Proceedings of I. Vekua Institute of Applied Mathematics
Vol. 65, 2015

DERIVATION OF NONLINEAR EQUATIONS FOR SHALLOW SHELLS, CONSISTING OF A MIXTURE OF TWO ISOTROPIC MATERIALS

Janjgava R.

Abstract. In the paper we consider geometrically nonlinear equations of elastic balance for binary mixture of two isotropic materials. In the literature the considered model is called Green-Naghdi-Steel's model. The main three-dimensional equations of static balance corresponding to the considered model are recorded in any curvilinear system of coordinates. The main two-dimensional relations for the shallow shells consisting of binary mixture are obtained from these equations using I. Vekua's reduction method and basing on T. Meunargia's works.

Keywords and phrases: Binary mixture of two isotropic materials, shallow shells, I. Vekua method.

AMS subject classification (2010): 74E30.

1. Introduction

Fundamentals of the theory of elastic mixtures were founded in the sixties of the last century. The basic mechanical principles of the deformable medium with complex internal structure were formulated in works of K. Truesdell and R. Toupin [1,2]. Subsequenty this theory was generalized and developed in many directions. The nonlinear model of mixture of two solid isotropic materials considered in this work was developed in [3-7]. The mentioned model of elastic mixtures relating to three-dimensional spatial coordinates, in honor of her founders, bears the name of Green-Nagdi-Steel's model. A lot of works were devoted to the study of linear problems of a statics, dynamics and elastic fluctuations for this model by the various mathematical methods (see eg. [8-11]). Nonlinear problems within the considered model were also studied [12, 13]. In works of the author [14, 15] two-dimensional equations for shallow shells were obtained from a linear three-dimensional theory of mixtures. The method of reduction of I. Vekua was used [16].

The purpose of this work is to obtain the main equations for geometrically nonlinear shells consisting of mixture of two isotropic materials. For this purpose the corresponding main three-dimensional relations are recorded in any curvilinear system of coordinates. The same method of Vekua generalized in T. V. Meunargia's works [17-19] for nonlinear problems (as geometrically and physically) was applied for the reduction.

2. Complete system of the equations for binary mixture of elastic materials

Suppose we have a body consisting of two elastic isotropic materials the reference configuration of which occupies the domain $\overline{\Omega} \subset R^3$. If x_1, x_2, x_3 is the rectangular Cartesian system of coordinates, the nonlinear equations of static equilibrium will have the form [1-3]

$$\begin{cases} \partial_i[\sigma'_{ij} - \delta_{ij}\Pi + (\sigma'_{ik} - \delta_{ik}(\Pi - \alpha))\partial_k u'_j] + \rho_1 F'_j = 0, \\ \partial_i[\sigma''_{ij} + \delta_{ij}\Pi + (\sigma''_{ik} + \delta_{ik}(\Pi - \alpha))\partial_k u''_j] + \rho_2 F''_j = 0, \end{cases} \quad \text{in } \Omega \qquad (1)$$

Reaction functions or nonlinear relations corresponding to the generalized Hooke's law have the form

$$\begin{aligned} \sigma'_{ij} &= (-\alpha + \lambda_1\varepsilon'_{kk} + \lambda_3\varepsilon''_{kk})\delta_{ij} + 2\mu_1\varepsilon'_{ij} + 2\mu_3\varepsilon''_{ij} - 2\lambda_5 h_{ij} \\ \sigma''_{ij} &= (\alpha + \lambda_4\varepsilon'_{kk} + \lambda_2\varepsilon''_{kk})\delta_{ij} + 2\mu_3\varepsilon'_{ij} + 2\mu_2\varepsilon''_{ij} + 2\lambda_5 h_{ij}, \end{aligned} \quad \text{in } \overline{\Omega} \qquad (2)$$

where $\partial_i \equiv \frac{\partial}{\partial x_i}$; $\sigma'_{ij}, \sigma''_{ij}$ are components of a tensor of stresses of two components of mixture; δ_{ij} is Kronecker-Delta; $\Pi_j \equiv \partial_j \Pi$ are so-called interaction forces of interaction between the two components of mixture

$$\Pi = \frac{\alpha\rho_2}{\rho}\varepsilon'_{kk} + \frac{\alpha\rho_1}{\rho}\varepsilon''_{kk}, \quad \rho = \rho_1 + \rho_2;$$

