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ON A VIBRATION OF AN ISOTROPIC ELASTIC CUSPED PLATES UNDER ACTION OF AN INCOMPRESSIBLE VISCOUS FLUID

Chinchaladze N.

I.Vekua Institute of Applied Mathematics

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This paper deals with the bending vibration caused by interaction of a viscous fluid with a cusped plate considered in [1] (see this issue). In what follows all refeneces to formulas from [1] will be indicated by the index p, e.i., $(1)_p$.

We consider the interface problem of the interaction of a plate whose projection on $x_3 = 0$ occupies the domain Ω

$$\Omega = \{(x_1, x_2, x_3) : \; -\infty < x_1 < \infty, \;\; 0 < x_2 < l, \;\; x_3 = 0\},$$

and thickness is given by the following equation

$$2h(x_2) = h_0 x_2^{\alpha/3}(l - x_2)^{\beta/3}, \;\; h_0, \; l \; \alpha, \; \beta = \text{const}, \;\; h_0, \; l > 0, \;\; \alpha, \; \beta \geq 0, \qquad (1)$$

and of a flow of the fluid. Let the flow of the fluid be independent of x_1, parallel to the plane $0x_2x_3$, i.e. $v_1 \equiv 0$, and generating bending of the plate. Let at infinity, for pressure we have

$$p(x_2, x_3, t) \to p_\infty(t), \;\; \text{when} \;\; |x| \to \infty, \qquad (2)$$

and let for the velocity components conditions at infinity be

$$v_j(x_2, x_3, t) = O(1), \;\; j = 2, 3, \qquad (3)$$

where $v := (v_2, v_3)$ is a velocity vector of the fluid, $p(x_2, x_3, t)$ is a pressure, and $p_\infty(t)$ is given functions.

We suppose the fluid occupies the whole space R^3 but the middle plane Ω of the plate.

Let,

$$I := \{[0, l] \times 0\},$$
$$\Omega^f := \left\{x_1, x_2, x_3 : x_1 = 0, \; x := (x_2, x_3) \in \mathbb{R}^2 \backslash I\right\},$$
$$v_2, \;\; v_3 \in C^1(\Omega^f) \cap C^1(t > 0).$$

Transmission conditions for $v_j(x_2, x_3, t)$, $j = 2, 3$, can be written in the following form (compear with [2], [3], [4])

$$\begin{aligned} &v_2(x_2, 0, t) = 0, \quad x_2 \in]0, l[, \quad t \geq 0, \\ &v_3(x_2, 0, t) = \frac{\partial w(x_2, t)}{\partial t}, \quad x_2 \in]0, l[, \quad t \geq 0. \end{aligned} \tag{4}$$

Because of incompressibility we have

$$\operatorname{div} v(x_2, x_3, t) = 0, \quad (x_2, x_3) \in \Omega^f, \quad t \geq 0, \tag{5}$$

and (see e.g., [5], p.5)

$$\sigma^f_{jk} = -p\delta_{jk} + \mu \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right), \quad j, k = \text{const} = 2, 3, \tag{6}$$

where σ^f_{jk} is a stress tensor, μ is a coefficient of viscosity, δ_{jk} is Kroneker delta.

From (5) and (6) we obtain

$$\begin{aligned} \sigma^f_{33}(x_2, x_3, t) &= -p(x_2, x_3, t) + 2\mu \frac{\partial v_3(x_2, x_3, t)}{\partial x_3} \\ &= -p(x_2, x_3, t) - 2\mu \frac{\partial v_2(x_2, x_3, t)}{\partial x_2}. \end{aligned} \tag{7}$$

In virtue of (7) and (4) yields

$$\sigma^{f\,(\pm)}_{33}(x_2, 0, t) = p^{\pm}(x_2, 0, t).$$

Transmission conditions for p, taking into account of smallness of the thickness, we rewrite as follows

$$-p^+(x_2, 0, t) + p^-(x_2, 0, t) = q_0(x_2, t), \quad x_2 \in]0, l[. \tag{8}$$

Let the motion of the fluid be sufficiently slow, i.e., v_j and $v_{j,k}$ $(i, k = 2, 3)$ be so small that linearization of Navier-Stokes equations (see [2], [3], [4]) be admissible. Hence,

