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THE BASIC BOUNDARY VALUE PROBLEM FOR THE PLANE THEORY OF ELASTICITY OF POROUS COSSERAT MEDIA WITH TRIPLE-POROSITY

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Abstract. The static equilibrium of porous elastic materials with triple-porosity is considered in the case of an elastic Cosserat medium. A two-dimensional system of equations of plane deformation is written in the complex form and its general solution is represented by means of three analytic functions of a complex variable and three solutions of Helmholtz equations. Concrete problem are solved for the circle.

Keywords and phrases: Triple-porosity, the elastic Cosserat medium.

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Introduction

The first theory of consolidation for elastic materials with double porosity was presented by Wilson and Aifantis [1]. This theory unifies the earlier proposed models of porous media with single [2] and double [3, 4] porosities. More general models of double porosity materials based on Darcy's law are introduced in [5-9] and studied by several authors [10-22].

The first mathematical formulation of flow through triple porosity media is introduced by Liu [23] and several new triple porosity models for single-phase flow in a fracture-matrix system are presented by Liu et al. [24], Abdassah and Ershaghi [25], Al Ahmadi and Wattenbarger [26], Wu et al. [27].

The mathematical models of multi-porosity media have found applications in many branches of civil engineering, geotechnical engineering, technology and biomechanics. The intended applications of the theories of elasticity and thermoelasticity for materials with a multi-porosity structure are to geological materials such as oil and gas reservoirs, rocks and soils, manufactured porous materials such as ceramics and pressed powders, and biomaterials such as bone [28-30].

It should be noted that all the papers mentioned above dealt with a classical (symmetric) medium. We consider the problem of elasticity for solids with triple-porosity in the case of an elastic Cosserat medium.

1. Basic equations

Let V be a bounded domain in the Euclidean two-dimensional space E^2 bounded by the contour S. Suppose that $S \in C^{1,\beta}$, $0 < \beta \leq 1$. Let

$x = (x_1, x_2)$ be the points of space E^2, $\partial_i = \frac{\partial}{\partial x_i}$. Let us assume that the domain V is filled with an isotropic triple-porosity material.

The basic homogeneous system of equations for isotropic materials with triple porosity has the form [22, 32, 33]:

$$\partial_\alpha \sigma_{\alpha\beta} = 0, \quad \partial_\alpha \mu_{\alpha 3} + (\sigma_{12} - \sigma_{21}) = 0, \quad (\alpha, \beta = 1, 2) \tag{1}$$

$$\begin{aligned}
&\sigma_{11} = -\beta_1 p_1 - \beta_2 p_2 - \beta_3 p_3 + \lambda\theta + 2\mu\partial_1 u_1,\\
&\sigma_{22} = -\beta_1 p_1 - \beta_2 p_2 - \beta_3 p_3 + \lambda\theta + 2\mu\partial_2 u_2,\\
&\sigma_{12} = (\mu + \alpha)\partial_1 u_2 + (\mu - \alpha)\partial_2 u_1 - 2\alpha\omega,\\
&\sigma_{21} = (\mu + \alpha)\partial_2 u_1 + (\mu - \alpha)\partial_1 u_2 + 2\alpha\omega,\\
&\mu_{13} = (\nu + \beta)\partial_1 \omega, \quad \mu_{23} = (\nu + \beta)\partial_2 \omega,
\end{aligned} \tag{2}$$

$$(\theta := \partial_1 u_1 + \partial_2 u_2),$$

where $\sigma_{\alpha\beta}$ are stress tensor components, $\mu_{\alpha 3}$ are moment stress tensor components, u_α are components of the displacement vector, ω is the component of the rotation vector, p_i $(i = 1, 2, 3)$ are the pressures in the fluid phase, λ and μ are the Lamé parameters, α, β, μ are the constants characterizing the microstructure of the considered elastic medium, β_i $(i = 1, 2, 3)$ are the effective stress parameters.