$\rho_1 > 0$, $\rho_2 > 0$ are partial densities of components of mixture; F'_{ij}, F''_{ij} are components of vectors of mass forces; $\alpha, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \mu_1, \mu_2, \mu_3$ are the elastic constants characterizing mechanical properties of mixture, when $\alpha = \lambda_3 - \lambda_4$; $\varepsilon'_{ij} = \varepsilon'_{ji}, \varepsilon''_{ij} = \varepsilon''_{ji}$ are components of tensors of deformations expressed by means of the formulas

$$\varepsilon'_{ij} = \frac{1}{2}(\partial_i u'_j + \partial_j u'_i + \partial_i u'_k \partial_j u'_k), \quad \varepsilon''_{ij} = \frac{1}{2}(\partial_i u''_j + \partial_j u''_i + \partial_i u''_k \partial_j u''_k); \quad (3)$$

$h_{ij} = -h_{ji}$ are components of rotation tensor of components of mixture

$$h_{ij} = \frac{1}{2}(\partial_i u'_j - \partial_j u'_i + \partial_j u''_i - \partial_i u''_j + \partial_i u'_k \partial_j u''_k - \partial_j u'_k \partial_i u''_k); \qquad (4)$$

$\vec{u}\,' = (u'_1, u'_2, u'_3)$, $\vec{u}\,'' = (u''_1, u''_2, u''_3)$ are partial vectors of displacement of components of mixture.

In the above formulas Latin indexes take the value 1,2,3 and we assume summation on the repeating indexes. We will assume the same below, but the Greek indexes will take the value 1,2.

Thus, in the theory of binary mixture two vectors of displacement and two tensors of deformations and stresses are considered in each point of a body. In view of the fact that the antisymmetric tensor h_{ij}, participates in the record of Hooke's law the symmetry of magnitudes σ'_{ij} and σ''_{ij} is broken.

Let's introduce the following notation

$$P'_{ij} := \sigma'_{ij} - \delta_{ij}(\Pi - \alpha), \quad P''_{ij} := \sigma''_{ij} + \delta_{ij}(\Pi - \alpha). \tag{5}$$

To simplifly the record we will enter also following notation (matrix columns)

$$P_{ij} :=(P'_{ij}, P''_{ij})^T,\ U_j :=(u'_j, u''_j)^T,\ \epsilon_{ij} :=(\varepsilon'_{ij}, \varepsilon''_{ij})^T,\ H_{ij} := (h_{ij}, h_{ji})^T. \tag{6}$$

The relations (1) and (2) considering notation (5) and (6) can be written as follows

$$\partial_i(P_{ij} + P_{ik} \circ \partial_k U_j) + \Phi_j = 0 \quad \text{in } \Omega, \tag{7}$$

$$P_{ij} = \Lambda \epsilon_{kk} \delta_{ij} + 2M\epsilon_{ij} - 2\lambda_5 H_{ij} \quad \text{in } \overline{\Omega}, \tag{8}$$

where

$$\Phi_j := (\rho_1 F'_j, \rho_2 F''_j)^T, \ \ \vec{U} := (\vec{u}\,', \vec{u}\,'')^T,$$

$$\Lambda = \begin{pmatrix} \lambda_1 - \dfrac{\alpha\rho_2}{\rho} & \lambda_3 - \dfrac{\alpha\rho_1}{\rho} \\ \lambda_4 + \dfrac{\alpha\rho_2}{\rho} & \lambda_1 + \dfrac{\alpha\rho_1}{\rho} \end{pmatrix}, \quad M = \begin{pmatrix} \mu_1 & \mu_3 \\ \mu_3 & \mu_2 \end{pmatrix};$$

the symbol $\circ$ denotes the following operation

$$(a_1, a_2)^T \circ (b_1, b_2)^T = (a_1 b_1, a_2 b_2)^T.$$

Using the obtained notation we represent formulas (3) as follows

$$\epsilon_{ij} = \frac{1}{2}(\partial_i U_j + \partial_j U_i + \partial_i U_k \circ \partial_j U_k), \tag{9}$$

and in view of formula (4) we have

$$H_{ij} = \frac{1}{2} S(\partial_i U_j - \partial_j U_i + \partial_i U_k \circ (E\partial_j U_k)), \tag{10}$$

where the following matrixes are denoted by S and E

$$S = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Substituting the formulas (9) and (10) into formula (8), we will have

$$\begin{aligned} P_{ij} = {} & \Lambda \partial_k U_k \delta_{ij} + (M - \lambda_5 S)\partial_i U_j + (M + \lambda_5 S)\partial_j U_i \\ & + \frac{1}{2}\Lambda(\partial_m U_k \circ \partial_m U_k)\delta_{ij} + M(\partial_i U_k \circ \partial_j U_k) - \lambda_5 S(\partial_i U_k \circ (E\partial_j U_k)). \end{aligned} \tag{11}$$

