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A CUSPED ELASTIC PLATE-IDEAL INCOMPRESSIBLE FLUID INTERACTION PROBLEM

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Last years the direct and inverse problems connected with the interaction between difference vector fields have received much attention in the mathematical and engineering scientific literature and have been intensively investigated. A lot of authors have considered and studied in deteil the interaction problems of interaction between an elastic isotropic body, which occupies a bounded region and where a three-dimensional elastic vector field is to be defined, and some isotropic medium (e.g, fluid), which occupies the unbounded exterior region. But interaction problems when the profile of an elastic part is cusped one on some part or on the whole boundary was not considered. The present work is devoted to such problems.

Let us consider the interface problem of the interaction of a plate whose projection on $x_3 = 0$ occupies the domain Ω

$$\Omega = \{(x_1, x_2, x_3) : \; -\infty < x_1 < \infty, \;\; 0 < x_2 < l, \;\; x_3 = 0\},$$

thickness is given by the following equation

$$2h(x_2) = h_0 x_2^{\alpha/3}(l - x_2)^{\beta/3}, h_0, \; l \; \alpha, \; \beta = \text{const}, \;\; h_0, \; l > 0, \;\; \alpha, \; \beta \geq 0, \qquad (1)$$

and of a flow of the fluid. Let the flow of the fluid be independent of x_1, parallel to the plane $0x_2x_3$, i.e. $v_1 \equiv 0$, and generating bending of the plate. Let at infinity, for pressure we have

$$p(x_2, x_3, t) \to p_\infty(t), \;\; \text{when} \;\; |x| \to \infty, \qquad (2)$$

and let for the velocity components conditions at infinity

$$v_2(x_2, x_3, t) = O(1), \;\; v_3(x_2, x_3, t) \to v_{3\infty}(t), \qquad (3)$$

where $v := (v_2, v_3)$ is a velocity vector of the fluid, $p(x_2, x_3, t)$ is a pressure, and $v_{3\infty}(t)$, $p_\infty(t)$ are given functions.

In what fellows we suppose that the plate is so thin that, we can assume: the fluid occupies the whole space R^3 but the middle plane Ω of the plate.

Let,

$$I := \{[0,l] \times 0\},$$
$$\Omega^f := \left\{x_1, x_2, x_3 : x_1 = 0,\ x := (x_2, x_3) \in \mathbb{R}^2 \backslash I\right\},$$
$$v_2,\ \ v_3 \in C^1(\Omega^f) \cap C^1(t > 0).$$

Transmission conditions for $v_3(x_2, x_3, t)$ we can write in the following form (compear with [1], [2], [3])

$$v_3(x_2, 0, t) = \frac{\partial w(x_2, t)}{\partial t},\ \ x_2 \in]0, l[,\ \ t \geq 0. \tag{4}$$

Because of incompressibility we have

$$\operatorname{div} v(x_2, x_3, t) = 0,\ \ (x_2, x_3) \in \Omega^f,\ \ t \geq 0, \tag{5}$$

In case of ideal fluid in virtue of $\sigma^f_{jk} = -p\delta_{jk}$ we get

$$\sigma^f_{33}(x_2, \overset{(\pm)}{h}(x_2), t) = -p(x_2, \overset{\pm}{h}(x_2), t),$$

where σ^f_{jk} is a stress tensor, $j, k = 2, 3$

Therefore, the transmission condition for p has the following form

$$\begin{aligned} &- \ \ p(x_2, \overset{(-)}{h}(x_2), t) \cos(\overrightarrow{n}(x_2, \overset{(-)}{h}(x_2)), x_3) \\ &- \ \ p(x_2, \overset{(+)}{h}(x_2), t) \cos(\overrightarrow{n}(x_2, \overset{(+)}{h}(x_2)), x_3) = q(x_2, t),\ \ x_2 \in]0, l[, \end{aligned} \tag{6}$$

where $q(x_2, t)$ is a lateral load of the plate.

