Proceedings of I. Vekua Institute
of Applied Mathematics
Vol. 67, 2017

IMPACT OF THE WIND MOVING FROM THE MOUNTAIN CREST DOWN THE SNOW-COVERED SLOPE ON THE SNOW AND AVALANCHE FORMATION

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Abstract. A differential equation showing the cold wind current flowing over the mountain ridge crest and its movement down the inclination covered with snow as a cold density current (wind). In order to obtain the explicit expression of the law of the wind velocity variation to obtain the desired expression, an approach different from the traditional one to solve quite a complex equation is developed and the law of the wind velocity variation depending on the relevant parameter is obtained.

The value of aerodynamic resistance playing a role of a shift force for the upper surface of the snow cover is obtained. It is the component of the gravity force acting on the slope surface, which, together with the forces of seismotechnical origin and snow melting filtration forces (or individually), can cause a shift of the snow cover and origination of a snow avalanche. In addition, a formula to calculate the distance from the mountain top to the site on the sloping, near which the impact of the tail wind velocity is virtually imperceptible.

Keywords and phrases: Slope, snow, wind velocity, shearing force, avalanche.

AMS subject classification (2010): 76E99.

1. Introduction

Such natural phenomena as snow precipitations and formation of a snow cover on vast mountain slopes are the major factors in avalanche formation. They are usually moved either under the influence of seismic impacts, or internal overload, or filtration forces [1, 2]. Besides, in order to identify the criteria of stability or instability of the snow cover, alongside with the mentioned factors, an impact of cold winds on the snow cover is essential to consider. The equations giving the analytical description of the dynamic processes of snow avalanches must include the summands envisaging the resistance at the upper limit beforehand, i.e. along the contact contour of the snow with air, as it is under the impact of the tangent pressures acting on the snow-air contact surface, the forces are passed from the katabatic wind to the snow cover. Quite often, the impact of cold winds on the snow cover results in the origination of a strong snow avalanche. Let us consider this problem.

2. Main body

Problem 1. Let us determine the overflow of the cold air accumulated in front of the mountain ridge and expression to calculate the law of the variation of the velocity of the wind originated with the cold currents moving in the direction of sloping over the snow-covered slope (Fig. 1).

The expression to calculate the velocity of the wind originated by the action of cold air currents moving from the mountain peaks down the snow-covered slopes can be gained based on the analogy of a hydraulic approach considered in [3, 4].

The equation given in [4] for the established regime of the turbulent current flowing into the sea canyon can be written down as follows:

$$\frac{v}{g}\frac{dv}{dx} + \left(1 - \frac{\gamma_\omega}{\gamma_c}\right)\left(\cos\psi \cdot \frac{dh}{dx} + f \cdot S\cos\psi - i\right) + \left(1 - S + \aleph_h \frac{\gamma_\omega}{\gamma_c} \cdot \frac{C^2}{g}\right) \cdot \frac{Q^2}{K^2} = 0, \tag{1}$$

where v is the mean velocity of the density current with layer of h thickness and γc volumetric weight; γ_ω is the volumetric weight of the sea water free off drift with thickness: $H \gg h$; Q is the discharge of the density current flowing down the slope; K is the discharge modulus ($K = \omega C\sqrt{h}$, here ω is the area of section of the density current; C is Chez coefficient, which is associated with λ, Darcy turbulent friction coefficient, with formula: $C^2 = \frac{8g}{\lambda}$); S is the concentration of the river drift in the flowing density current; ψ is the angle of inclination of the underwater slope ground to the horizon; $\aleph_h$ is the hydrodynamic friction coefficient; f is the Coulomb friction coefficient between the current density drift and underwater slope ground with slope: $i = \sin\psi$. In addition, equation (1) considers that the density current is subject to turbulent friction (i.e. it is drift-carrier).

Figure 1 shows the calculation diagram showing the accumulation of cold air on a windy mountain ridge, its flow over the ridge crest and its flow as a density cold current down the snow-covered slope (a similar problem is considered in [2] from the position of an ideal liquid hydrodynamics).

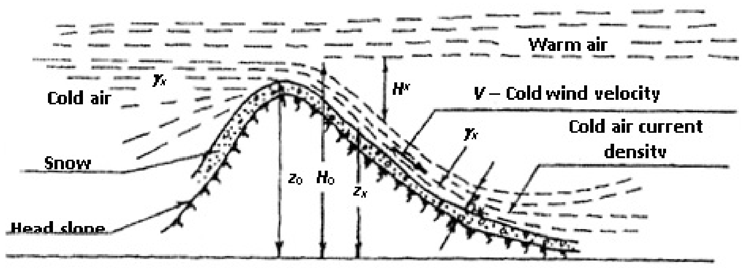

Fig. 1.

