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THE BASIC BOUNDARY VALUE PROBLEMS OF STATICS OF THE THEORY OF ELASTIC TRANSVERSALLY ISOTROPIC MIXTURES FOR AN INFINITE PLANE WITH AN ELLIPTIC HOLE

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Abstract

In the paper the basic two-dimensional boundary value problems (BVPs) of statics of elastic transversally isotropic binary mixtures are investigated for an infinite plane with elliptic hole. Using the potential method and the theory of singular integral equations, Fredholm type equations are obtained for all the considered problems. By the aid of these equations, Poisson type formulas of explicit solution are constructed for an infinite plane with elliptic hole.

Key words and phrases: boundary value problems, transversally-isotropic elastic mixtures, explicit solution.

AMS subject classification: 74E30,74G05

Introduction

Since the early sixties, the theory of elastic mixtures has become very popular in mechanics and engineering sciences. A lot of important results heve been obtained concerning mathematical problems of three-dimensional models (see Rushchitski [1] and references cited therein). As to the corresponding two-dimensional problems, they are not deeply investigated so far. The purpose of this paper is to consider the two-dimensional version of statics of the theory of elastic transversally-isotropic binary mixtures, which is the simplest anisotropic one and for which we can do explicit computations (it is assumed that the second component of the three-dimensional partial displacement vectors are equal to zero and the other components depend only on the variables x_1 and x_3). The fundamental and some other matrices of singular solutions for the system of equations of statics of a transversally-isotropic elastic mixtures are constructed in [1]. Using these matrices, the potentials are composed and the solution of basic BVPs for half-plane are constructed in [2,3].

In this paper we will explicity construct solutions to the BVPs for an infinite plane with elliptic hole. Applying a special integral representation formula for the displacement vector the problems are reduced to a simple system of integral equations.By the aid of these equations, Poisson type formula of explicit solutions are constructed for an infinite plane with elliptic hole.

Some previous results

Let D denote an infinite plane with elliptic hole. The boundary S of D is an ellipse with the semi-axis a and b.

We say that a body is subject to a plane deformation if the second components u_2' and u_2'' of the partial displacements vectors $u'(u_1', u_2', u_3')$ and $u''(u_1'', u_2'', u_3'')$ vanish and the other components are functions of the variables only x_1, x_3. Then the basic equations of statics of a transversally isotropic elastic mixtures in the case of plane deformation read as [1]

$$C(\partial x)U = \begin{pmatrix} C^{(1)}(\partial x) & C^{(3)}(\partial x) \\ C^{(3)}(\partial x) & C^{(2)}(\partial x) \end{pmatrix} U = 0, \tag{1}$$

where

$$C^{(j)}(\partial x) = \begin{pmatrix} c_{11}^{(j)} \dfrac{\partial^2}{\partial x_1^2} + c_{44}^{(j)} \dfrac{\partial^2}{\partial x_3^2} & (c_{13}^{(j)} + c_{44}^{(j)}) \dfrac{\partial^2}{\partial x_1 \partial x_3} \\ (c_{13}^{(j)} + c_{44}^{(j)}) \dfrac{\partial^2}{\partial x_1 \partial x_3} & c_{44}^{(j)} \dfrac{\partial^2}{\partial x_1^2} + c_{33}^{(j)} \dfrac{\partial^2}{\partial x_3^2} \end{pmatrix}, j = 1, 2, 3.$$

$U(x) = U(u', u'')$ -is four-dimensional displacement vector, $u'(u_1', u_3')$ and $u''(u_1'', u_3'')$ are partial displacement vectors, depending on the variables x_1, x_3. $c_{pq}^{(j)}$ are constants.