In case of the potential motion of the flow there exists a complex function $\Phi = \psi + i\varphi$ such that

$$\begin{aligned} \frac{\partial \varphi(x_2, x_3, t)}{\partial x_2} &= \frac{\partial \psi(x_2, x_3, t)}{\partial x_3} = v_2(x_2, x_3, t), \\ \frac{\partial \varphi(x_2, x_3, t)}{\partial x_3} &= -\frac{\partial \psi(x_2, x_3, t)}{\partial x_2} = v_3(x_2, x_3, t). \end{aligned} \tag{7}$$

The pressure is given by the formula

$$p(x_2, x_3, t) = \rho^f \left[\frac{v^2_\infty}{2} + \frac{p_\infty}{\rho^f} + \frac{\partial \varphi_\infty}{\partial t} - \frac{\partial \varphi}{\partial t} - \frac{1}{2}(v_2^2 + v_3^2)\right]. \tag{8}$$

In case under consideration $w(x_2, t)$ is given by the equation [4]

$$(h^3(x_2) w,_{22}(x_2, t)),_{22} = q(x_2, t) - 2\rho^s h(x_2) \frac{\partial^2 w(x_2, t)}{\partial t^2},\qquad 0 < x_2 < l, \tag{9}$$

where ρ^s is a density of the plate.

Taking into account transmission condition (6), we have

$$\left(x_2^\alpha (l - x_2)^\beta w,_{22}(x_2, t)\right),_{22} = -\frac{2h(x_2)\rho^s}{h_0^3} w,_{tt}(x_2, t) +$$

$$+\frac{p(x_2, \overset{(-)}{h}(x_2), t)\cos(\overrightarrow{n}(x_2, \overset{(-)}{h}(x_2)), x_3) + p(x_2, \overset{(+)}{h}(x_2), t)\cos(\overrightarrow{n}(x_2, \overset{(+)}{h}(x_2)), x_3)}{h_0^3}.$$

For $\Phi_{,2}(x_2, x_3, t) = -v_3 + iv_2$, in view of (7), (4) and (3), we get the following expression (see [5])

$$\Phi_{,2} = -\frac{1}{\pi i\sqrt{(x_2+ix_3)(x_2+ix_3-l)}}\int\limits_0^l \frac{\sqrt{(\xi_2+ix_3)(\xi_2+ix_3-l)}}{(\xi_2-x_2)-ix_3} w_{,t}(\xi_2, t)d\xi_2$$

$$+v_{3\infty}(t)\frac{x_2+ix_3-l/2}{\sqrt{(x_2+ix_3)(x_2+ix_3-l)}}. \tag{10}$$

From (10), we have expressions for v_2 and v_3 as follows

$$\begin{aligned} v_2(x_2, x_3, t) &= -\frac{1}{\pi}\int\limits_0^l R_1(\xi, x_2, x_3)w_{,t}(\xi, t)d\xi + v_{3\infty}(t)R_3(x_2, x_3) \\ v_3(x_2, x_3, t) &= \frac{1}{\pi}\int\limits_0^l R_2(\xi, x_2, x_3)w_{,t}(\xi, t)d\xi + v_{3\infty}(t)R_4(x_2, x_3), \end{aligned}$$

where

$$R_1(\xi, x_2, x_3) = \frac{\sqrt{r(\xi, x_3)}}{\sqrt{r(x_2, x_3)}}$$
$$\times\frac{(x_2-\xi)\cos[(\phi(\xi, x_3)-\phi(x_2, x_3))/2] + x_3\sin[(\phi(\xi, x_3)-\phi(x_2, x_3))/2]}{(\xi-x_2)^2+x_3^2},$$
$$R_2(\xi, x_2, x_3) = \frac{\sqrt{r(\xi, x_3)}}{\sqrt{r(x_2, x_3)}}$$
$$\times\frac{(x_2-\xi)\sin[(\phi(\xi, x_3)-\phi(x_2, x_3))/2] + x_3\cos[(\phi(\xi, x_3)-\phi(x_2, x_3))/2]}{(\xi-x_2)^2+x_3^2},$$