$$\begin{aligned} &\frac{\partial v_2}{\partial t} = -\frac{1}{\rho^f} \frac{\partial p}{\partial x_2} + \nu \Delta v_2, + F_2(x_2, x_3, t), \\ &\frac{\partial v_3}{\partial t} = -\frac{1}{\rho^f} \frac{\partial p}{\partial x_3} + \nu \Delta v_3 + F_3(x_2, x_3, t), \end{aligned} \tag{9}$$

where $\nu = \mu/\rho^f$, $\Delta := \dfrac{\partial^2}{\partial x_2^2} + \dfrac{\partial^2}{\partial x_3^2}$, $F := (F_2, F_3)$ is a volume force. Let

$$\begin{aligned} &v_i \in C^2(\Omega^f) \cap C(\mathbb{R}^2) \cap C(t > 0), \quad i = 2, 3; \\ &p \in C^2(\Omega^f); \\ &q_{,2}(\cdot, t) \in H([0, l]), \end{aligned}$$

and

$$F_i \in C^2(\Omega^f), \quad i = 2, 3.$$

where H is the class of Hölder continious functions.

Let

$$A^\infty(t) := \lim_{|x|\to\infty} \int\limits_0^{x_2} F_2(\xi_2, x_3, t) d\xi_2,$$

and

$$F_3(x_2, x_3, t)|_{|x|\to\infty} = O(1).$$

After differentiation of the first equation of (9) with respect to x_2, of the second equation of (9) with respect to x_3 and termwise summation, by virtue of (5), we obtain that $p(x_2, x_3, t)$ is satisfying the following equation

$$\Delta p(x_2, x_3, t) = \left(\frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3} \right) \rho^f. \tag{10}$$

In case of harmonic vibration in the fluid part, from (5), (9), (10) we obtain the following system (see formulaes $(11)_p$, $(12)_p$)

$$\Delta p_0(x_2, x_3) = \rho^f \left(\frac{\partial F_2^0}{\partial x_2} + \frac{\partial F_3^0}{\partial x_3} \right), \tag{11}$$

$$-\omega^2 u_j^0 = -\frac{1}{\rho^f} \frac{\partial p_0}{\partial x_j} + \nu i \omega \Delta u_j^0 + F_j^0(x_2, x_3), \quad j = 2, 3, \tag{12}$$

where $F_j(x_2, x_3) = e^{i\omega t} F_j^0(x_2, x_3)$.

Transmission conditions (8), (4), conditions at infinity (2) and (3) have the following forms

$$-p_0^+(x_2) + p_0^-(x_2) = q_0(x_2), \quad x_2 \in]0, l[, \tag{13}$$

$$u_3^0(x_2, 0) = w_0(x_2), \quad u_2^0 = 0, \quad x_2 \in]0, l[, \tag{14}$$

$$p_0\big|_{|x|\to\infty} = p_0^\infty, \quad u_j^0\big|_{|x|\to\infty} = O(1), \quad j = 2, 3. \tag{15}$$

After separating the real and imaginary parts in (12) we have

$$u_j^0 = \frac{1}{\omega^2 \rho^f} \frac{\partial p_0}{\partial x_j} - \frac{1}{\omega^2} F_j^0(x_2, x_3), \quad j = 2, 3, \tag{16}$$

$$\Delta u_j^0 = 0, \quad j = 2, 3. \tag{17}$$

Therefore, taking into account (11),

$$-\frac{\partial}{\partial x_j} \left(\frac{\partial F_2^0}{\partial x_2} + \frac{\partial F_3^0}{\partial x_3} \right) + \Delta F_j^0 = 0, \quad j = 2, 3. \tag{18}$$

Therefore, taking into account (13),

$$-\frac{\partial}{\partial x_j}\left(\frac{\partial F_2^0}{\partial x_2}+\frac{\partial F_3^0}{\partial x_3}\right)+\Delta F_j^0=0, \quad j=2,3. \tag{19}$$

We can rewrite (19) in the following form

$$\frac{\partial}{\partial x_j}\left(\frac{\partial F_2^0}{\partial x_3}-\frac{\partial F_3^0}{\partial x_2}\right)=0, \quad j=2,3,$$

i.e.,

$$\frac{\partial F_2^0(x_2,x_3)}{\partial x_3}-\frac{\partial F_3^0(x_2,x_3)}{\partial x_2}=\text{const}, \tag{20}$$

The solution of the equation (11) under condition (13), (15), using formula (20), has the following form (see [6])

$$\begin{aligned} p_0(x_2,x_3) &= -\frac{x_3}{2\pi}\int_0^l \frac{q_0(\xi_2)d\xi_2}{(\xi_2-x_2)^2+x_3^2}+\rho^f\int_0^{x_2} F_2^0(\xi_2,x_3)d\xi_2 \\ &+ p_0^\infty-\rho^f A_0^\infty, \end{aligned} \tag{21}$$

where $A^\infty := A_0^\infty e^{i\omega t}$.

