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ON THE RELATION OF BOUNDARY VALUE PROBLEMS FOR CUSPED PLATES AND BEAMS TO THREE-DIMENSIONAL PROBLEMS

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This paper deals with the analysis of the physical (mechanical) sense of the quantities considered in the theory of cusped plates [1] and beams [2]. The relation of boundary value problems of the three-dimensional model of elasticity to boundary value problems of two- and one-dimensional models is also discussed.

1. Physical and Mathematical Moments of Stresses

Let $X_{ij}, \quad i,j=1,2,3,$ be stress tensor. The k-th order mathematical moment of $X_{ij} \in C(\Omega \cup \overset{(+)}{h} \cup \overset{(-)}{h})$ is defined as follows (see [3]):

$$\overset{k}{X}_{ij}(x_1,x_2) := \int\limits_{\overset{(-)}{h}(x_1,x_2)}^{\overset{(+)}{h}(x_1,x_2)} X_{ij}(x_1,x_2,x_3) P_k(ax_3 - b) dx_3, \quad (x_1,x_2) \in \omega, \qquad (x_1,x_2,x_3) \in \Omega, \tag{1}$$

where

$$P_k(t) := \sum_{l=0}^{\left[\frac{k}{2}\right]} (-1)^l \frac{(2k-2l)!}{2^k l!(k-l)!(k-2l)!} t^{k-2l}, \quad k=0,1,..., \tag{2}$$

are Legendre polynomials (see [4], Section 15.1), $\left[\frac{k}{2}\right]$ is an integer part of $\frac{k}{2}$, Ω is a domain occupied by a plate of the variable thickness, ω is its projection on the plane $x_3=0$,

$\overset{(\pm)}{h} := \{(x_1,x_2,x_3) \in R^3 : (x_1,x_2) \in \omega, \quad x_3 = \overset{(\pm)}{h}(x_1,x_2)\}$
are the upper and lower face surfaces of the plate,

$$a := \frac{1}{h}, \quad b := \frac{\tilde{h}}{h},$$

$$2h(x_1,x_2) := \overset{(+)}{h}(x_1,x_2) - \overset{(-)}{h}(x_1,x_2),$$

$$2\tilde{h}(x_1,x_2) := \overset{(+)}{h}(x_1,x_2) + \overset{(-)}{h}(x_1,x_2).$$

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Physical moments are defined as follows:

$$S_{ij}(x_1,x_2) := \overset{0}{M}_{ij}(x_1,x_2), \quad i,j=1,2,3,$$

$$\overset{k}{M}_{ij}(x_1,x_2) := \int\limits_{\overset{(-)}{h}(x_1,x_2)}^{\overset{(+)}{h}(x_1,x_2)} X_{ij}(x_1,x_2,x_3)x_3^k dx_3, \quad k=0,1,..., \quad i,j=1,2,3, \qquad (3)$$

where, S_{23}, S_{13} are so called intersecting forces, $S_{\alpha\beta}$, $\alpha,\beta=1,2$ are so called membrane (or normal and tangent) forces, $\overset{1}{M}_{11}$, $\overset{1}{M}_{22}$ are bending moments, $\overset{1}{M}_{12}$ is a twisting moment. In what follows, generalizing these definitions, $\overset{k}{M}_{ij}(x_1,x_2)$ will be called physical moments of the k-th order. In particular, as it was just mentioned, zero moments coincide with the intersecting and membrane forces; the first moments coincide with the bending and twisting moments and the splitting couple of forces (for the last notion see [3]).

Evidently, (1) and (3) have the sence for $2h>0$.

