} + \sqrt{a_2a_3a_4}) > 0,$$

If we choose D so that

$$Dm = E,$$

where E is the unit matrix, from the equation (6) we have $g = C - F$ and (3) takes the form

$$u(x) = -\frac{1}{\pi} Im \sum_{k=1}^{4} R^{(k)T} m \int_{-\infty}^{+\infty} \frac{F(t)dt}{z_k - t} - -Im \sum_{k=1}^{4} R^{(k)T} mCi. \tag{7}$$

By solving the matrix equation we obtain

$$m = \frac{1}{b_4^2 \Delta_3} \begin{pmatrix} m_{11} & m_{12} & 0 & m_{14} \\ 0 & m_{22} & 0 & m_{24} \\ 0 & m_{32} & m_{11} & m_{34} \\ 0 & m_{24} & 0 & m_{44} \end{pmatrix},$$

$$m_{11} = \frac{1}{b_4}, m_{24} = \frac{-L_{24}}{\Delta_3}, m_{22} = \frac{L_{44}}{\Delta_3}, m_{44} = \frac{L_{22}}{\Delta_3},$$
$$m_{12} = \frac{-a_{12}}{b_4} - \frac{1}{b_4\Delta_3}[-b_4 m_2(a_{22} + a_{11}\alpha_1\alpha_2\alpha_3\alpha_4) + b_4(b_{22} + 2a_{12})a_{11}\alpha_1\alpha_2\alpha_3\alpha_4$$
$$+\Delta q_4[a_{11}a_{22}(b_{11} + 2a_{34}) + (b_{22} + 2a_{12})a_{13}a_{24} - (a_{14} + a_{23} - b_{33})(a_{13}a_{22} + a_{11}a_{24})]],$$
$$m_{14} = \frac{-a_{14}}{b_4} - \frac{1}{b_4\Delta_3}[-b_4 m_2(a_{24} + a_{13}\alpha_1\alpha_2\alpha_3\alpha_4) + b_4(-b_{33} + a_{14})a_{11}\alpha_1\alpha_2\alpha_3\alpha_4$$
$$+\Delta q_4[a_{11}a_{24}(b_{11} + 2a_{34}) + (b_{22} + 2a_{12})a_{13}a_{44} - (a_{14} + a_{23} - b_{33})(a_{13}a_{24} + a_{11}a_{44})]],$$
$$m_{32} = \frac{-a_{23}}{b_4} - \frac{1}{b_4\Delta_3}[-b_4 m_2(a_{24} + a_{13}\alpha_1\alpha_2\alpha_3\alpha_4) + b_4(a_{14} + a_{23} - b_{33})a_{11}\alpha_1\alpha_2\alpha_3\alpha_4$$
$$+\Delta q_4[a_{13}a_{22}(b_{11} + 2a_{34}) + (b_{22} + 2a_{12})a_{33}a_{24} - (a_{14} + a_{23} - b_{33})(a_{13}a_{24} + a_{22}a_{33})]],$$
$$m_{34} = \frac{-a_{34}}{b_4} - \frac{1}{b_4\Delta_3}[-b_4 m_2(a_{44} + a_{33}\alpha_1\alpha_2\alpha_3\alpha_4) + b_4(b_{11} + 2a_{34})a_{11}\alpha_1\alpha_2\alpha_3\alpha_4$$
$$+\Delta q_4[a_{13}a_{24}(b_{11} + 2a_{34}) + (b_{22} + 2a_{12})a_{33}a_{44} - (a_{14} + a_{23} - b_{33})(a_{13}a_{44} + a_{33}a_{24})]], \tag{8}$$