The stress vector is defined as follows [2]

$$T(\partial_x, n)U = \begin{pmatrix} T^{(1)}(\partial x, n) \, T^{(3)}(\partial x, n) \\ T^{(3)}(\partial x, n) \, T^{(2)}(\partial x, n) \end{pmatrix}, \tag{2}$$

$$T^{(j)}(\partial_x, n) = \begin{pmatrix} c_{11}^{(j)} n_1 \partial x_1 + c_{44}^{(j)} n_3 \partial x_3, & c_{13}^{(j)} n_1 \partial x_3 + c_{44}^{(j)} n_3 \partial x_1, \\ c_{44}^{(j)} n_1 \partial x_3 + c_{13}^{(j)} n_3 \partial x_1, & c_{44}^{(j)} n_1 \partial x_1 + c_{33}^{(j)} n_3 \partial x_3, \end{pmatrix}, j = 1, 2, 3,$$

$$\partial_x = (\partial x_1, \partial x_3), \partial x_k = \frac{\partial}{\partial x_k}, k = 1, 3.$$

where n_1, n_3 are components of outside normal vector.

Definition. A vector function U defined in the region D is called regular , if $u_k', u_k'' \in \in C^2(D) \cap C^1(\bar{D})$, and the following conditions at infinity $u_k' = O(1), u_k'' = O(1), \varrho \partial_k u' = = O(1), \varrho \partial_k u'' = O(1), k = 1, 3$ to be fulfilled with $\varrho^2 = x_1^2 + x_3^2, \partial_k = \dfrac{\partial}{\partial x_k}$.

For the equation (1), we pose the following basic (BVPs). Find a regular vector U satisfying the system of equations (1) in D, if on the boundary S one of the following boundary conditions is given:

Problem I. The displacement vector U is given on S

$$\lim_{x \to t} U(x) = [U(t)]^- = f(t), x \in D, t \in S,$$

Problem II. The stress vector TU is given on S

$$[T(\partial_t, n)U(t)]^- = F(t), t \in S,$$

where f and F are given vectors on S.

Solution of the first BVP

A solution of the first boundary value problem for an infinite plane with an elliptic hole will be sought in the form

$$u(x) = A_0 + \frac{1}{\pi}\int_S Im \sum_{k=1}^{4} N^{(k)} \frac{\partial}{\partial s} \ln \sigma_k(x,y) g(y) ds, \tag{3}$$

where g is unknown real vector function, A_0 is an arbitrary real constant vector which will be defined below, $\sigma_k = z_k - \xi_k$, $z_k = x_1 + i\alpha_k x_3$, $\xi_k = y_1 + i\alpha_k y_3$, $\frac{\partial}{\partial s} = n_1 \frac{\partial}{\partial y_3} - -n_3 \frac{\partial}{\partial y_1}$, $N^{(k)}$ is the following matrix.

$$N^{(k)} = A^{(k)} A, \quad A^{(k)} = \|A^{(k)}_{pq}\|_{4,4}, A^{(k)}_{pq} = A^{(k)}_{qp}, A = \tfrac{1}{\Delta_1}\|A_{pq}\|_{4,4},$$

$A^{(k)}$ is the constant matrix whose elements are given in [3] and the elements of matrix A are following

$$\begin{aligned}
A_{11} &= c^{(2)}_{11} q_4 C_1 + t_{11} B_1 + t_{12} A_1 + c^{(2)}_{44} q_3 D_1,\\
A_{13} &= t_{22} B_1 + t_{13} A_1 - c^{(3)}_{44} q_3 D_1 - c^{(3)}_{11} q_4 C_1,\\
A_{22} &= c^{(2)}_{44} q_1 C_1 + t_{44} B_1 + t_{42} A_1 + c^{(2)}_{33} q_4 D_1,\\
A_{24} &= -c^{(3)}_{44} q_1 C_1 + t_{66} B_1 + t_{62} A_1 - c^{(3)}_{33} q_4 D_1,\\
A_{33} &= c^{(1)}_{11} q_4 C_1 + t_{33} B_1 + t_{23} A_1 + c^{(1)}_{44} q_3 D_1,\\
A_{44} &= c^{(1)}_{44} q_1 C_1 + t_{55} B_1 + t_{52} A_1 + c^{(1)}_{33} q_4 D_1,
\end{aligned}$$

$$A_{12} = A_{14} = A_{21} = A_{23} = A_{34} = A_{43} = A_{41} = A_{32} = 0,$$
$$C_1 = \sum_{k=1}^{4} d_k, B_1 = \sum_{k=1}^{4} d_k \alpha_k^2, A_1 = \sum_{k=1}^{4} d_k \alpha_k^4, D_1 = \sum_{k=1}^{4} d_k \alpha_k^6.$$