If a point $P \in \Gamma := \partial\Omega \backslash (\overset{(+)}{h} \cup \overset{(-)}{h})$ belongs to the cusped edge $\Gamma_0 \bar{\subset} \Gamma$ of the plate, i.e., $2h(P_\omega)=0$, $P_\omega \in \gamma_0$, then the mathematical and physical moments we define as the following limits:

$$\overset{k}{X}_{ij}(P) := \overset{k}{X}_{ij}(P_\omega) := \lim_{\omega \ni Q_\omega \to P_\omega} \overset{k}{X}_{ij}(Q_\omega), \quad i,j=1,2,3,$$

$$\overset{k}{M}_{ij}(P) := \overset{k}{M}_{ij}(P_\omega) := \lim_{\omega \ni Q_\omega \to P_\omega} \overset{k}{M}_{ij}(Q_\omega), \quad i,j=1,2,3, \qquad (4)$$

where γ_0 is a projection of Γ_0 on the plane $x_3=0$, and P_ω and Q_ω are the projections of $P \in \Gamma$ and $Q \in \Omega$, respectively. When the cusped edge lies on $\partial\omega$, then obviously $P_\omega \equiv P$. For the sake of simplicity in what follows we suppose $P_\omega \equiv P$ unless otherwise stated (in this connection see Figures 1-3, where plane sections of the plate parallel to the plane $x_1=0$ are given).

If X_{ij} are bounded on Ω functions, then if $P \in \Gamma_0$, obviously, $\overset{k}{X}_{ij}(P_\omega)=0$ and $\overset{k}{M}_{ij}(P_\omega)=0$. Evidently, $\overset{k}{X}_{ij}(P_\omega)\neq 0$ and $\overset{k}{M}_{ij}(P_\omega)\neq 0$ only if $\lim\limits_{\Omega \ni Q \to P \in \Gamma_0} X_{ij}(Q)=\infty$.

If, e.g., the cusped edge belongs to the axis x_1 and in some neighborhood along $\partial\omega$ (i.e., axis $0x_1$) of the point P where $2h=0$, then $\overset{0}{M}_{2j}(P)dx_1$, $j=1,2,3$, are the components of a concentrated along linear element dx_1 force and $\overset{0}{M}_{2j}(P)$, $j=1,2,3$ are the components of a concentrated along unit of $\partial\omega$ force applied at P (see Fig 4). When the cusped edge is a smooth element ds of the line $\partial\omega$ with a normal n at the point P, then $\overset{0}{M}_{nj}(P)ds = \overset{0}{M}_{ij}(P)n_i(P)ds$, $j=1,2,3$, are the components of a concentrated along the line element ds force. Similarly, $\overset{1}{M}_{nj}(P)$ and $\overset{1}{M}_{nj}(P)ds$,

$j=1,2,3$ are the components of a concentrated along the line moment at the point P and along the element ds, respectively. We recall that $X_{ij}(P)=\infty$ in the case when either $\overset{k}{X}_{ij}$ or $\overset{k}{M}_{ij}$ are not zero at P (see Fig. 4).

Let, in general, $P\neq P_\omega$. Let further some neighborhoods of the point P along $\partial\omega$ and on the upper and lower surfaces are not loaded by the concentrated forces and moments along $\partial\omega$ and surface forces, respectively. Besides of forces and physical moments concentraded along lines we can define also concentraded at the point P of the cusped edge of the plate forces and physical moments as follows (see Fig. 5)

$$\begin{aligned}
\overset{k}{F}_i(P) &:= \lim_{\rho\to 0}\int\limits_S X_{ni}(Q)x_3^k dS = \lim_{\rho\to 0}\int\limits_S X_{ni}(Q_\omega,x_3)x_3^k dS\\
&= \lim_{\rho\to 0}\int\limits_{S_\omega} dS_\omega \int\limits_{\overset{(-)}{h}(x_1,x_2)}^{\overset{(+)}{h}(x_1,x_2)} X_{ni}(Q_\omega,x_3)x_3^k dx_3 = \lim_{\rho_\omega\to 0}\int\limits_{S_\omega}\overset{k}{M}_{ni}(Q_\omega)dS_\omega = \overset{k}{F}_i(P_\omega),
\end{aligned}$$

where S is a cylindrical surface parallel to x_3, S_ω is its projection on the plane $x_3=0$; ρ is maximum of the distances between P and $Q\in S$ and ρ_ω is maximum of of the distances between P_ω and $Q_\omega\in S_\omega$.