Substituting (21) into (12), for u_2^0 and u_3^0 we obtain

$$u_2^0(x_2,x_3) = \frac{x_3}{\pi\omega^2\rho^f}\int_0^l \frac{q_0(\xi_2)(\xi_2-x_2)d\xi_2}{[(\xi_2-x_2)^2+x_3^2]^2}, \tag{22}$$

$$\begin{aligned} u_3^0 &= \frac{1}{2\pi\omega^2\rho^f}\int_0^l \frac{q_0(\xi_2)[x_3^2-(\xi_2-x_2)^2]}{[(\xi_2-x_2)^2+x_3^2]^2}d\xi_2+\frac{1}{\omega^2}\int_0^{x_2}\frac{\partial F_2^0}{\partial x_3}(\xi_2,x_3)d\xi_2 \\ &- \frac{1}{\omega^2}F_3^0(x_2,x_3). \end{aligned} \tag{23}$$

After consideration the limit of $u_2^0(x_2,x_3)$ when $x_3\to 0$, $x_2\in]0,l[$, we have

$$\begin{aligned} \lim_{x_3\to 0} u_2^0(x_2,x_3) &= \frac{1}{2\pi\omega^2\rho^f}\lim_{x_3\to 0} x_3\int_0^l q_0(\xi_2)\left(\frac{1}{(\xi_2-x_2)^2+x_3^2}\right)_{,\xi_2} d\xi_2 \\ &= \frac{1}{2\pi\omega^2\rho^f}\lim_{x_3\to 0}\left\{\frac{x_3q_0(l)}{(l-x_2)^2+x_3^2}-\frac{x_3q_0(0)}{x_2^2+x_3^2}-x_3\int_0^l\frac{q_0'(\xi_2)}{(\xi_2-x_2)^2+x_3^2}d\xi_2\right\} \\ &= -\frac{1}{2\pi\omega^2\rho^f}\lim_{x_3\to 0}\int_0^l\frac{q_0'(\xi_2)(x_3+(\xi_2-x_2)-(\xi_2-x_2))}{(\xi_2-x_2)^2+x_3^2}d\xi_2 \\ &= -\frac{1}{2\pi\omega^2\rho^f}\lim_{x_3\to 0}\left[\int_0^l\frac{q_0'(\xi_2)(x_3+(\xi_2-x_2))}{(\xi_2-x_2)^2+x_3^2}d\xi_2-\int_0^l\frac{q_0'(\xi_2)(\xi_2-x_2)}{(\xi_2-x_2)^2+x_3^2}d\xi_2\right] \end{aligned}$$

$$= -\frac{1}{2\pi\omega^2\rho^f}\left[\int_0^l \frac{q_0'(\xi_2)}{\xi_2 - x_2}d\xi_2 - \int_0^l \frac{q_0'(\xi_2)}{\xi_2 - x_2}d\xi_2\right] = 0.$$

The last expression means that transmission condition (14), for $u_2^0(x_2, x_3)$ is fullfiled.

Let now consider the following limit, when $x_2 \in]0, l[$,

$$\lim_{x_3\to 0}\int_0^l \frac{q_0(\xi_2)[x_3^2 - (\xi_2 - x_2)^2]}{[(\xi_2 - x_2)^2 + x_3^2]^2}d\xi_2 = \lim_{x_3\to 0}\left\{q_0(l)\frac{l - x_2}{(l - x_2)^2 + x_3^2}\right.$$

$$\left. + q_0(0)\frac{x_2}{x_2^2 + x_3^2} - \int_0^l \frac{q_0(\xi_2)(\xi_2 - x_2)}{(\xi_2 - x_2)^2 + x_3^2}d\xi_2\right\} = \lim_{x_3\to 0}\left\{q_0(l)\frac{l - x_2}{(l - x_2)^2 + x_3^2}\right.$$

$$+ q_0(0)\frac{x_2}{x_2^2 + x_3^2} - \int_0^l \frac{[q_0'(\xi_2) - q_0'(x_2)]\,(\xi_2 - x_2)}{(\xi_2 - x_2)^2 + x_3^2}d\xi_2$$