$$R_4(x_2, x_3) = \left\{(x_2-l/2)\cos\frac{\phi(x_2, x_3)}{2} + x_3\sin\frac{\phi(x_2, x_3)}{2}\right\}\frac{1}{\sqrt{r(x_2, x_3)}},$$
$$R_3(x_2, x_3) = \left\{(x_2-l/2)\sin\frac{\phi(x_2, x_3)}{2} + x_3\cos\frac{\phi(x_2, x_3)}{2}\right\}\frac{1}{\sqrt{r(x_2, x_3)}},$$

here $\phi(x_2, x_3)$ is defined by either

$$cos\phi(x_2, x_3) = (x_2^2 - x_3^2 - lx_2)/r(x_2, x_3)$$

or

$$sin\phi(x_2, x_3) = (2x_2 - l)x_3/r(x_2, x_3)$$

and

$$r(x_2,x_3)=\sqrt{(x_2^2-x_3^2-lx_2)^2+((2x_2-l)x_3)^2}.$$

By means of the latter, in view of (7), we can calculate φ which we have to substitute in (8)

$$\begin{aligned}
p(x_2,x_3,t) &= \frac{\rho^f}{\pi}\int_0^l w_{,tt}(\xi,t)\int_0^{x_3} R_2(\xi,x_2,x_3)dx_3d\xi + v_{3\infty}(t)\rho^f\int_0^{x_3} R_4(x_2,x_3)dx_3\\
&+ \rho^f\left[\frac{v_\infty^2(t)}{2}+\frac{p_\infty(t)}{\rho^f}+\frac{\partial\varphi_\infty(t)}{\partial t}\right]\\
&- \frac{\rho^f}{2}\left\{\left(\frac{1}{\pi}\int_0^l R_1(\xi,x_2,x_3)w_{,t}(\xi,t)d\xi + v_{3\infty}(t)R_3(x_2,x_3)\right)^2\right.\\
&+ \left.\left(\frac{1}{\pi}\int_0^l R_2(\xi,x_2,x_3)w_{,t}(\xi,t)d\xi + v_{3\infty}(t)R_4(x_2,x_3)\right)^2\right\}.
\end{aligned}$$

Let

$$w(x_2,t)=e^{i\omega t}w_0(x_2),\ \ q(x_2,t)=e^{i\omega t}q_0(x_2), \tag{11}$$

$$\begin{gathered}
p(x_2,x_3,t)=e^{i\omega t}p_0(x_2,x_3),\\
u_2(x_2,x_3,t)=e^{i\omega t}u_2^0(x_2,x_3),\ \ u_3(x_2,x_3,t)=e^{i\omega t}u_3^0(x_2,x_3),
\end{gathered} \tag{12}$$

where $\omega=\text{const}>0$, $v_2=u_{2,t}$ $(v_3=u_{3,t})$. Further,

$$\varphi(x_2,x_3,t)=ie^{i\omega t}\varphi_0(x_2,x_3),\ \ \psi(x_2,x_3,t)=ie^{i\omega t}\psi_0(x_2,x_3),$$

$$v_2(x_2,x_3,t)=ie^{i\omega t}v_2^0(x_2,x_3),\ \ v_3(x_2,x_3,t)=ie^{i\omega t}v_3^0(x_2,x_3),$$

$$p_\infty(t)=e^{i\omega t}p_\infty^0,\ \ v_{3\infty}(t)=ie^{i\omega t}v_{3\infty}^0,\ \ p_\infty^0,\ v_{3\infty}^0=\text{const}.$$