By virtue of (2), we have

$$\begin{aligned}
P_k(ax_3-b) &= P_k\left(\frac{x_3-\tilde h}{h}\right) = \frac{1}{2^k}\sum_{l=0}^{\left[\frac{k}{2}\right]}(-1)^l\frac{1}{l!(k-l)!}\frac{(2k-2l)!}{(k-2l)!}(x_3-\tilde h)^{k-2l}h^{2l-k}\\
&= \frac{1}{2^k}\sum_{l=0}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l}(-1)^{l+r}\frac{1}{l!(k-l)!}\frac{(2k-2l)!}{(k-2l)!}\frac{(k-2l)!}{r!(k-2l-r)!}x_3^{k-2l-r}\tilde h^r h^{2l-k}\\
&= \frac{1}{2^k}\sum_{l=0}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l}(-1)^{l+r}\frac{(2k-2l)!}{l!(k-l)!r!(k-2l-r)!}x_3^{k-2l-r}\tilde h^r h^{2l-k}\\
&= \frac{(2k)!}{2^k(k!)^2}\cdot\frac{x_3^k}{h^k}+\frac{1}{2^k}\left\{\sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l}+\sum_{\substack{r=1\\ l=0}}^{k}\right\}(-1)^{l+r}\\
&\times\frac{(2k-2l)!}{l!(k-l)!r!(k-2l-r)!}x_3^{k-2l-r}\tilde h^r h^{2l-k},\quad \sum_{l=1}^{0}(\cdot)=0,
\end{aligned}\tag{5}$$

since

$$\sum_{l=0}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} c_{lr} = \sum_{r=0}^{k} c_{0r} + \sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} c_{lr} = c_{00} + \sum_{r=1}^{k} c_{0r} + \sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} c_{lr} = c_{00}$$

$$+ \left\{ \sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} + \sum_{\substack{r=1\\ l=0}}^{k} \right\} c_{lr}.$$

Hence,

$$x_3^k = \frac{2^k (k!)^2}{(2k)!} h^k P^k (ax_3 - b) - \frac{(k!)^2}{(2k)!} \left\{ \sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} + \sum_{\substack{r=1\\ l=0}}^{k} \right\} (-1)^{l+r}$$
$$\times \frac{(2k-2l)!}{l!(k-l)!r!(k-2l-r)!} \tilde{h}^r h^{2l} x_3^{k-2l-r}. \tag{6}$$

From (1), (6), (3) we get

$$\overset{k}{M}_{ij}(x_1,x_2) = \int\limits_{\overset{(-)}{h}}^{\overset{(+)}{h}} x_3^k X_{ij}(x_1,x_2,x_3)dx_3 = \frac{2^k (k!)^2}{(2k)!} h^k \overset{k}{X}_{ij}(x_1,x_2)$$
$$- \frac{(k!)^2}{(2k)!} \left\{ \sum_{l=1}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l} + \sum_{\substack{r=1\\ l=0}}^{k} \right\} (-1)^{l+r} \frac{(2k-2l)!}{l!(k-l)!r!(k-2l-r)!} \tag{7}$$
$$\times \tilde{h}^r h^{2l} \overset{k-2l-r}{M_{ij}} (x_1,x_2), \quad k = 0,1,2,\cdots.$$

Which gives the recurence formulae for calculating of $\overset{k}{X}_{ij}$ by means of $\overset{s}{M}_{ij}$, $s = = 0,1,...,k$, and of $\overset{k}{M}_{ij}$ by means of $\overset{s}{X}_{ij}$, $s = 0,1,...,k$.