In the stationary case, the values $p = (p_1, p_2, p_3)^T$ satisfy the following equation

$$\Delta p - Ap = 0, \quad A = \begin{pmatrix} b_1/a_1 & -a_{12}/a_1 & -a_{13}/a_1 \\ -a_{21}/a_2 & b_2/a_2 & -a_{23}/a_2 \\ -a_{31}/a_3 & -a_{32}/a_3 & b_3/a_3 \end{pmatrix} \tag{3}$$

where $a_i = \frac{k_i}{\mu'}$ (for the fluid phase, each phase i carries its respectively permeability k_i, μ' is fluid viscosity), a_{ij} is the fluid transfer rate between phase i and phase j, Δ is the 2D Laplace operator, $b_1 = a_{12} + a_{13}$, $b_2 = a_{21} + a_{23}$, $b_3 = a_{31} + a_{32}$.

On the plane x_1x_2, we introduce the complex variable $z = x_1 + ix_2 = re^{i\vartheta}$, $(i^2 = -1)$ and the operators $\partial_z = 0.5(\partial_1 - i\partial_2)$, $\partial_{\bar z} = 0.5(\partial_1 + i\partial_2)$, $\bar z = x_1 - ix_2$, and $\Delta = 4\partial_z\partial_{\bar z}$.

If relations (2) are substituted into system (1), then system (1) is written in the complex form

$$\begin{aligned}
&2(\mu + \alpha)\partial_{\bar z}\partial_z u_+ + (\lambda + \mu - \alpha)\partial_{\bar z}\theta - 2\alpha i\partial_{\bar z}\omega\\
&-\partial_{\bar z}(\beta_1 p_1 + \beta_2 p_2 + \beta_3 p_3) = 0,\\
&2(\nu + \beta)\partial_{\bar z}\partial_z \omega + \alpha i(\theta - 2\partial_{\bar z} u_+) - 2\alpha\omega = 0,
\end{aligned} \tag{4}$$

$$(u_+ = u_1 + iu_2).$$

2. The general solution of system (3)-(4)

In this section, we construct the analogues of the Kolosov-Muskhelishvili formulas [34] for system (4).

Equations (3) imply that

$$p_i = f'(z) + \overline{f'(z)} + l_{i1}\chi_1(z,\bar{z}) + l_{i2}\chi_2(z,\bar{z}),$$

where $f(z)$ is an arbitrary analytic functions of a complex variable z in the domain V and $\chi_\alpha(z,\bar{z})$ is an arbitrary solution of the Helmholtz equation

$$\Delta\chi_\alpha(z,\bar{z}) - \kappa_\alpha\chi_\alpha(z,\bar{z}) = 0,$$

κ_α are eigenvalues and (l_{11}, l_{21}, l_{31}), (l_{12}, l_{22}, l_{32}) are eigenvectors of the matrix A.

Theorem. The general solution of the system of equations (4) is represented as follows:

$$2\mu u_+ = \varkappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} + \frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}(f'(z) + \overline{f'(z)})$$

$$+\frac{4\mu}{\lambda+2\mu}\partial_{\bar{z}}[\delta_1\chi_1(z,\bar{z}) + \delta_2\chi_2(z,\bar{z})],$$

$$2\mu\omega = \frac{2\mu}{\nu+\beta}\chi(z,\bar{z}) - \frac{\varkappa+1}{2}i(\varphi'(z) + \overline{\varphi'(z)}),$$

where $\varkappa = \frac{\lambda+3\mu}{\lambda+\mu}$, $\delta_\alpha := \frac{l_{1\alpha}}{\kappa_\alpha}\beta_1 + \frac{l_{2\alpha}}{\kappa_\alpha}\beta_2 + \frac{l_{3\alpha}}{\kappa_\alpha}\beta_3$, $\varphi(z)$ and $\psi(z)$ are arbitrary analytic functions of a complex variable z in the domain V, $\chi(z,\bar{z})$ is an arbitrary solution of the Helmholtz equation