Then substituting the obtained expression of $p(x_2,x_3,t)$ in (6), by virtue of (11) and (12) we get the following expression for $q_0(x_2)$

$$\begin{aligned}
q_0(x_2) &= \frac{\omega^2\rho^f}{\pi}\int_0^l w_0(\xi)\int_0^{\overset{(-)}{h}(x_2)} R_1(\xi,x_2,x_3)\cdot\cos(\overrightarrow{n}(x_2,\overset{(-)}{h}(x_2)),x_3)dx_3d\xi\\
&+ \int_0^l w_0(\xi)\int_0^{\overset{(+)}{h}(x_2)} R_1(\xi,x_2,x_3)\cdot\cos(\overrightarrow{n}(x_2,\overset{(+)}{h}(x_2)),x_3)dx_3d\xi
\end{aligned}$$

$$
\begin{aligned}
- & \; v^0_{3\infty}\omega^2\rho^f \left\{ \int\limits_0^{\overset{(-)}{h}(x_2)} R_2(x_2,x_3)\cdot\cos(\overrightarrow{n}(x_2,\overset{(-)}{h}(x_2)),x_3)dx_3 \right. \\
+ & \left. \int\limits_0^{\overset{(-)}{h}(x_2)} R_2(x_2,x_3)\cdot\cos(\overrightarrow{n}(x_2,\overset{(-)}{h}(x_2)),x_3)dx_3 \right\} \qquad (13)
\end{aligned}
$$

Taking into account (11), (12), from (9) after four times integration with respect to x_2 we get the following relation

$$
\begin{aligned}
w_0(x_2) & - 2\rho^s\omega^2 \int\limits_{x_2^0}^{x_2} h(\xi)K(x_2,\xi)w_0(\xi)d\xi = \int\limits_{x_2^0}^{x_2} (c_1\xi + c_2)(x_2-\xi)D^{-1}(\xi)d\xi \\
& + c_3x_2 + c_4 + \int\limits_{x_2^0}^{x_2} K(x_2,\xi)q_0(\xi)d\xi, \qquad (14)
\end{aligned}
$$

where

$$
x_2^0 \in]0,l[, \quad K(x_2,\xi) = -\int\limits_{\xi}^{x_2} (x_2-\eta)(\xi-\eta)D^{-1}(\eta)d\eta.
$$

Constants c_i $(i = 1,...,4)$ should be defined from the admissible boundary value conditions [4]

Problem 1. Let $\alpha < 1$, $\beta < 1$. Find $w \in C^4(]0,l[) \cap C^1([0,l])$ satisfying (9) and the following boundary conditions (BCs):

$$
w_0(0) = g_{11}, \;\; w_{0,2}(0) = g_{21}, \;\; w_0(l) = g_{12}, \;\; w_{0,2}(l) = g_{22};
$$

Problem 2. Let $\alpha < 1$, $\beta < 1$. Find $w \in C^4(]0,l[) \cap C^1([0,l])$ satisfying (9) and BCs:

$$
w_0(0) = g_{11}, \;\; w_{0,2}(0) = g_{21} \;\; w_{0,2}(l) = g_{22} \;\; Q_2(l) = h_{22};
$$

Problem 3. Let $0 \le \alpha < 1$, $\quad 0 \le \beta < 2$. Find $w \in C^4(]0,l[) \cap C^1([0,l[) \cap C([0,l])$ satisfying (9) and BCs:

$$
w_0(0) = g_{11}, \;\; w_{0,2}(0) = g_{21}, \;\; w_0(l) = g_{12}, \;\; M_2(l) = h_{12};
$$

Problem 4. Let $0 \le \alpha < 1$, $\beta \ge 0$. Find $w \in C^4(]0,l[) \cap C^1([0,l[)$ satisfying (9) and the following BCs:

$$
w_0(0) = g_{11}, \;\; w_{0,2}(0) = g_{21} \;\; M_2(l) = h_{12}, \;\; Q_2(l) = h_{22};
$$

Problem 5. Let $0 \le \alpha$, $\beta < 1$. Find $w \in C^4(]0,l[) \cap C^1([0,l])$ satisfying (9) and the following BCs:

$$
w_{0,2}(0) = g_{21} \;\; Q_2(0) = h_{21}, \;\; w_0(l) = g_{12}, \;\; w_{0,2}(l) = g_{22};
$$