Therefore, for $k = 0,1$,

$$S_{ij}(x_1,x_2) := \overset{0}{M}_{ij}(x_1,x_2) = \overset{0}{X}_{ij}(x_1,x_2), \tag{8}$$

$$\overset{1}{M}_{ij}(x_1,x_2) = h\overset{1}{X}_{ij}(x_1,x_2) + \tilde{h} S_{ij}(x_1,x_2), \tag{9}$$

respectively.

Now, tending Q_ω to $P = P_\omega$, from (8), (9), and (7) for $k \geq 2$ we obtain

$$S_{ij}(P) = \overset{0}{X}_{ij}(P), \tag{10}$$

$$\overset{1}{M}_{ij}(P) = \lim_{\omega \ni Q_\omega \to P} h(Q_\omega)\overset{1}{X}_{ij}(Q_\omega), \tag{11}$$

and

$$\overset{k}{M}_{ij}(P) = \frac{2^k (k!)^2}{(2k)!} \lim_{\omega \ni Q_\omega \to P} h^k(Q_\omega)\,\overset{k}{X}_{ij}(Q_\omega), \tag{12}$$

because of

$$\lim_{\omega \ni Q_\omega \to P} \tilde{h}^r h^{2l} \overset{k-2l-r}{M_{ij}}(Q_\omega) = 0$$

since $r + l > 0$ in sums of (7), and

$$h(P) = 0, \quad \tilde{h}(P) = 0.$$

Taking into accaunt (10), (11), we conclude that (12) is valid for $k \geq 0$.

In the case $P \neq P_\omega$ (see Fig.1), in view of (4), (7), we have

$$\overset{k}{M}_{ij}(P) := \overset{k}{M}_{ij}(P_\omega) = \frac{2^k (k!)^2}{(2k)!} \lim_{\omega \ni Q_\omega \to P_\omega} h^k(Q_\omega)\,\overset{k}{X}_{ij}(Q_\omega)$$

$$-k! \sum_{\substack{r=1 \\ l=0}}^{k} (-1)^r \frac{1}{(k-r)!r!} \tilde{h}^r(P_\omega) \overset{k-r}{M_{ij}}(P_\omega).$$

Thus, when we consider boundary conditions in stresses, i.e., forces and moments concentrated along the cusped edge are given, for mathematical moments at cusped edges in the N-th approximation from (12) we get the following boundary conditions:

$$\lim_{\omega \ni Q_\omega \to P_\omega} h^k(Q_\omega)\,\overset{k}{X}_{ij}(Q_\omega), \quad \text{are prescribed for } \ k = \overline{0, N}, \tag{13}$$

which are weighted boundary conditions for $k \geq 1$.

The homogeneous boundary conditions (13) for $i = 2$ at cusped edges belonging to axis x_1 of two-dimensional model correspond to the three-dimensional model, when on the face surfaces the stresses and on the lateral non-cusped edge (boundary) $\Gamma \backslash \bar{\Gamma}_0$ either the displacements or the stresses are prescribed. In this case homogenoues boundary conditions (13) are automatically satisfied for the bounded stresses or for $u_i \in H^1(\Omega)$ since in the last case $X_{ij} \in L_2(\Omega)$ and by Fubini theorem the summability of X_{ij} along x_3 in (1) can be shown. Therefore, $\lim\limits_{\omega \ni Q_\omega \to P} \overset{k}{X}_{ij}(Q_\omega) = 0$ since limits of integration in (1) tend to 0. In the case of sharp cusped edges the homogeneous boundary conditions (13) at cusped edges can be replaced by the boundednes in ω of weighted moments of displacements (see below Section 2) or by belonging to some space on ω without boundary conditions at cusped edge.

The nonhomogeneous boundary conditions (13) for $i = 2$ at cusped edges belonging to axis x_1 of two-dimensional model correspond to the three-dimensional model, when at above-mentioned cusped edges Γ_0 forces and physical moments concentrated along the cusped edges are applied and on the other parts $\overset{(+)}{h}$, $\overset{(-)}{h}$, $\Gamma \backslash \bar{\Gamma}_0$ of the body boundary the same conditions as in the above-formulated case of homogeneous boundary conditions (13) are given.