$$4\partial_z\partial_{\bar{z}}\chi(z,\bar{z}) - \xi^2\chi(z,\bar{z}) = 0,$$

where

$$\xi^2 := \frac{2\mu\alpha}{(\nu+\beta)(\mu+\alpha)} > 0.$$

For combinations of stress tensor components we obtain the following formulas

$$
\begin{aligned}
&\sigma_{11}+\sigma_{22}+i(\sigma_{12}-\sigma_{21})=2\left[\varphi'(z)+\overline{\varphi'(z)}+2i\partial_z\partial_{\bar z}\chi(z,\bar z)\right]\\
&\quad+\frac{2\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}(f'(z)+\overline{f'(z)})\\
&\quad-\frac{8\mu}{\lambda+2\mu}\partial_z\partial_{\bar z}\left[\delta_1\chi_1(z,\bar z)+\delta_2\chi_2(z,\bar z)\right],\\
&\sigma_{11}-\sigma_{22}+i(\sigma_{12}+\sigma_{21})=2\left[-z\overline{\varphi''(z)}-\overline{\psi'(z)}+2i\partial_{\bar z}\partial_{\bar z}\chi(z,\bar z)\right]\\
&\quad+\frac{2\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}z\overline{f''(z)}+\frac{8\mu}{\lambda+2\mu}\partial_{\bar z}\partial_{\bar z}\left[\delta_1\chi_1(z,\bar z)+\delta_2\chi_2(z,\bar z)\right],\\
&\mu_{13}+i\mu_{23}=2\partial_{\bar z}\chi(z,\bar z)+\frac{(\varkappa+1)(\nu+\beta)}{2\mu}i\overline{\varphi''(z)},\\
&\mu_{31}+i\mu_{32}=\frac{2(\nu-\beta)}{\nu+\beta}\partial_{\bar z}\chi(z,\bar z)+\frac{(\varkappa+1)(\nu-\beta)}{2\mu}i\overline{\varphi''(z)}.
\end{aligned}
\tag{5}
$$

Let mutually perpendicular unit vectors $\mathbf{l}$ and $\mathbf{s}$ be such that

$$\mathbf{l}\times\mathbf{s}=\mathbf{e}_3,$$

where $\mathbf{e}_3$ is the unit vector directed along the x_3-axis. The vector $\mathbf{l}$ forms the angle ϑ with the positive direction of the x_1-axis. Then the displacement components $u_l=\mathbf{u}\cdot\mathbf{l}$, $u_s=\mathbf{u}\cdot\mathbf{s}$, as well as the stress and moment stress components acting on an area of arbitrary orientation are expressed by the formulas

$$
\begin{gathered}
u_l+iu_s=e^{-i\vartheta}u_+,\\
\sigma_{ll}+i\sigma_{ls}=0.5[\sigma_{11}+\sigma_{22}+i(\sigma_{12}-\sigma_{21})+(\sigma_{11}-\sigma_{22}+i(\sigma_{12}+\sigma_{21}))e^{-2i\vartheta}],\\
\mu_{l3}=0.5[(\mu_{13}+i\mu_{23})e^{-i\vartheta}+(\mu_{13}-i\mu_{23})e^{i\vartheta}].
\end{gathered}
\tag{6}
$$

3. A problem for a circle

In this section, we solve a concrete boundary value problem for a circle with radius R (see fig. 1). On the boundary of the considered domain stresses, moment stresses and the values of pressures p_1, p_2, p_3 are given.

We consider the following problem

$$p_i=\sum_{-\infty}^{+\infty}A_{in}e^{in\vartheta},\quad |z|=R,\quad i=1,2,3 \tag{7}$$

$$\sigma_{rr}-i\sigma_{r\vartheta}=\sum_{-\infty}^{+\infty}B_ne^{in\vartheta},\quad |z|=R, \tag{8}$$

$$\mu_{r3}=\sum_{-\infty}^{+\infty}C_ne^{in\vartheta},\quad |z|=R. \tag{9}$$

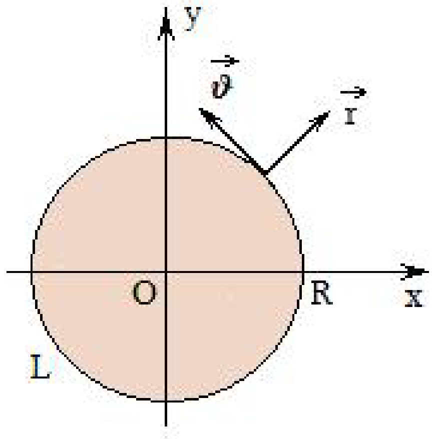

Fig. 1.