$$\left. - \frac{q_0'(x_2)}{2}\int_0^l \left\{\ln\left[(\xi_2 - x_2)^2 + x_3^2\right]\right\}_{,\xi_2} d\xi_2\right\} = \lim_{x_3\to 0}\left\{q_0(l)\frac{l - x_2}{(l - x_2)^2 + x_3^2}\right.$$

$$\left. + q_0(0)\frac{x_2}{x_2^2 + x_3^2} - \frac{q_0'(x_2)}{2}\ln\frac{(l - x_2)^2 + x_3^2}{x_2^2 + x_3^2} - \int_0^l \frac{[q_0'(\xi_2) - q_0'(x_2)]\,(\xi_2 - x_2)}{(\xi_2 - x_2)^2 + x_3^2}d\xi_2\right\}$$

(becouse of $q_0' \in H([0, l])$)

$$= \frac{q_0(l)}{l - x_2} + \frac{q_0(0)}{x_2} - q_0'(x_2)\ln\frac{l - x_2}{x_2} - \int_0^l \frac{q_0'(\xi_2) - q_0'(x_2)}{\xi_2 - x_2}d\xi_2.$$

On the other hand if we define the following supersingular integral as H'adamard integral, we analoguosly obtain

$$\int_0^l \frac{q_0(\xi_2)}{(\xi_2 - x_2)^2}d\xi_2 = \lim_{\varepsilon\to 0}\left(\int_0^{x_2-\varepsilon} \frac{q_0(\xi_2)}{(\xi_2 - x_2)^2}d\xi_2 + \int_{x_2+\varepsilon}^l \frac{q_0(\xi_2)}{(\xi_2 - x_2)^2}d\xi_2 + \frac{2q_0(x_2)}{\varepsilon}\right)$$

$$= \frac{q_0(l)}{l - x_2} + \frac{q_0(0)}{x_2} - q_0'(x_2)\ln\frac{l - x_2}{x_2} - \int_0^l \frac{q_0'(\xi_2) - q_0'(x_2)}{\xi_2 - x_2}d\xi_2.$$

Hence, using transmission condition (14) for u_3^0, we get the following expression

$$w_0(x_2) = -\frac{1}{2\pi\omega^2\rho^f}\int_0^l \frac{q_0(\xi_2)}{(\xi_2 - x_2)^2}d\xi_2 + \frac{1}{\omega^2}\int_0^{x_2} \frac{\partial F_2^0}{\partial x_3}(\xi_2, 0)d\xi_2 - \frac{1}{\omega^2}F_3^0(x_2, 0), \quad x_2 \in]0, l[, \qquad (24)$$

where the supersingular integral on the right hand side we define as H'adamard integral (see [9], [10]).

For the diflection we have the following equation (compear with $(9)_p$ from [1])

$$\left(h^3(x_2)w_0''(x_2)\right)'' \quad = \quad q_0(x_2) + 2\omega^2\rho^s h(x_2)w_0(x_2), \tag{25}$$

where ρ^s is a density of the plate. This equation we solve under boundary conditions giving in [1] (see this issue), Problems 1_p-10_p. In [7] and [8] is shown that equation (25) cab be reduced to the integral equation with symmetric positive definite kernel, and it solution has the following form

$$\begin{aligned} w_o(x_2) \quad &= \quad \int_0^l K(x_2,\xi)q_0(\xi)d\xi + \omega^2 \int_0^l \left(\int_0^l \Gamma(x_2,\eta,\lambda)g(\eta)K(\eta,\xi)d\eta \right) q_0(\xi)d\xi \\ &:= \quad \int_0^l K_1(x_2,\xi)q_0(\xi)d\xi, \end{aligned} \tag{26}$$

where $\Gamma(x_2,\xi,\lambda)$ is a resolvent of the symmetric kernel $K(x_2,\eta)\sqrt{g(x_2)g(\eta)}$ (for the explicit form of $K(x_2,\eta)$ (see [7], [8])). Kx_2,ξ is a continious function with respect to x_2 and ξ, and it is defined from the equation $(9)_p$ and depends on Problems 1_p-10_p.

Substituting (24) into (26), for $q_0(x_2)$ we obtain the following supersingular integral equation

$$\begin{aligned} &\int_0^l \frac{q_0(\xi_2)}{(\xi_2 - x_2)^2} d\xi_2 + 2\pi\omega^2\rho^f \int_0^l K_1(x_2,\xi_2)q_0(\xi_2)d\xi_2 \\ &= 2\pi\rho^f \left[\int_0^{x_2} \frac{\partial F_2}{\partial x_3}(\xi_2,0)d\xi_2 + F_3(0,0) - F_3^0(x_2,0) \right] =: f(x_2). \end{aligned} \tag{27}$$

We will find approximate solution of (27) using the method of solving given in books [9], [10] for $q_0'(x_2) := (dq_0(x_2)/dx_2) \in H([0,l])$.