In the N-th approximation the stress tensor is given in the form

$$X_{ij}(x_1,x_2,x_3) \cong \sum_{k=0}^{N} a\left(k+\frac{1}{2}\right) \overset{k}{X}_{ij}(x_1,x_2)P_k(ax_3-b), \quad i,j=1,2,3. \tag{14}$$

In this case the relation (1) between X_{ij} and $\overset{k}{X}_{ij}$ is correct when the symbol of approximate equality in (14) can be replaced by the exact equality symbol. This occurs if either X_{ij} are N-th order polynomials with respect to x_3 or $N=+\infty$ (i.e., we consider the Fourier-Legendre series representation for

$$X_{ij}(x_1,x_2,\cdot) \in C^2\left(\left[\overset{(+)}{h}(x_1,x_2), \overset{(-)}{h}(x_1,x_2)\right]\right).$$

Obviously, surface forces $X_{ni}(x_1,x_2,x_3)$ can be considered only at points of blunt cusped edges (in this case the union of the upper and lower surfaces is a smooth surface and there exist normals) and, as it follows from (14), they become infinite as $Q \to P$ if boundary conditions (13) (see also (9)) are inhomogeneous.

Let us remark that the inhomogeneous boundary conditions (13), i.e., (12), mean that along the cusped edge the concentrated along the edge forces and moments are given (see Figures 6,7, where the plane sections of the three-dimensional problems are given). Some such problems in two- and one-dimensional formulations are solved in [5], [6], [2]. The similar analysis can be carried out for cusped beams.

2. Displacement Vector. Weighted Moments

The k-th order moments of the displacement vector components are defined similar to (1):

$$\overset{k}{u}_i(x_1,x_2) := \int_{\overset{(-)}{h}(x_1,x_2)}^{\overset{(+)}{h}(x_1,x_2)} u_i(x_1,x_2,x_3)P_k(ax_3-b)dx_3, \quad i=1,2,3, \quad k=0,1,2,\cdots$$

In the N-th approximation, by virtue of (5), u_i are expressed by $\overset{k}{u}_i$ as follows:

$$\begin{aligned} u_i(x_1,x_2,x_3) &\cong \sum_{k=0}^{N} a\left(k+\frac{1}{2}\right) \overset{k}{u}_i(x_1,x_2)P_k(ax_3-b) \\ &= \sum_{k=0}^{N}\left(k+\frac{1}{2}\right) h^k \overset{k}{v}_i(x_1,x_2)P_k(ax_3-b) \\ &= \sum_{k=0}^{N}\frac{1}{2^k}\left(k+\frac{1}{2}\right)\sum_{l=0}^{\left[\frac{k}{2}\right]}\sum_{r=0}^{k-2l}(-1)^{l+r}\frac{(2k-2l)!}{l!(k-l)!r!(k-2l-r)!} \\ &\times \overset{k}{v}_i(x_1,x_2)x_3^{k-2l-r}\tilde{h}^r h^{2l}, \quad i=1,2,3, \end{aligned} \tag{15}$$

where

$$\overset{k}{v}_i(x_1,x_2) := \frac{\overset{k}{u}_i(x_1,x_2)}{h^{k+1}}, \quad i=1,2,3,..., \quad k=\overline{0,N},$$

are weighted moments of the displacement vector components.

In particular, in the $N=0$ and $N=1$ approximations

$$u(x_1,x_2,x_3) \cong \frac{1}{2}\overset{0}{v}_i(x_1,x_2), \quad i=1,2,3,$$

and

$$u_i(x_1,x_2,x_3) \cong \frac{1}{2}\overset{0}{v}_i(x_1,x_2) + \frac{3}{2}\overset{1}{v}_i(x_1,x_2)(x_3-\tilde{h}), \quad i=1,2,3,$$

respectively.