$t_{11}, t_{12}, t_{13}, t_{44}, t_{33}, t_{66}, t_{62}, t_{55}, t_{52}$ - are given in [3], and

$$\Delta_1 = \frac{B_0}{\sqrt{a_1a_2a_3a_4}}[m_2 q_3 q_4(\delta_{11} + \sqrt{a_1a_2a_3a_4}\delta_{22}) + q_4(\delta_{11}\delta_{22} - k_2) +$$
$$m_3 m_1 q_3 q_4 (q_4 + q_3\sqrt{a_1a_2a_3a_4}) + q_3\sqrt{a_1a_2a_3a_4}(\delta_{11}\delta_{22} - k_1)],$$
$$\delta_{11} = c^{(1)}_{11}c^{(2)}_{44} + c^{(2)}_{11}c^{(1)}_{44} - 2c^{(3)}_{44}c^{(3)}_{11}, \ \delta_{22} = c^{(1)}_{33}c^{(2)}_{44} + c^{(2)}_{33}c^{(1)}_{44} - 2c^{(3)}_{33}c^{(3)}_{44},$$
$$B_0^{-1} = -\prod_{k=2}^{4}(\sqrt{a_1} + \sqrt{a_k})(\sqrt{a_2} + \sqrt{a_3})(\sqrt{a_2} + \sqrt{a_4})(\sqrt{a_3} + \sqrt{a_4}),$$
$$m_1 = \sum_{k=1}^{4}\sqrt{a_k}, \quad m_3 = (\sqrt{a_1a_2a_3} + \sqrt{a_1a_2a_4} + \sqrt{a_1a_3a_4} + \sqrt{a_2a_3a_4}) > 0,$$
$$m_2 = \sqrt{a_1a_2} + \sqrt{a_1a_3} + \sqrt{a_1a_4} + \sqrt{a_4a_2} + \sqrt{a_3a_2} + \sqrt{a_3a_4},$$
$$k_1 = \frac{1}{c^{(2)2}_{44}}[c^{(2)2}_{44}c^{(3)}_{13} - 2c^{(3)}_{13}c^{(2)}_{44}c^{(3)}_{44} + c^{(3)2}_{44}c^{(2)}_{13} + c^{(2)}_{44}q_4]^2 +$$
$$\frac{2q_4}{c^{(2)2}_{44}}[c^{(2)}_{44}c^{(3)}_{13} - c^{(3)}_{44}c^{(2)}_{13}]^2 + \frac{q_4^2}{c^{(2)2}_{44}}(c^{(2)}_{13} + c^{(2)}_{44})^2, \ q_3 = c^{(1)}_{33}c^{(2)}_{33} - c^{(3)2}_{33}, q_4 = c^{(1)}_{44}c^{(2)}_{44} - c^{(3)2}_{44},$$
$$k_2 = \alpha^{(2)2}_{13}c^{(1)}_{11}c^{(1)}_{33} + \alpha^{(1)2}_{13}c^{(2)}_{11}c^{(2)}_{33} + \alpha^{(3)2}_{13}(c^{(2)}_{33}c^{(1)}_{11} + 2c^{(3)}_{11}c^{(3)}_{33} + c^{(2)}_{11}c^{(1)}_{33}) -$$
$$-2\alpha^{(1)}_{13}\alpha^{(3)}_{13}(c^{(2)}_{11}c^{(3)}_{33} + c^{(3)}_{11}c^{(2)}_{33}) - 2\alpha^{(2)}_{13}\alpha^{(3)}_{13}(c^{(3)}_{11}c^{(1)}_{33} + c^{(1)}_{11}c^{(3)}_{33}) + 2\alpha^{(1)}_{13}\alpha^{(2)}_{13}c^{(3)}_{11}c^{(3)}_{33},$$

We will need to give the function $\frac{\partial}{\partial s}\ln\sigma_k ds$ in the form of series in the exterior elliptic domain. Since y belongs to the boundary of the domain, we have $y_1 = a\cos t$ and $y_3 = b\sin t$.