Thus, the identity of the processes of flow of the density drift-carrier current from the river plain over the underwater sea canyon slope and the cold air layer flow over the snow-covered downwind slope is clear and needs no corroboration.

It is clear that for this problem, in equation (1) we must admit that $s = f = 0$, as the wind current is merely a falling layer of the cold air forming strong wind, but not containing hard admixtures and not being able to cause Coulomb (dry) friction when it comes in contact with a snow-covered surface.

Thus, equation (1) is true for the cold wind when $f = 0$ and $s = 0$. As a result, equation (1) will be written down as follows:

$$\frac{v}{g}\frac{dv}{dx} + \left(1 - \frac{\gamma_1}{\gamma_2}\right)\left(\cos\psi \cdot \frac{dh}{dx} - i\right) + \left(1 + \aleph_h \frac{\gamma_1}{\gamma_2} \cdot \frac{C^2}{g}\right) \cdot \frac{Q^2}{K^2} = 0, \tag{2}$$

where γ_1 and γ_2 are the volumetric weights of warm and cold air, respectively, which are proportional to the densities. $\frac{\gamma_1}{\gamma_2}$ ratio can be written down as follows: $\frac{\rho_1}{\rho_2}$. Clearly, this ratio can also be written down as follows: $\frac{T_2}{T_1}$, where T_2 and T_1 are the temperatures of cold and warm air, respectively, what ensues from Clapeyron equation and from the fact that the dividing surface between the cold and relatively warm air masses is a contact rupture face, and the values of hydrodynamic pressure on it are equal. So, according to Clapeyron equation:$\rho_1 T_1 = \rho_2 T_2$.

If considering that $i = -\frac{dz_x}{dx} = \sin\psi, \frac{c^2}{g} \cdot \frac{Q^2}{K^2} = \frac{v^2}{gh}$ and fact that we can consider the conditions of both environments, i.e. the density and temperature of wind current and warmer air mass above it ($\rho_2 > \rho_1$ and $T_2 < T_1$, respectively) are constant, then equation (2) can be written down as follows:

$$\frac{d}{dx}\left[\frac{v^2}{2g} + \sigma\left(h + z_x\right)\right] + \frac{v^2}{C^2 h} + \aleph_h \cdot \frac{\gamma_1}{\gamma_2} \cdot \frac{v^2}{gh} = 0, \tag{3}$$

where $\sigma = 1 - \frac{\gamma_1}{\gamma_2}$.

The last two members of the equation above represent the intensity of tangent pressures passed by the cold wind current onto the snow-covered surface (the first one) and to the upper immovable mass of warmer air (the second one).

Following the integration, the first top of the two yields the value of tangent force, which acts on the snow cover, in the direction of its movement down the slope, and which can play a role of the initiation mechanism to cause snow avalanche under certain conditions. As for the second summand, its role is relatively less and can be ignored.

In this case, by using a simple transformation, equation (3) can be deduced as follows:

$$\frac{v^3 - \sigma g q}{g v^2} \cdot dv = \frac{\sigma i q C^2 - v^3}{q C^2} \cdot dx. \tag{4}$$

Let us integrate equation (4) with initial terms $t = t_0 = 0$, $x = x_0 = 0$, $v = v_0$, where t is the time and v is the initial velocity. Then, we will have:

$$x = \frac{q\left(v_0 - v\right)}{i v_0 v} + \frac{q\left(g - iC^2\right)}{3ig}\left[\ln\left|\frac{v - \sqrt[3]{a}}{v_0 - \sqrt[3]{a}}\right| \sqrt{\frac{v_0^2 + v_0\sqrt[3]{a} + \sqrt[3]{a^2}}{v^2 + v\sqrt[3]{a} + \sqrt[3]{a^2}}} \right.$$
$$\left. + \sqrt{3}\,\mathrm{arctg}\,\frac{2v + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}} - \sqrt{3}\,\mathrm{arctg}\,\frac{2v_0 + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right], \tag{5}$$

where $a = \sigma i q C^2$.

As it can be seen from equation (5), an attempt to represent the law of the wind velocity change in a visible and suitable form fails to yield an explicit expression describing the law of the velocity change.