Let divide interval $[0,l]$ into N parts as follows

$$y_N' := \frac{lk}{N}, \quad k = \overline{0,N}, \quad y_k := \frac{lk}{N} + \frac{l}{2N}, \quad k = \overline{0,N-1},$$

$$q_{0N} := (q_0(y_0), ..., q_0(y_{N_1})),$$

we will call q_{0N} approximate solution of (27). For q_{0N} we get the following system of linear equations

$$\begin{aligned} &-\frac{4N}{l}q_0(y_i) - \sum_{\substack{j=0 \\ j\neq i}}^{N-1} q_0(y_j) \left[\frac{1}{y_{j+i}' - y_i} - \frac{1}{y_j' - y_i} \right] \\ &+ \frac{2\pi\omega^2\rho^f l}{N} \sum_{j=0}^{N-1} K_1(y_i,y_j)q_0(y_j) = f(y_i), \quad i = \overline{0,N-1}. \end{aligned} \tag{28}$$

It is well-known (see [10]) that the determinant of the system (28) is not zero. Therefore, (28) is uniquely solvable.

Now, we have to estimate the error of the approximate solution of the equation (27). Let us denote by q_0^* the solution of (27), by q_{0N}^* the solution of (28) and let $\hat{q}_{0N}^*$ be a projection of q_0^* on y_k. Further, we obtain

$$\left| -\frac{4N}{l}\left(q_{0N}^*(y_i)-\hat{q}_{0N}^*(y_i)\right)-\sum_{\substack{j=0\\ j\neq i}}^{N-1}\left\{q_{0N}^*(y_j)-\hat{q}_{0N}^*(y_j)\right\}\left\{\frac{1}{y'_{j+i}-y_i}-\frac{1}{y'_j-y_i}\right\}\right|$$
$$=\left|\int_0^l \frac{q_0^*(\xi)}{(\xi-y_i)^2}d\xi-\frac{4N}{l}\hat{q}_{0N}^*(y_i)+\sum_{\substack{j=0\\ j\neq i}}^{N-1}\hat{q}_{0N}^*(y_j)\left\{\frac{1}{y'_{j+i}-y_i}-\frac{1}{y'_j-y_i}\right\}\right|$$
$$\leq\left|\int_{y'_i}^{y'_{i+1}}\frac{q_0^*(\xi)-q_0^*(y_i)}{(\xi-y_i)^2}\right|+\sum_{\substack{j=0\\ j\neq i}}^{N-1}\left|\int_{y'_i}^{y'_{i+1}}\frac{q_0^*(\xi)-q_0^*(y_j)}{(\xi-y_j)^2}d\xi\right| =: I_1+I_2.$$

Therefore, since $q'_0(x_2)\in H([0,l])$, we have that there exist $A=\text{const}>0$, and $\alpha_1=\text{const}$, $0<\alpha_1<1$ such that

$$|q'_0(y_1)-q'_0(y_2)|\leq A|y_1-y_2|^{\alpha_1}.$$

Using the following expression

$$\int_{y'_i}^{y'_{i+1}}\frac{d\xi}{\xi-y_i}=\ln|\xi-y_i|\Big|_{y'_i}^{y'_{i+1}}=\ln\frac{l/(2N)}{l/(2N)}=0,$$

we obtain

$$\begin{aligned} I_1 &= \left|\int_{y'_i}^{y'_{i+1}}\frac{q_0^*(\xi)-q_0^*(y_i)-(\xi-y_i)\left\{\frac{dq_0^*(\xi)}{d\xi}|_{\xi=y_i}\right\}}{(\xi-y_i)^2}d\xi\right| \\ &= \left|\int_{y'_i}^{y'_{i+1}}\frac{\frac{dq_0^*(\xi)}{d\xi}-\frac{dq_0^*(\xi)}{d\xi}|_{\xi=y_i}}{\xi-y_i}d\xi\right|\leq A\left(\frac{2N}{l}\right)^{-\alpha_1}. \end{aligned}\tag{29}$$