Simple calculations lead to the relation [4]

$$\frac{\partial}{\partial s}\ln\sigma_k ds = i\sum_{n=1}^{\infty}[\lambda_k^n e^{-int} - e^{int}]\tau_{k1}^n dt,$$

where

$$\lambda_k = \frac{a+\alpha_k ib}{a-\alpha_k ib}, \quad \tau_{k1}^{-1} = \frac{z_k + \sqrt{z_k^2 - a^2 + b^2 a_k}}{a - ib\alpha_k}.$$

It can be shown that when z belongs to the exterior of the ellips then $|\tau_{k1}| < 1$. Therefore

$$\begin{aligned} u(x) &= A + \frac{1}{\pi}\int_s Im\sum_{n=1}^{\infty}\sum_{k=1}^{4} iN^{(k)}[\lambda_k^n e^{-int} - e^{int})\tau_{k1}^n g(y)dt = \\ &= A + 2Re\sum_{n=1}^{\infty}\sum_{k=1}^{4} N^{(k)}[Q_n g_{-n} - Eg_n]\tau_{k1}^n, \end{aligned} \tag{4}$$

where

$$g_{-n} = \frac{1}{2\pi}\int_0^{2\pi} ge^{-int}dt, \quad g_n = \frac{1}{2\pi}\int_0^{2\pi} ge^{int}dt, n = 1,2,3,...$$

$$Q_n = \sum_{k=1}^{4} N^{(k)}\lambda_k^n, \quad N^{(k)}\lambda_k^n = N^{(k)}Q_n.$$

Taking into account the boundary condition, (4) can be rewritten as

$$A + 2Re\sum_{n=1}^{\infty}\sum_{k=1}^{4} N^{(k)}[Q_n g_{-n} - Eg_n]e^{-in\phi} = f(\phi).$$

From there we define the unknown coefficients

$$A = f_0, \quad Q_n g_{-n} - g_n = f_n, \tag{5}$$

where

$$f_n = \frac{1}{2\pi}\int_0^{2\pi} f(t)e^{int}dt \quad , n = 1,2,3,...$$

putting (5) into (4) ,it takes the form

$$\begin{aligned} u(x) &= f_0 + 2Im\sum_{k=1}^{4} N^{(k)} i\sum_{n=1}^{\infty}\tau_{k1}^n f_n = \frac{1}{2\pi}\int_0^{2\pi}[1 + 2Im\sum_{k=1}^{4} iN^{(k)}\sum_{n=1}^{\infty}\tau_{k1}^n e^{-int}]f(t)dt = \\ &= \frac{1}{2\pi}\int_0^{2\pi} Re\sum_{k=1}^{4} N^{(k)}[\frac{1+\tau_{k1}e^{it}}{1-\tau_{k1}e^{it}}f(t)dt, \quad x \in D. \end{aligned}$$

Thus we have obtained the Poisson formula for the solution of the first BVP for the infinite plane with an elliptic hole.For the regularity of the displacement vector in the domain D^- it is sufficient that $f \in C^{1,\alpha}(s), \dfrac{1}{2} < \alpha \leq 1$.