Let $\overset{k}{v}_i$ be bounded then for

$$k-2l-r+r+2l=k>0$$

the limits in the right hand side of (15) are zero as $\Omega \ni Q \to P = P_\omega$, i.e., as $\omega \ni Q_\omega \to P_\omega = P$, there remains only the summend for $k=0$, i.e.,

$$\lim_{\omega \ni Q \to P} u_i(Q) = \frac{1}{2}\overset{0}{v}_i(P) \ \text{ if } \ I_0 := \int_P^Q \frac{dn}{h} < +\infty, \tag{16}$$

where n is an inward normal to $\partial\omega$ at the point P, since for $I_0 = +\infty$ the $\lim\limits_{\omega \ni Q \to P} \overset{0}{v}_i(Q)$, in general, does not exist (see [1], [2], [7-9] and references therein).

This will be not the fact in case $P \neq P_\omega$ (see Fig.1), since now $x_3 \to x_3^0 \neq 0$ as $Q_\omega \to P_\omega$ (when $P \equiv P_\omega$, evidently $x_3^0 = 0$). Taking into account that (see [10] and also Remark 3 below)

$$\lim_{\omega \ni Q_\omega \to P_\omega} \overset{k}{v}_i(Q_\omega), \quad k=0,1,...,N,$$

there exist for

$$\int_{P_\omega}^{Q_\omega} \frac{dn}{h^{2k+1}} < +\infty, \quad k=0,1,...,N,$$

from (15) we conclude that

$$\lim_{\omega \ni Q \to P} u_i(Q) = \sum_{k=0}^{N} \frac{1}{2^k}\left(k+\frac{1}{2}\right)\sum_{r=0}^{k}(-1)^r \frac{(2k)!}{k!r!(k-r)!}$$
$$\times \overset{k}{v}_i(P_\omega)(x_3^0)^{k-r}\tilde{h}^r, \ \ i=1,2,3, \ \text{ for } \int_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty. \tag{17}$$

From (15), evidently,

$$\frac{\partial^j u_i(Q)}{\partial x_3^j} = \sum_{k=j}^{N} \frac{1}{2^k}\left(k+\frac{1}{2}\right) \sum_{l=0}^{\left[\frac{k-j}{2}\right]} \sum_{r=0}^{k-2l-j} (-1)^{l+r} \frac{(2k-2l)!}{l!(k-l)!(k-2l-r)!r!}$$
$$\times \overset{k}{v}_i(Q_\omega)(k-2l-r)(k-2l-r-1)\cdots(k-2l-r-j+1)$$
$$\times x_3^{k-2l-r-j}\tilde{h}^r h^{2l}, \quad j=1,2,...,N. \tag{18}$$

Whence,

$$\lim_{\Omega\ni Q\to P} \frac{\partial^j u_i(Q)}{\partial x_3^j} = \sum_{k=j}^{N} \frac{1}{2^k}\left(k+\frac{1}{2}\right) \sum_{r=0}^{k} (-1)^r \frac{(2k)!}{k!r!(k-r)!} \overset{k}{v}_i(P_\omega)$$
$$\times (k-r)(k-r-1)\cdots(k-r-j+1)(x_3^0)^{k-r-j}\tilde{h}^r$$
$$j=1,2,\cdots,N, \quad i=1,2,3, \quad \text{when} \int_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty. \tag{19}$$

In the case $P \equiv P_\omega$, by virtue of $\tilde{h}(P_\omega)=0$ and $x_3^0=0$, from (17), (19) we get

$$\lim_{\Omega\ni Q\to P} \frac{\partial^j u_i(Q)}{\partial x_3^j} = \frac{1}{2^j}\left(j+\frac{1}{2}\right)\frac{(2j)!}{j!}\overset{j}{v}_i(P), \tag{20}$$

$$j=1,2,\cdots,N, \quad i=1,2,3, \quad \textit{when} \int_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty.$$