The analytic function $f(z)$ and the metaharmonic functions $\chi_1(z,\bar{z})$, $\chi_2(z,\bar{z})$ are represented as the series

$$f(z)=\sum_{n=1}^{+\infty}c_nz^n,\quad \chi_1(z,\bar{z})=\sum_{n=0}^{+\infty}\alpha_nI_n(r\kappa_1),\qquad \chi_2(z,\bar{z})=\sum_{n=0}^{+\infty}\beta_nI_n(r\kappa_2), \tag{10}$$

where $I_n(r\zeta)$ are modified Bessel function of n-th order, $z=re^{i\vartheta}$, and are substituted in the boundary conditions (7) we have

$$\sum_{n=1}^{+\infty}nR^{n-1}\left(c_ne^{i(n-1)\vartheta}+\bar{c}_ne^{-i(n-1)\vartheta}\right)+l_{i1}\sum_{-\infty}^{+\infty}\alpha_nI_n(R\kappa_1)e^{in\vartheta}$$
$$+l_{i2}\sum_{-\infty}^{+\infty}\beta_nI_n(R\kappa_2)e^{in\vartheta}=\sum_{-\infty}^{+\infty}A_{in}e^{in\vartheta},\quad i=1,2,3. \tag{11}$$

Compare the coefficients at identical degrees. We obtain the following system of equations

$$\begin{cases} c_1+\bar{c}_1+l_{i1}I_0\alpha_0+l_{i2}I_0\beta_0=A_{i0}, & i=1,2,3 \\ nR^{n-1}a_n+l_{i1}I_{n-1}\alpha_{n-1}+l_{i2}\beta_{n-1}=A_{in-1}, & i=1,2,3. \end{cases} \tag{12}$$

The coefficients $c_n,\ \alpha_n,\ \beta_n$ are found by solving (11).

Let us now satisfy the boundary conditions (8), (9). Due to the general representations (5) and (6) obtained above we have

$$\begin{aligned}\sigma_{rr}-i\sigma_{r\vartheta}&=\varphi'(z)+\overline{\varphi'(z)}-e^{2i\vartheta}\left(\bar{z}\varphi''(z)+\psi'(z)\right)\\&+\frac{i\xi^2}{2}\left(\chi(z,\bar{z})-\frac{4}{\xi^2}\partial_z\partial_z\chi(z,\bar{z})e^{2i\vartheta}\right)\\&-\frac{8\mu}{\lambda+2\mu}\left(\frac{\delta_1\kappa_1}{4}\chi_1(z,\bar{z})+\frac{\delta_2\kappa_2}{4}\chi_2(z,\bar{z})\right)\\&+\frac{8\mu}{\lambda+2\mu}\partial_z\partial_z(\delta_1\chi_1(z,\bar{z})+\delta_2\chi_2(z,\bar{z}))e^{2i\vartheta}\\&-\frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}\left(f'(z)+\overline{f'(z)}-\bar{z}\overline{f''(z)}e^{2i\vartheta}\right)\\&=\sum_{-\infty}^{+\infty}B_ne^{in\vartheta},\end{aligned}\tag{13}$$

$$\begin{aligned}\mu_{r3}&=\frac{(\varkappa+1)(\nu+\beta)i}{4\mu}\left(\overline{\varphi''(z)}e^{-i\vartheta}-\varphi''(z)e^{i\vartheta}\right)\\&+\partial_{\bar{z}}\chi(z,\bar{z})e^{-i\vartheta}+\partial_z\chi(z,\bar{z})e^{i\vartheta}=\sum_{-\infty}^{+\infty}C_ne^{in\vartheta}.\end{aligned}\tag{14}$$