Solution of the second BVP

A solution of the second BVP is sought in the domain D in term of the simple layer potential

$$U(x) = \frac{1}{\pi} Im \sum_{k=1}^{4} \int_S R^{(k)T} L \ln \sigma_k h(y) ds, \quad (x_1, x_3) \in D, \quad z_k = x_1 + \alpha_k x_3, \tag{6}$$

where h is an unknown real vector-function, $R^{(k)T}$ denote transposition of matrix $R^{(k)}$,

$$\begin{aligned}
R^{(k)} &= \|R^{(k)}_{pq}\|_{4,4},\\
R^{(k)}_{1j} &= c^{(1)}_{44}(A^{(k)}_{1j}\alpha_k + A^{(k)}_{j2}) + c^{(3)}_{44}(A^{(k)}_{j3}\alpha_k + A^{(k)}_{j4}), R^{(k)}_{1j} = -\alpha_k R^{(k)}_{2j},\\
R^{(k)}_{2j} &= (c^{(1)}_{33}A^{(k)}_{2j} + c^{(3)}_{33}A^{(k)}_{j4})\alpha_k + c^{(1)}_{13}A^{(k)}_{1j} + c^{(3)}_{13}A^{(k)}_{j3}, R^{(k)}_{(3j} = -\alpha_k R^{(k)}_{4j},\\
R^{(k)}_{3j} &= c^{(3)}_{44}(A^{(k)}_{1j}\alpha_k + A^{(k)}_{j2}) + c^{(2)}_{44}(A^{(k)}_{j3}\alpha_k + A^{(k)}_{j4}),\\
R^{(k)}_{4j} &= (c^{(3)}_{33}A^{(k)}_{2j} + c^{(2)}_{33}A^{(k)}_{j4})\alpha_k + c^{(3)}_{13}A^{(k)}_{1j} + c^{(2)}_{13}A^{(k)}_{j3}, j = 1, 2, 3, 4.
\end{aligned}$$

$$L = -\frac{\Delta q_4}{\Delta_2\sqrt{a_1a_2a_3a_4}} \begin{pmatrix} L_{11} & 0 & L_{13} & 0\\ 0 & L_{22} & 0 & L_{24}\\ L_{13} & 0 & L_{33} & 0\\ 0 & L_{24} & 0 & L_{44} \end{pmatrix},$$

where

$$\begin{aligned}
L_{11} &= a_{22}B_1 + (b_{22} + 2a_{12})A_1 + a_{11}D_1,\\
L_{13} &= a_{24}B_1 + (-b_{33} + a_{14} + a_{23})A_1 + a_{13}D_1,\\
L_{22} &= \sqrt{a_1a_2a_3a_4}[a_{22}C_1 + (b_{22} + 2a_{12})B_1 + a_{11}A_1],\\
L_{33} &= a_{44}B_1 + (b_{11} + 2a_{34})A_1 + a_{33}D_1],\\
L_{24} &= \sqrt{a_1a_2a_3a_4}[a_{24}C_1 + (-b_{33} + a_{14} + a_{23})B_1 + a_{13}A_1],\\
L_{44} &= \sqrt{a_1a_2a_3a_4}[a_{44}C_1 + (b_{11} + 2a_{34})B_1 + a_{33}A_1],\\
\Delta_2 &= q_3(m_1m_3 - 2\sqrt{a_1a_2a_3a_4}) + \Delta(a_{11}a_{44} + a_{22}a_{33} - 2a_{13}a_{24}) > 0,
\end{aligned}$$

From (6) we obtain

$$T(\partial x, n)U = -\frac{1}{\pi} Im \sum_{k=1}^{4} \int_S L^{(k)} L \frac{\partial}{\partial s} \ln \sigma_k h(y) ds, , \quad (x_1, x_3) \in D, \tag{7}$$