Problem 6. Let $0 \le \alpha < 1$, $0 \le \beta < 2$. Find $w \in C^4(]0,l[) \cap C^1([0,l[) \cap C([0,l])$ satisfying (9) and the following BCs:

$$w_{0,2}(0) = g_{21}, \;\; Q_2(0) = h_{21}, \;\; w_0(l) = g_{12}, \;\; M_2(l) = h_{12};$$

Problem 7. Let $0 \le \alpha < 2, \quad 0 \le \beta < 1$. Find $w \in C^4(]0,l[) \cap C^1(]0,l]) \cap C([0,l])$ satisfying (9) and the following BCs:

$$w_0(0) = g_{11}, \;\; M_2(0) = h_{11}, \;\; w_0(l) = g_{12}, \;\; w_{0,2}(l) = g_{22};$$

Problem 8. Let $0 \le \alpha < 2$, $0 \le \beta < 1$. Find $w \in C^4(]0,l[) \cap C([0,l]) \cap C^1(]0,l])$ satisfying (9) and the following BCs:

$$w_0(0) = g_{11}, \;\; M_2(0) = h_{11}, \;\; w_{0,2}(l) = g_{22}, \;\; Q_2(l) = h_{22}; \tag{15}$$

Problem 9. Let $0 \le \alpha, \; \beta < 2$. Find $w \in C^4(]0,l[) \cap C([0,l])$ satisfying (9) and the following BCs:

$$w_0(0) = g_{11}, \;\; M_2(0) = h_{11} \;\; w_0(l) = g_{12}, \;\; M_2(l) = h_{12};$$

Problem 10. Let $\alpha \ge 0, \quad 0 \le \beta < 1$. Find $w \in C^4(]0,l[) \cap C^1(]0,l])$ satisfying (9) and the following BCs:

$$M_2(0) = h_{11}, \;\; Q_2(0) = h_{22} \;\; w_0(l) = g_{12}, \;\; w_{0,2}(l) = g_{22}.$$

In all these problems $g_{i,j}$, h_{ij} $(i,j = 1,2)$ are given constants. By $M_2(x_2)$ and $Q_2(x_2)$ are denote bending moment and intersecting force

$$M_2(x_2) := -h^3(x_2) w_{0,22}(x_2), \quad Q_2(x_2) := M_{2,2}(x_2).$$

Let consider, e.g., boundary conditions (15). Then for $w_0(x_2)$ we get the following equation [6]

$$\begin{aligned} w_0(x_2) &- \omega^2 \int_0^l K_1(x_2,\xi) w_0(\xi) d\xi \\ &- 2\rho^s \omega^2 \Bigg\{ \int_{x_2^0}^{x_2} h(\xi) K(x_2,\xi) w_0(\xi) d\xi + \int_{x_2^0}^{l} h(\xi) K_l(x_2,\xi) w_0(\xi) d\xi \\ &+ \int_0^{x_2^0} h(\xi) K_0(x_2,\xi) w_0(\xi) d\xi \Bigg\} \\ &= f(x_2), \end{aligned} \tag{16}$$

where

$$K_0(x_2,\xi) = \xi\left\{\int_l^{x_2} x_2 D^{-1}(\eta)d\eta - \int_0^{x_2}\eta D^{-1}(\eta)d\eta\right\} - K(0,\xi),$$

$$K_l(x_2,\xi) = x_2\int_l^{x_2}\eta D^{-1}(\eta)d\eta - \int_0^{x_2}\eta^2 D^{-1}(\eta)d\eta + x_2\int_\xi^{l}(\eta-\xi)D^{-1}(\eta)d\eta,$$

$$K_1(x_2,\xi) = \frac{\rho^f}{\pi}\left\{\int_{x_2^0}^{l} K_l(x_2,\zeta)\int_0^{\overset{(-)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(-)}{h}(\zeta)),x_3)dx_3d\zeta\right.$$

$$+\int_{x_2^0}^{l} K_l(x_2,\zeta)\int_0^{\overset{(+)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(+)}{h}(\zeta)),x_3)dx_3d\zeta$$

$$+\int_{x_2^0}^{0} K_0(x_2,\zeta)\int_0^{\overset{(-)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(-)}{h}(\zeta)),x_3)dx_3d\zeta$$