If additionally $\overset{k}{v}_i$ are bounded, from (18) we get (20) under the corresponding condition

$$\int_{P}^{Q_\omega} \frac{dn}{h^{2j+1}} < +\infty. \tag{21}$$

Thus, from (20) we can define $\overset{j}{v}_i$, $j=0,1,\cdots,N, i=1,2,3$, provided the left-hand sides of (20), i.e., derivatives with respect to x_3 up to the N-th order of the displacements u_i at point $P \in \Gamma_0$ are known. In the same way we can define $\overset{j}{v}_i(P_\omega)$, $j=0,1,\cdots,N$, $i=1,2,3$, from equalities (19), (17) by means of their left-hand sides.

To this end we set in (19) sequently $j=N$, $j=N-1,...,$ $j=1$ and obtain the values for $\overset{j}{v}_i(P_\omega)$, using the results of the previous steps. Finally, we define $\overset{0}{v}_i$ from (17).

Remark 1. (20) when (21) holds signifies that boundary conditions of two-dimensional models, when on $\gamma_0 \equiv \Gamma_0$

$$\overset{j}{v}_i(P) \ \text{if} \int_{P}^{Q_\omega} \frac{dn}{h^{2j+1}} < +\infty, \ j=0,1,...,N, \ i=1,2,3, \ \text{are prescribed,}$$

correspond to the boundary conditions of the three-dimensional model, when on $\Gamma_0 \equiv$ $\equiv \gamma_0$

$$\frac{\partial^j u_i(P)}{\partial x_3^j}, \text{ if} \int\limits_{P_\omega}^{Q_\omega} \frac{dn}{h^{2j+1}} < +\infty, \; j=0,1,...,N, \; i=1,2,3, \text{ are prescribed.}$$

Similarly, (17), (19) signify that boundary conditions of the two-dimensional models, when on γ_0

$$\overset{j}{v}_i(P_\omega) \text{ if} \int\limits_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty, \; j=0,1,...,N, \; i=1,2,3, \text{are prescribed} \tag{22}$$

correspond to the boundary conditions of the three-dimensional model, when on Γ_0

$$\frac{\partial^j u_i(P)}{\partial x_3^j} \text{ if} \int\limits_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty, j=0,1,...,N, \; i=1,2,3, \text{are prescribed.} \tag{23}$$

Such boundary conditions fall outside the limits of the classicall three-dimensional theory of elastisity. If $N = +\infty$ (i.e.,we actually have three-dimensional model), then $\int\limits_{P_\omega}^{Q_\omega} \frac{dn}{h^{2N+1}} = +\infty$ and therefore, boundary conditions (22), (23) disappear like of classical three-dimensional model.

Remark 2. When $\overset{0}{v}_i(P) = 0$ (see (16)) along the cusped edge (because of $I_0 < +\infty$ this cusped end is blunt one [1]), it means that in the corresponding three-dimensional problem the cusped edge is fixed; on the face surfaces the stresses and on the lateral non-cusped edge either the displacements or the stresses are prescribed (see Fig.8). The physical sense of $\overset{0}{v}_i(P) \neq 0$ is evident. Such formulation of the three-dimensional boundary value problem is not usual and falls outside the limits of the classical three-dimensional theory of elastisity.

Remark 3. If we considor admissible (i.e., correct) boundary value problems in corresponding weighted Sobolev spaces, then generalizing known results [10], [7] (see also [11] and references theirin and [12]) for the N-th approximation at a cusped edge we will get the following boundary conditions in weighted moments of displacements in the sence of traces [8]:

$\overset{k}{v}_i(P)$ if $\int\limits_{P}^{Q_\omega} \frac{dn}{h^{2k+l}} < +\infty$, $k = 0,1,...,N$, $i = 1,2,3$, are given.