In this connection, below, we are giving a solution of equation (3) different from the traditional one, in particular, differential equation (4) (unlike a traditional approach using x and h variables) as a result of transformations, will be deduced to the equation with x and v variables. We will have:

$$\frac{q}{v^2}\left(\frac{v^3}{gq} - \sigma\right) \cdot \frac{dv}{dx} = \sigma i - \frac{v^3}{qC^2} - \aleph_h \frac{\gamma_1}{\gamma_2} \cdot \frac{v^3}{gq}. \tag{6}$$

By considering that$q = h_k v_k = h_k\sqrt{\sigma g h_k}$, where $h_k$is the critical depth, when Lagrangian perturbations do not spread opposite the perturbed current movement, equation (6) can be presented in the following dimensionless form:

$$\left(\tilde{v}^3 - 1\right) \cdot \frac{d\tilde{v}}{d\tilde{x}} = \tilde{v}^2 \left(i - \mu\tilde{v}^3\right), \tag{7}$$

where $\tilde{v} = \frac{v}{v_k}$; $\tilde{x} = \frac{x}{h_k}$; $\mu = \frac{\lambda}{8}$, and $\lambda = \frac{8g}{C^2}$ is the dimensionless coefficient of linear Darcy resistance in case of turbulent movement.

One must consider that the inclination of a mountain slope along which the cold air mass flows down, must be quite big, because only in such a case is the wind velocity high. Usually, the wind velocity is 15-25 m/sec on average, but sometimes, it reaches 40-50 m/sec.

In order to obtain derivative $\frac{d\tilde{v}}{d\tilde{x}}$ more than zero and have the velocity from the ridge peaks increase along the inclination, it is clear that the following inequalities must be true: $\tilde{v}^3 > 1$ and $i > \mu\tilde{v}^3$. As v and v_k are the values of the same series, series$\mu = \frac{\lambda}{8}$ (λ is close to $0,001 \approx 0,1$) is much less than 1 and therefore, the values of $i$must be of the same series as $\frac{v}{v_k}$, but surely, it must always be less than 1, as $i = \sin\psi$. So, the given inequalities are true indeed, as the slopes over which the snow avalanches are originated, are quite steep and therefore, $\frac{d\tilde{v}}{d\tilde{x}} > 0$.

First, the solution of equation (7) is considered in terms of ignoring the force of resistance. In this case, its integral will be written down as follows (when $t = t_0 = 0$, $\tilde{x} = \tilde{x}_0 = 0$, $\tilde{v} = v_0$):

$$\frac{\tilde{v}^3}{2} - \left(i\tilde{x} + \frac{\tilde{v}_0^2}{2} + \frac{1}{\tilde{v}_0}\right)\tilde{v} + 1 = 0. \tag{8}$$

As it is known, the Cardan formula can be used to solve cubic equation$x^3 + px + q = 0$, when its discriminant $D = \frac{q^2}{4} + \frac{p^3}{27} \geq 0$.

In our case, the verification of equation (8) shows that:

$$\begin{aligned} D &= 1 - \left(\frac{2}{3}\right)^3 \left(i\tilde{x} + \frac{\tilde{v}_0^2}{2} + \frac{1}{\tilde{v}_0}\right)^3 = 1 - \left(\frac{2}{3}\right)^3 \left(i\tilde{x} + \frac{\tilde{v}_0^2}{2} + \frac{1}{2\tilde{v}_0} + \frac{1}{2\tilde{v}_0}\right)^3 \\ &\leq 1 - \left(\frac{2}{3}\right)^3 \left(i\tilde{x} + 3\sqrt[3]{\frac{\tilde{v}_0^2}{2} \cdot \frac{1}{2\tilde{v}_0} \cdot \frac{1}{2\tilde{v}_0}}\right)^3 = 1 - \left(\frac{2}{3}\right)^3 \left(ix + \frac{3}{2}\right)^3 \\ &< 1 - \left(\frac{2}{3}\right)^3 \left(\frac{3}{2}\right)^3 = 0, \quad \text{when } \tilde{x} > 0. \end{aligned}$$

Therefore, in this case, not only the Cardan formula is valid, but there is no formula to use to solve the cubic equation in radicals. Besides, as per inequality$D < 0$, all roots of equation (8) are true and different. If rewriting equation (8) as follows: $\frac{\tilde{v}^2}{2} - \left(i\tilde{x} + \frac{\tilde{v}_0^2}{2} + \frac{1}{\tilde{v}_0}\right) = -\frac{1}{\tilde{v}}$, and considering that the root of the equation for maximal values $\frac{1}{\tilde{v}}$ is quite little, it can be ignored.