$$+\int_{x_2^0}^{0} K_0(x_2,\zeta)\int_0^{\overset{(+)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(+)}{h}(\zeta)),x_3)dx_3d\zeta$$

$$+\int_{x_2^0}^{x_2} K(x_2,\zeta)\int_0^{\overset{(-)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(-)}{h}(\zeta)),x_3)dx_3d\zeta$$

$$+\int_{x_2^0}^{x_2} K(x_2,\zeta)\int_0^{\overset{(+)}{h}(\zeta)} R_1(\xi,\zeta,x_3)\cdot\cos(\overrightarrow{n}(\zeta,\overset{(+)}{h}(\zeta)),x_3)dx_3d\zeta\ ,$$

$$f(x_2) = x_2\left(g_{22} + h_{22}\int_{x_2^0}^{l}\xi D^{-1}(\xi)d\xi + h_{11}\int_{x_2^0}^{l} D^{-1}(\xi)d\xi\right) + g_{11} + h_{22}\int_0^{x_2^0}\xi^2 D^{-1}(\xi)d\xi$$

$$-h_{11}\int_0^{x_2^0}\xi D^{-1}(\xi)d\xi - \int_{x_2^0}^{x_2}(h_{22}\xi + h_{11})(x_2-\xi)D^{-1}(\xi)d\xi$$

$$
\begin{aligned}
-\omega^2\rho^f v_3^0\varphi \Bigg\{ \int\limits_{x_2^0}^{l} K_l(x_2,\xi) & \left[\int\limits_0^{\overset{(-)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(-)}{h}(\xi)),x_3)dx_0 \right. \\
& \left. + \int\limits_0^{\overset{(+)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(+)}{h}(\xi)),x_3)dx_3 \right] d\xi \\
-\int\limits_{x_2^0}^{0} K_0(x_2,\xi) & \left[\int\limits_0^{\overset{(-)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(-)}{h}(\xi)),x_3)dx_3 \right. \\
& \left. + \int\limits_0^{\overset{(+)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(+)}{h}(\xi)),x_3)dx_3 \right] d\xi \\
-\int\limits_{x_2^0}^{x_2} K(x_2,\xi) & \left[\int\limits_0^{\overset{(-)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(-)}{h}(\xi)),x_3)dx_3 \right. \\
& \left. + \int\limits_0^{\overset{(+)}{h}(\xi)} R_3(\xi,x_3)\cdot\cos(\overrightarrow{n}(\xi,\overset{(+)}{h}(\xi)),x_3)dx_3 \right] d\xi \Bigg\}.
\end{aligned}
$$

It is easy to show that $2\rho^s h(\xi)K(x_2,\xi)$, $2\rho^s h(\xi)K_0(x_2,\xi)$, $2\rho^s h(\xi)K_l(x_2,\xi)$, $K_1(x_2,\xi) \in$ $\in C([0,l])$ (in our case $0 \le \alpha < 2$, $0 \le \beta < 1$).

The integral equation (16) can be solved by method of successive approximations.

Remark. In case of the other above boundary conditions (see problems 1-7, 9, 10), the problem under consideration is solved analogously and in all cases we get (16) type integral equations.

Thus, the following Preposition is valid.

Proposition *Problem of the harmonic vibration of the plate with two cusped edges under action of the incompressible ideal fluid (i.e., equations (7), (8), (9), under transmission conditions (4), (6) and under conditions at infinity (2), (3) and BCs have) has a unique solution when*

$$\omega^2 < \frac{1}{Ml},$$

where

$$
\begin{aligned}
M := \max_{x_2,\xi\in[0,l]} & \left\{ |2\rho^s h(\xi)K(x_2,\xi)|,\ |2\rho^s h(\xi)K_0(x_2,\xi)|, \right. \\
& \left. |2\rho^s h(\xi)K_l(x_2,\xi)|,\ |K_1(x_2,\xi)| \right\}.
\end{aligned}
$$

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