The analytic functions $\varphi(z)$ and $\psi(z)$ are represented as the series

$$\varphi'(z)=\sum_{n=1}^{\infty}a_nz^n,\quad \psi'(z)=\sum_{n=0}^{\infty}b_nz^n,\quad \chi(z,\bar{z})=\sum_{-\infty}^{\infty}\gamma_nI_n(\xi r)e^{in\vartheta}$$

and are substituted in the boundary conditions (8), (9) so we have

$$\begin{aligned}&\left[\sum_{n=1}^{+\infty}(1-n)r^na_ne^{in\vartheta}+\sum_{n=1}^{+\infty}(1-n)r^n\bar{a}_ne^{-in\vartheta}-\sum_{n=0}^{+\infty}r^{n-2}b_{n-2}e^{-in\vartheta}\right.\\&\left.+\frac{i\xi^2}{2}\sum_{-\infty}^{\infty}(I_n(\xi r)-I_{n-2}(\xi r))\gamma_ne^{-in\vartheta}\right]_{r=R}=B_n',\end{aligned}\tag{15}$$

$$\begin{aligned}&\left[\frac{(\varkappa+1)(\nu+\beta)i}{4\mu}\sum_{n=1}^{+\infty}nr^{n-1}\bar{a}_ne^{-in\vartheta}+\sum_{n=1}^{+\infty}nr^{n-1}a_ne^{in\vartheta}\right.\\&\left.+\frac{\xi}{2}\sum_{-\infty}^{\infty}(I_{n+1}(\xi r)+I_{n-1}(\xi r))\gamma_ne^{in\vartheta}\right]_{r=R}=C_n',\end{aligned}\tag{16}$$

where

$$B_0'=B_0-\frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1}{4}\alpha_0I_0(\kappa_1R)+\frac{\delta_2\kappa_2}{4}\beta_0I_0(\kappa_2R)\right]$$

$$+\frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1^2}{4}\alpha_0I_2(\kappa_1R)+\frac{\delta_2\kappa_2^2}{4}\beta_0I_2(\kappa_2R)\right]+\frac{\mu(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}6R^2\bar{c}_3,$$

$$B_1' = B_1 - \frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1}{4}\alpha_1 I_1(\kappa_1 R) + \frac{\delta_2\kappa_2}{4}\beta_1 I_1(\kappa_2 R)\right]$$

$$+\frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1^2}{4}\alpha_1 I_1(\kappa_1 R) + \frac{\delta_2\kappa_2^2}{4}\beta_1 I_1(\kappa_2 R)\right] + \frac{2\mu R(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}(c_2-\bar{c}_2),$$

$$B_n' = B_n - \frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1}{4}\alpha_n I_n(\kappa_1 R) + \frac{\delta_2\kappa_2}{4}\beta_n I_n(\kappa_2 R)\right]$$

$$+\frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1^2}{4}\alpha_1 I_{n-2}(\kappa_1 R) + \frac{\delta_2\kappa_2^2}{4}\beta_1 I_{n-2}(\kappa_2 R)\right]$$

$$+\frac{\mu R^n(\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}(n+1)c_{n+1}, \quad n>1,$$

$$B_n' = B_n - \frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1}{4}\alpha_n I_n(\kappa_1 R) + \frac{\delta_2\kappa_2}{4}\beta_n I_n(\kappa_2 R)\right]$$

$$+\frac{8\mu}{\lambda+2\mu}\left[\frac{\delta_1\kappa_1^2}{4}\alpha_1 I_{n-2}(\kappa_1 R) + \frac{\delta_2\kappa_2^2}{4}\beta_1 I_{n-2}(\kappa_2 R)\right]$$

$$+\frac{\mu\beta_1+\beta_2+\beta_3)}{\lambda+2\mu}[(n+1)R^n\bar{c}_{n+1} - (n+2)(n+3)R^{n+2}\bar{c}_{n+3}), \quad n<0.$$