Analogously, we get

$$I_2\leq A(N-1)\left(\frac{2N}{l}\right)^{-\alpha_1}.\tag{30}$$

From (29) and (30) we obtain that the error of this method might be too large. For getting the most better results instead of the system (28) we consider the following system

$$a_{ii}q_0(y_i) - \sum_{j=0}^{N-1}{}' q_0(y_j)\left[\frac{1}{y'_{j+i}-y_i} - \frac{1}{y'_j - y_i}\right]$$
$$+\frac{2\pi\omega^2\rho^f l}{N}\sum_{j=0}^{N-1} K_1(y_i,y_j)q_0(y_j) = f(y_i), \quad i=\overline{0,N-1}. \tag{31}$$

where

$$a_{ii} := -\frac{4N}{l}\int\limits_{\Delta_{ii}} \frac{d\xi}{(\xi-y_i)^2}, \quad \Delta_{ii} := [0,l]\cap\left[y'_i - \frac{n}{N}, y'_{i+1}+\frac{n}{N}\right],$$

$$n := \sqrt{N} \quad \sum{}' := \sum_{\substack{j=0 \\ j\neq i-1,\ i,\ i+1}}^{N-1}.$$

After repeating above calculation we get

$$|q_0^* - q_{0N}^*| \leq A\left(\frac{2n}{l}\right)^{-\alpha_1},$$

where q_0^* and q_{0N}^* are the solutions of the equations (27) and (31) respectively.

After calculating q_{0N}, from (19) and (24) we get approximate expressions for $p_0(x_2,x_3)$ and $w_0(x_2)$, as follows

$$\begin{aligned} p_0(x_2,x_3) &= -\frac{x_3 l}{2\pi N}\sum_{j=0}^{N-1}\frac{q_0(y_j)}{(y_j-x_2)^2+x_3^2} + \rho^f\int\limits_0^{x_2} F_2(\xi_2,x_3)d\xi_2 \\ &+ p_0^\infty + A_0^\infty, \quad (x_2,x_3)\in\Omega^f; \end{aligned}$$

$$\begin{aligned} w_0(y_i) &= -\frac{1}{2\pi\omega^2\rho^f}\left\{a_{ii}q_0(y_i) - \sum_{j=0}^{N-1}{}' q_0(y_j)\left[\frac{1}{y'_{j+i}-y_i} - \frac{1}{y'_j-y_i}\right]\right\} \\ &+ \frac{1}{\omega^2}\left(\int\limits_0^{y_i}\frac{\partial F_2}{\partial x_3}(\xi_2,0)d\xi_2 - F_3^0(y_i,0)\right), \quad x_2\in]0,l[, \end{aligned}$$

Let us denote by $\bar{w}_0(y_i)$ the projection of w_0 on y_i and let estimate the error of the approximate solution of deflection. If we repeat the above calculation we get

$$|\bar{w}_0(y_i) - w_0(y_i)| \leq \frac{A}{2\rho^f\pi\omega^2}\left(\frac{2n}{l}\right)^{-\alpha_1}.$$

Further, after Substituting $p_0(x_2,x_3)$ in (16) we obtain $u_j^0(x_2,x_3)$.

$$u_3^0(x_2,x_3) = -\frac{1}{2\pi N\omega^2\rho^f}\sum_{j=0}^{N-1}\frac{q_0(y_j)(x_3^2-(y_j-x_2)^2)}{[(y_j-x_2)^2+x_3^2]^2}$$

$$+\frac{1}{\omega^2}\left(\int\limits_0^{x_2}\frac{\partial F_2}{\partial x_3}(\xi_2,x_3)d\xi_2-F_3^0(x_2,x_3)\right),$$

$$u_2^0(x_2,x_3) = \frac{x_3 l}{\pi N\omega^2\rho^f}\sum_{j=0}^{N-1}\frac{q_0(y_j)(y_j-x_2)}{[(y_j-x_2)^2+x_3^2]^2}, \quad (x_2,x_3)\in\Omega^f.$$

Proposition *In case of the harmonic vibration of the plate with two cusped edges under action of the incompressible viscous fluid [i.e., equations (11), (12), (25) under transmission conditions (13), (14) conditions at infinity (15) and BCs (see Problems* 1_p*-*10_p *in[1])] all quantities can be expressed by lateral load* $(q_0(x_2))$ *[see formulas (21)-(24)] and for the calculating of* $q_0(x_2)$ *we get (27) type super singular integral equation, where supersingular integral is defined as H'adamard integral. This equation has solution in class* $q_0' \in H([0,l])$.

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