Solution of the fourth BVP

Let us look for the solution to the fourth BVP in the form

$$u(x) = \frac{1}{\pi} Im \sum_{k=1}^{4} R^{(k)T} \kappa \int_{-\infty}^{+\infty} \frac{h(t)dt}{z_k - t}, \tag{9}$$

where h is unknown real vector function, κ is an arbitrary real constant matrix which will be defined below. From (10) we obtain

$$T(\partial x, n)U = \frac{1}{\pi} Im \sum_{k=1}^{4} \int_{-\infty}^{+\infty} L^{(k)} \kappa \frac{\partial}{\partial x_1} \frac{h(t)dt}{z_k - t}, \tag{10}$$
$$(x_1, x_3) \in D,$$

For determining h we have the integral equation

$$ImD_4\kappa(\pi g(x_1)i+\int\limits_{-\infty}^{+\infty}\frac{h(t)dt}{t-x_1})=C-F(x_1),x_1\in S. \qquad (11)$$

where $F=(F_1,f_2,F_3,f_4)^T,C=(c_1,0,c_3,0)^T,F_1=\int\limits_0^{x_1}f_1(t)dt,F_3=\int\limits_0^{x_1}f_3(t)dt$, D_4 is the following matrix

$$D_4=\begin{pmatrix} L_{11} & 0 & L_{13} & 0\\ R_{12} & b_4 & R_{32} & 0\\ L_{13} & 0 & L_{33} & 0\\ R_{14} & 0 & R_{34} & b_4\end{pmatrix},$$

If we choose κ so that $D_4\kappa=E$,where E is the unit matrix, rom the equation (12) we have $h=C-F$, and (11) takes the form

$$u(x)=-\frac{1}{\pi}Im\sum_{k=1}^{4}R^{(k)T}\kappa\int\limits_{-\infty}^{+\infty}\frac{F(t)dt}{z_k-t}-Im\sum_{k=1}^{4}R^{(k)T}\kappa Ci. \qquad (12)$$

where $C=(c_1,0,c_3,0)^T$ is an arbitrary constant vector.

By solving the matrix equation we obtain

$$\kappa=\frac{1}{b_4^2\Delta_3}\begin{pmatrix} \kappa_{11} & 0 & \kappa_{13} & 0\\ \kappa_{21} & \kappa_{22} & \kappa_{23} & 0\\ \kappa_{13} & 0 & \kappa_{33} & 0\\ \kappa_{41} & 0 & \kappa_{43} & \kappa_{22}\end{pmatrix},$$

where

$$\kappa_{11}=\tfrac{L_{33}}{\Delta_3},\kappa_{13}=-\tfrac{L_{13}}{\Delta_3},\kappa_{22}=\tfrac{1}{b_4},\kappa_{33}=\tfrac{L_{11}}{\Delta_3},$$

$$\kappa_{21}=-m_{12},\kappa_{23}=-m_{32},\kappa_{41}=-m_{14},\kappa_{43}=-m_{34}.$$

R E F E R E N C E S

1. Rushchitski Ya.Ya. Elements of Mixture Theory, Naukova Dumka, Kiev (1991).
2. Bitsadze L. On some solutions of statics of the theory of elastic transversally-isotropic mixtures, Reports of enlarged session of the seminar of I. Vekua Institute of Applied Mathematics, **16**, 1-3 (2001), 41-45.
3. Bitsadze L. The basic BVPs of statics of the theory of elastic transversally-isotropic mixtures, Seminar of I. Vekua Institute of Applied Mathematics, Reports, **26** – **27** (2001), 79-87.
4. Bitsadze L. The basic BVPs of statics of the theory of elastic transversally-isotropic mixtures for an infinite plane with an elliptic hole, Seminar of I. Vekua Institute of Applied Mathematics, Reports, **29** (2003), 46-52.
5. Muskhelishvili N.I. Singular integral equations, Boundary problems of the theory of functions and some of their applications in mathematical physics, (Russian) 3rd ed. *Nauka, Moscow*, 1968; English translation from 2nd Russian ed. (1946): *P. Noordhoff, Groningen*, 1953, corrected reprint *Dover Publications, Inc., N.Y.*, 1992.

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