Equating the coefficients of $e^{in\vartheta}$ in (15) and (16), we obtain the following system of equations:

$$(1-n)R^n a_n - R^{n-2}b_{n-2} + \frac{i\xi^2}{2}(I_n(\xi R) - I_{n-2}(\xi R)\gamma_n = B_n', \tag{17}$$

$$R^n a_n - \frac{i\xi^2}{2}(I_n(\xi R) - I_{n+2}(\xi R))\gamma_n = \bar{B}_{-n}', \tag{18}$$

$$-\frac{(\varkappa+1)(\nu+\beta)i}{4\mu}nR^{n-1}a_n + \frac{\xi}{2}(I_{n-1}(\xi R) + I_{n+1}(\xi R))\gamma_n = \bar{C}_{-n}. \tag{19}$$

The coefficients a_n and γ_n are found by solving (18), (19):

$$a_n = \frac{\Delta_1}{\Delta}, \quad \gamma_n = \frac{\Delta_2}{\Delta},$$

where

$$\Delta = \frac{\xi R^n}{2}(I_{n-1}(\xi R)+I_{n+1}(\xi R)) + \frac{(\varkappa+1)(\nu+\beta)\xi^2 nR^{n-1}}{8\mu}(I_n(\xi R)-I_{n+2}(\xi R)),$$

$$\Delta_1 = \frac{\xi\bar{B}_{-n}'}{2}(I_{n-1}(\xi R) + I_{n+1}(\xi R)) + \frac{\bar{C}_{-n}i\xi^2}{2}(I_n(\xi R) - I_{n+2}(\xi R)),$$

$$\Delta_2 = R^n\bar{C}_{-n} + \frac{(\varkappa+1)(\nu+\beta)inR^{n-1}}{4\mu}\bar{B}_{-n}'.$$

The coefficients b_n may be found from formulae (17)

$$b_n = (1-n)R^2 a_{n+2} + \frac{i\xi^2}{2R^n}(I_{n+2}(\xi R) - I_n(\xi R))\gamma_{n+2} - \frac{B_{n+2}'}{R^n}.$$

It is easy to prove the absolute and uniform convergence of the series obtained in the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

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REFERENCES

1. Wilson R.K., Aifantis E.C. On the theory of consolidation with double porosity-I. *International Journal of Engineering Science,* **20** (1982), no. 9, 1009–1035.

2. Biot M.A.: General theory of three-dimensional consolidation. *J. Appl. Phys.* **12** (1941), 155–164.

3. Barenblatt G.I., Zheltov I.P., Kochina I.N.: Basic concept in the theory of seepage of homogeneous liquids in fissured rocks (strata). *J. Appl. Math. Mech.* **24** (1960), 1286–1303.

4. Warren J., Root P. The behavior of naturally fractured reservoirs. *Soc. Pet. Eng. J.* **3** (1963), 245–255.

5. Gelet R., Loret B., Khalili N. Borehole stability analysis in a thermoporoelastic dual-porosity medium. *Int. J. Rock Mech. Min. Sci.* **50** (2012), 65–76.

6. Khalili N., Selvadurai A.P.S. A fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity. *Geophys. Res. Lett.* **30**, 2268 (2003).

7. Khalili N., Habte M.A., Zargarbashi S. A fully coupled flow deformation model for cyclic analysis of unsaturated soils including hydraulic and mechanical hysteresis. *Comput. Geotech.* **35** (2008), 872–889.

8 Pride S.R., Berryman J.G. Linear dynamics of double-porosity dual-permeability materials I. Governing equations and acoustic attenuation. *Phys. Rev.* E **68**, 036603 (2003).

9. Zhao Y., Chen M. Fully coupled dual-porosity model for anisotropic formations. *Int. J. Rock Mech. Min. Sci.* **43** (2006), 1128-1133.

10. Ciarletta M., Passarella F., Svanadze M. Plane waves and uniqueness theorems in the coupled linear theory of elasticity for solids with double porosity. *J. Elast.* **114** (2014), 55-68.

11. Gentile M., Straughan B. Acceleration waves in nonlinear double porosity elasticity. *Int. J. Eng. Sci.* **73** (2013), 10-16.

12. Scarpetta E., Svanadze M., Zampoli V. Fundamental solutions in the theory of thermoelasticity for solids with double porosity. *J. Therm. Stresses* **37** (2014), 727-748.