Maximum root of the given equation can be determined from the following condition:

$$\tilde{v}^2 = 2i\tilde{x} + \tilde{v}_0^2 + \frac{2}{\tilde{v}_0}. \tag{9}$$

Second approximate solution, which considers the resistance of aerohydrodynamic friction, is obtained by inserting the value of $\tilde{v}$obtained from equation (9) by the first approximation in the relevant summand of equation (7) what yields the following expression:

$$\frac{\tilde{\upsilon}^3}{2} - \left[i\tilde{x} - \frac{\mu}{5i}\left(2\tilde{x}i + \tilde{\upsilon}_0^2 + \frac{2}{\tilde{\upsilon}_0}\right)^{5/2} \right] \tilde{\upsilon} + 1 = 0. \tag{10}$$

An approximate solution of equation (10) is as follows:

$$\tilde{\upsilon}^2 = 2i\tilde{x} - \frac{2}{5} \cdot \frac{\mu}{i}\left(2i\tilde{x} + \tilde{C}_0\right)^{5/2}, \tag{11}$$

where $\tilde{C}_0 = \tilde{\upsilon}_0^2 + \frac{1}{\tilde{\upsilon}_0}$.

Transition to the dimension (physical) values yields:

$$\upsilon = \sqrt{2g\sigma \cdot ix - \frac{\sigma g \lambda}{20i}\left(2ixh_k^{3/2} + C_0 h_k^{5/2}\right)^{5/2}}, \tag{12}$$

which can be used in the following form:

$$\upsilon = \sqrt{2g\sigma H - \frac{\sigma g \lambda}{20i}\left(2Hh_k^{3/2} + C_0 h_k^{5/2}\right)^{5/2}}, \tag{13}$$

where $xi = H$ is the distance on the vertical from the horizontal plane, which passes across the ridge crest to the dividing surface (to the cold air current and warmer air mass above it).

If ignoring aerohydrodynamic resistance in expression (13), we will obtain the following expression:

$$\upsilon = \sqrt{2g\sigma H}, \tag{14}$$

which is given for the velocities of the cold winds moving over the sloping by L. Prandtl in "Hydro air mechanics" [2].

Problem 2. Let us determine the value of shearing forces transferred to the snow cover from the wind and distance from the top to the site on the sloping, near which the impact of the tail wind velocity is virtually imperceptible.

Thus, according to expressions (12) and (13), the consideration of the forces of aerohydrodynamic resistance reduces the velocity of tail wind what is fully compliant with the physics of the given phenomenon.

The value of shear pressure caused by the wind acting on the snow cover on the downwind mountain slope is determined with the following expression:

$$\tau = C_f \cdot \frac{\gamma}{g} \cdot \frac{\upsilon^2}{2} = \frac{\lambda}{4}\rho\upsilon^2, \tag{15}$$

While the friction force on the streamlined surface will be:

$$F_f = \Omega \cdot \tau = \frac{\lambda}{4} \cdot \rho \cdot \upsilon^2 \cdot \Omega, \tag{16}$$

where υ is determined by equation (13).

The aerohydrodynamic resistance force, which plays the role of a shearing force for the upper surface of snow cover, is determined with the following expression:

$$F = \frac{\lambda}{4}\rho\Omega \left[2g\sigma H - \frac{\sigma g \lambda}{20i}\left(2Hh_k^{3/2} + C + h_k^{5/2}\right)^{5/2} \right]. \tag{17}$$

It is this force being an additional component of the force of gravity acting on the slope surface, which, together with the forces of seismotechnic origin and snow filtration (or with either of them separately), may cause the shift of the snow cover and origination of snow avalanche.

The use of expression (17) can be considerably simplified if ignoring the impact of aerohydrodynamic friction coefficient on the velocity of tail winds, i.e. if ignoring the second summand of the expression given in square brackets as a little value what will enable us to avoid the determination of $h_k$and to use the simplified form of expression (17):

$$F = \frac{\lambda}{2} g\rho\sigma x i\Omega, \tag{18}$$

to determine the value of shearing force caused by tail wind with the first approximation. When using expression (18), its h_k is determined simply, by using Blanje-Bress formula $h_k = 0.67H_0$, where H_0 is the depth of the cold air layer on top of the mountain crest [5].

Finally, it must be noted that despite an approximate value of expression (17), it is useful in that it allows determining the distance from the mountain top down the sloping to the spot near which the velocity of tail wind is virtually insensible in respect of snow cover. This distance is determined by expression obtained from expression (17):

$$x^* = \left(3.6\frac{i}{\lambda}\right)^{2/3} \frac{0.67H_0}{i}. \tag{19}$$

3. Conclusions

According to expression (17), the consideration of the forces of aerohydrodynamic resistance reduces the velocity of tail wind what is fully compliant with the physics of the given phenomenon.

The system of equations describing the dynamic processes of snow avalanches must include the summands envisaging the resistance along the contact contour of the snow layer with the air, as it is under the impact of the tangent pressures acting on the snow-air contact surface, the forces are passed from the wind to the snow cover.

One must consider that the presence of the ice-drift on the surface of the snow cover raises the coefficient of aerodynamic resistance by one rank, and the impact forces of tail winds increase consequently.

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Received 12.03.2017; accepted 21.05.2017.

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