where

$$
\begin{aligned}
&L^{(k)}=\|L^{(k)}_{pq}\|_{4,4},\\
&L^{(k)}_{11}=\alpha_k^2L^{(k)}_{22},\ L^{(k)}_{12}=\alpha_kL^{(k)}_{22},\ L^{(k)}_{13}=\alpha_k^2L^{(k)}_{24},\ L^{(k)}_{14}=-\alpha_kL^{(k)}_{42},\\
&L^{(k)}_{23}=-\alpha_kL^{(k)}_{24},\ L^{(k)}_{34}=-\alpha_kL^{(k)}_{44},\ L^{(k)}_{33}=\alpha_k^2L^{(k)}_{44},\\
&L^{(k)}_{22}=-\Delta q_4d_k[a_{44}+\alpha_k^2(b_{11}+2a_{34})+a_{33}\alpha_k^4],\\
&L^{(k)}_{24}=\Delta q_4d_k[a_{24}+\alpha_k^2(-b_{33}+a_{14}+a_{23})+a_{13}\alpha_k^4],\\
&L^{(k)}_{44}=-\Delta q_4d_k[a_{22}+\alpha_k^2(b_{22}+2a_{12})+a_{11}\alpha_k^4],\\
&\Delta=(c^{(1)}_{13}c^{(2)}_{13}-c^{(3)2}_{13})^2-q_1q_3+\Delta(c^{(1)}_{11}a_{11}+c^{(2)}_{11}a_{33}+2c^{(3)}_{11}a_{13})>0,
\end{aligned}
$$

$a_{11},...,a_{44}$ are the real constant values which characterise mechanical properties of the elastic mixture in queastion and satisfy following conditions.

$$
\begin{aligned}
&a_{11}a_{44}+a_{22}a_{33}-2a_{13}a_{24}=\frac{1}{a_{11}a_{44}}[(a_{11}a_{14}-a_{13}a_{24})^2+\frac{a_{11}a_{33}q_1+a_{14}^2q_3}{\Delta}]>0,\\
&m_1m_3-2\sqrt{a_1a_2a_3a_4}>0,\ a_{11}\Delta=c^{(2)}_{11}q_3-c^{(1)}_{33}c^{(2)2}_{13}+2c^{(3)}_{33}c^{(2)}_{13}c^{(3)}_{13}-c^{(2)}_{33}c^{(3)2}_{13}>0,\\
&a_{13}\Delta=-c^{(3)}_{11}q_3+c^{(2)}_{33}c^{(1)}_{13}c^{(3)}_{13}+c^{(1)}_{33}c^{(2)}_{13}c^{(3)}_{13}-c^{(3)}_{33}(c^{(1)}_{13}c^{(2)}_{13}+c^{(3)2}_{13}),\\
&a_{22}\Delta=c^{(2)}_{33}q_1-c^{(1)}_{11}c^{(2)2}_{13}+2c^{(3)}_{11}c^{(2)}_{13}c^{(3)}_{13}-c^{(2)}_{11}c^{(3)2}_{13}>0,\\
&a_{33}\Delta=c^{(3)}_{11}q_3-c^{(2)}_{33}c^{(1)2}_{13}+2c^{(3)}_{33}c^{(1)}_{13}c^{(3)}_{13}-c^{(1)}_{33}c^{(3)2}_{13})>0,\\
&a_{44}\Delta=c^{(1)}_{33}q_1-c^{(2)}_{11}c^{(1)2}_{13}+2c^{(3)}_{11}c^{(1)}_{13}c^{(3)}_{13}-c^{(1)}_{11}c^{(3)2}_{13}>0,\\
&a_{12}\Delta=c^{(2)}_{13}(c^{(1)}_{13}c^{(2)}_{13}-c^{(3)2}_{13})-c^{(2)}_{11}c^{(1)}_{13}c^{(2)}_{33}-c^{(3)}_{33}c^{(2)}_{13}c^{(3)}_{11}+c^{(3)}_{13}(c^{(3)}_{33}c^{(2)}_{11}+c^{(3)}_{11}c^{(2)}_{33}),\\
&a_{14}\Delta=-c^{(3)}_{13}(c^{(1)}_{13}c^{(2)}_{13}-c^{(3)2}_{13})+c^{(2)}_{11}c^{(1)}_{13}c^{(3)}_{33}+c^{(1)}_{33}c^{(2)}_{13}c^{(3)}_{11}-\\
&c^{(3)}_{13}(c^{(1)}_{33}c^{(2)}_{11}+c^{(3)}_{11}c^{(3)}_{33}),\ a_{23}\Delta=-c^{(3)}_{13}(c^{(1)}_{13}c^{(2)}_{13}-c^{(3)2}_{13})+c^{(3)}_{11}c^{(1)}_{13}c^{(2)}_{33}+\\
&c^{(3)}_{33}c^{(2)}_{13}c^{(1)}_{11}-c^{(3)}_{13}(c^{(2)}_{33}c^{(1)}_{11}+c^{(3)}_{11}c^{(3)}_{33}),\ a_{34}\Delta=c^{(1)}_{13}(c^{(1)}_{13}c^{(2)}_{13}-c^{(3)2}_{13})-\\
&c^{(1)}_{11}c^{(2)}_{13}c^{(1)}_{33}-c^{(3)}_{33}c^{(1)}_{13}c^{(3)}_{11}+c^{(3)}_{13}(c^{(3)}_{33}c^{(1)}_{11}+c^{(3)}_{11}c^{(1)}_{33}),
\end{aligned}
$$