13. Straughan B. Stability and uniqueness in double porosity elasticity. *Int. J. Eng. Sci.* **65** (2013), 1-8.

14. Svanadze, M. Uniqueness theorems in the theory of thermoelasticity for solids with double porosity. *Meccanica* **49** (2014), 2099-2108.

15. Svanadze M. On the theory of viscoelasticity for materials with double porosity. *Discrete Contin. Dyn. Syst., Ser.* B **19** (2014), 2335-2352.

16. Svanadze M., De Cicco S. Fundamental solutions in the full coupled linear theory of elasticity for solid with double porosity. *Arch. Mech.* **65** (2013), 367-390.

17. Svanadze M., Scalia A. Mathematical problems in the coupled linear theory of bone poroelasticity. *Comput. Math. Appl.* **66** (2013), 1554-1566.

18. Tsagareli I., Svanadze M.M. The solution of the stress boundary value problem of elastostatics for double porous plane with a circular hole. *Reports of Enlarged sess. of Sem. of Appl. Math.*, **24**, (2010), 130–133.

19. Basheleishvili M., Bitsadze L., Explicit solution of the BVP of the theory of consolidation with double porosity for half-plane. *Georgian Mathematical Journal,* **19**, 1 (2012), 41–49.

20. Basheleishvili M., Bitsadze L., Explicit solutions of the BVPs of the theory of consolidation with double porosity for the half-space. *Bulletin of TICMI,* **14** (2010), 9–15.

21. Tsagareli I., Bitsadze L. The boundary value problems in the full coupled theory of elasticity for plane with double porosity with a circular hole. *Semin. I. Vekua Inst. Appl. Math. Rep.* **40** (2014), 68-79.

22. Janjgava R. Elastic equilibrium of porous Cosserat media with double porosity. *Adv. Math. Phys.*, (2016), Article ID 4792148, 9 pages http://dx.doi.org/10.1155/2016/4792148.

23. Liu C.Q. Exact solution for the compressible flow equations through a medium with triple-porosity. *Appl. Math. Mech.* **2** (1981), 457-462.

24. Liu J.C., Bodvarsson G.S., Wu Y.S. Analysis of pressure behaviour in fractured lithophysical reservoirs. *J. Cantam. Hydrol.* **62–63** (2003), 189-211.

25. Abdassah D., Ershaghi I. Triple-porosity systems for representing naturally fractured reservoirs. *SPE Form. Eval.* **1** (1986), 113-127. SPE-13409-PA.

26. Al Ahmadi H.A., Wattenbarger R.A. Triple-porosity models: one further step towards capturing fractured reservoirs heterogeneity. *Saudi Aramco J. Technol.* **2011** (2011), 52-65.

27. Wu Y.S., Liu H.H., Bodavarsson G.S.: A triple-continuum approach for modelling flow and transport processes in fractured rock. *J. Contam. Hydrol.* **73** (2004), 145-179.

28. Cowin S.C. (ed.) Bone Mechanics Handbook. *Informa Healthcare USA Inc.,* New York (2008).

29. Cowin S.C., Cardoso L. Blood and interstitial flow in the hierarchical pore space architecture of bone tissue. *J. Biomech.* **48** (2015), 842-854.

30. Cowin S.C., Gailani G., Benalla M. Hierarchical poroelasticity: movement of interstitial fluid between levels in bones. *Philos. Trans. R. Soc. Lond.* A **367** (2009), 3401-3444.

31. Cosserat E., Cosserat F. Theorie des Corps Deformables. Hermann, Paris, France, 1909.

32. Svanadze M. Fundamental solutions in the theory of elasticity for triple porosity materials. *Meccanica*, **51** (2016), 1825–1837.

33. Svanadze M. On the linear theory of thermoelasticity for triple porosity materials. In: M. Ciarletta, V. Tibullo, F. Passarella, eds., *Proc. 11th Int. Congress on Thermal Stresses,* 5-9 June, 2016, Salerno, Italy, 259–262.

34. Muskhelishvili N.I. Some basic problems of the mathematical theory of elasticity. Fundamental equations, plane theory of elasticity, torsion and bending. *Noordhoff International Publishing, Leiden*, 1977.

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