Whence, by virtue of (15), for displacements we obtain that there exist traces

$$u_i(P), \;\; i=1,2,3, \text{ if} \int\limits_{P}^{Q_\omega} \frac{dn}{h^{2N+1}} < +\infty.$$

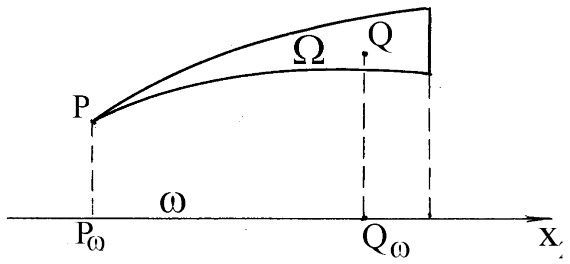

fig.1.

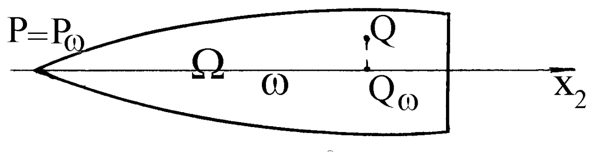

fig.2

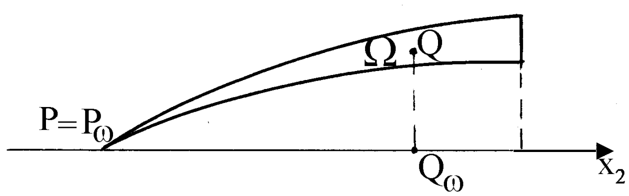

fig.3.

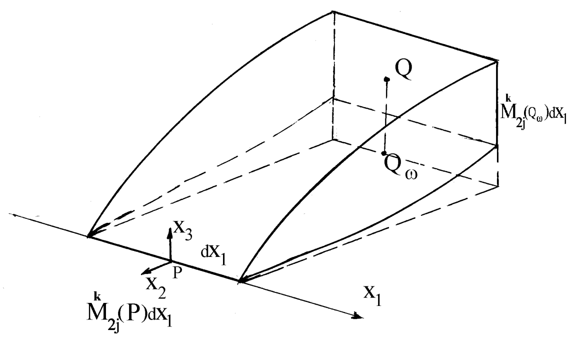

fig.4

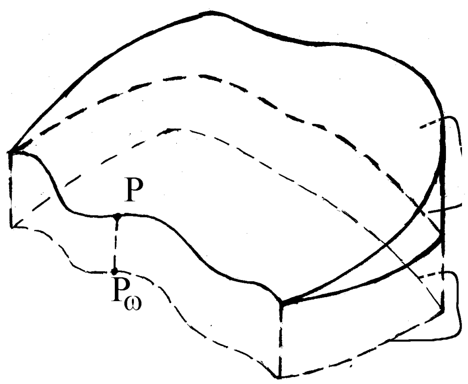

fig.5.

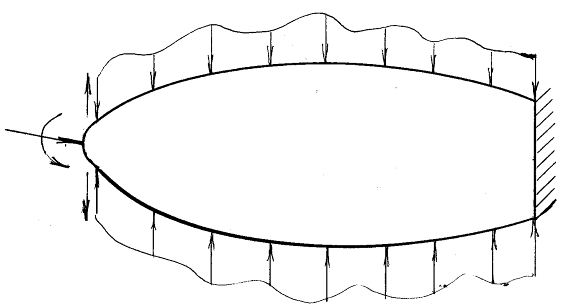

fig.6

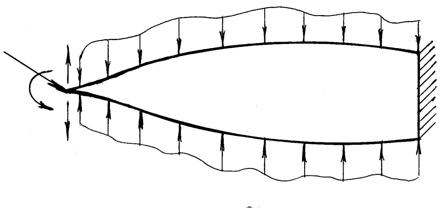

fig.7.

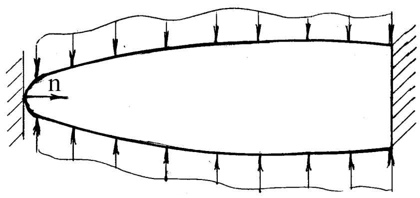

fig.8

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