For determining h, taking into account the following relation [4]

$$\frac{\partial}{\partial s}\ln\sigma_k=\frac{n_3-\alpha_kn_1}{\sqrt{z_k^2-a^2-b^2\alpha_k^2}}[1+\sum_{n=1}^{\infty}\tau^n_{k1}[\lambda^n_ke^{-int}+e^{int}],$$

from (7) we obtain the following equation

$$-\mu(t)+\frac{1}{\pi}Im\sum_{k=1}^{4}L^{(k)}L(-i)\int_0^{2\pi}[\frac{1}{1-e^{(\phi-t)i}}+\sum_{n=1}^{\infty}\lambda^n_ke^{-in(\phi-t)}]\mu(\phi)d\phi=f(t), \tag{8}$$

where $\mu(t)=\sqrt{b^2\cos^2t+a^2\sin^2t}h(t),\quad f(t)=\sqrt{b^2\cos^2t+a^2\sin^2t}F(t)$

Let's introduce the notations

$$X_0=\frac{1}{4\pi}\int_0^{2\pi}\mu dt,\ X_{-n}=\frac{1}{2\pi}\int_0^{2\pi}\mu e^{-int}dt,\ F_n=\frac{1}{2\pi}\int_0^{2\pi}Fe^{int}dt,\ n=1,2,3,...$$

Then (8) can be rewritten in the form

$$-\mu(t)-X_0-2Re\sum_{n=1}^{\infty}Q_ne^{-int}X_{-n}=F(t). \tag{9}$$

where

$$Q_n = \sum_{k=1}^{4} L^{(k)} L \lambda_k^n.$$

Direct calculations give

$$L^{(k)} L \lambda_k^n = L^{(k)} L Q_n.$$

From (9) it follows that

$$X_n + Q_n X_{-n} = -F_n. \tag{10}$$

It is obvious that for the compatibility of the equation (9) it is necesary that the condition $\int\limits_0^{2\pi} F dt = 0$ be satisfied.

Thus, if the principal vector of external stresses is equal to zero, then the displacement vectoris defined to within the rigid displacement, while the stress vector is defined uniquely. Substituting (10) in (7) we obtain

$$Tu = -2Im \sum_{k=1}^{4} L^{(k)} L \frac{n_3 - \alpha_k n_1}{\sqrt{z_k^2 - a^2 - b^2 \alpha_k^2}} \sum_{n=1}^{\infty} \tau_{k1}^n F_n. \tag{11}$$

Summating the last series, for the second BVP we have the Poisson type formula

$$u = -\frac{1}{\pi} Im \sum_{k=1}^{4} R^{(k)T} L \int\limits_0^{2\pi} \ln(1 - \tau_{k1} e^{-it}) F(t) dt, x \in D,$$

$$Tu = -\tfrac{1}{\pi} Im \sum_{k=1}^{4} L^{(k)} L \int\limits_0^{2\pi} \frac{\partial}{\partial s} \ln(1 - \tau_{k1} e^{-it}) F(t) dt.$$

For the regularity of the solution of the second BVP it is sufficient that

$$F \in C^{0,\alpha}(S), \ \int\limits_0^{2\pi} F(t) dt = 0, \ \alpha > 0,$$

R E F E R E N C E S

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