If $\vec{e}_1, \vec{e}_2, \vec{e}_3$ are unit vectors of the Cartesian system of coordinates, we can write the relations (7), (9) and (10) we in the vector form

$$\partial_i[\vec{P}^i + (\vec{e}_k \vec{P}^i) \circ \partial_k \vec{U})] + \vec{\Phi} = 0 \quad \text{in } \Omega, \tag{12}$$

$$\epsilon_{ij} = \frac{1}{2}(\vec{e}_j \partial_i \vec{U} + \vec{e}_i \partial_j \vec{U} + \partial_i \vec{U} \circ \partial_j \vec{U}), \tag{13}$$

$$H_{ij} = \frac{1}{2} S(\vec{e}_j \partial_i \vec{U} - \vec{e}_i \partial_j \vec{U} + \partial_i \vec{U} \circ (E \partial_j \vec{U})), \tag{14}$$

where $\vec{P}^i = P^{ij}\vec{e}_j = (P'^{ij}\vec{e}_j, P''^{ij}\vec{e}_j)^T$ contravariant components of stresses which coincide with covariant components of stresses in the Cartesian system of coordinates;

$$\vec{U} = (\vec{u}\,', \vec{u}\,'')^T = (u'^i \vec{e}_i, u''^i \vec{e}_i)^T; \quad \vec{\Phi} = (\Phi'^i \vec{e}_i, \Phi''^i \vec{e}_i)^T.$$

In the arbitrary curvilinear system of coordinates x^1, x^2, x^3 of the equation of equilibrium (12) will take the form

$$\frac{1}{\sqrt{\mathrm{g}}} \partial_i [(\sqrt{g} P^{ij} \vec{R}_j + P^{ik} \circ \partial_k \vec{U})] + \vec{\Phi} = 0 \quad \text{in } \Omega, \tag{15}$$

where g discriminant of the relative metric square form; $\vec{R}_j$- covariant basis vectors.

Really, in case of the Cartesian system of coordinates $\mathrm{g} = 1$ the system (15) passes into system (12). The first summand of the right part of the system (15) represents a divergence of the following tensor

$$T = T^{ij} \vec{R}_i \otimes \vec{R}_j := (P^{ij} + P^{ik} \circ \overset{\circ}{\nabla}_k U^j) \vec{R}_i \otimes \vec{R}_j, \tag{16}$$

where $\overset{\circ}{\nabla}_k$ symbol of a spatial covariant derivative ($\partial_i \vec{U} = \overset{\circ}{\nabla}_i U^k \vec{R}_k$); $\otimes$ a symbol of the tensor product. Thus,

$$div\, T = \frac{1}{\sqrt{\mathrm{g}}} \partial_k (\sqrt{g} T^{kj} \vec{R}_j).$$

This last expression is an invariant concerning a choice of spatial system of coordinates. In the arbitrary system of coordinates we will have also the formulas analogous to formulas (13) and (14)

$$\epsilon_{ij} = \frac{1}{2}(\vec{R}_j \partial_i \vec{U} + \vec{R}_i \partial_j \vec{U} + \partial_i \vec{U} \circ \partial_j \vec{U}), \tag{17}$$

$$H_{ij} = \frac{1}{2} S(\vec{R}_j \partial_i \vec{U} - \vec{R}_i \partial_j \vec{U} + \partial_i \vec{U} \circ (E \partial_j \vec{U})). \tag{18}$$

Taking into account the following known formulas [16]

$$\vec{R}_j \partial_i \vec{U} = \overset{\circ}{\nabla}_i U_j, \quad \partial_i \vec{U} = \overset{\circ}{\nabla}_i U^k \vec{R}_k,$$

we'll write relations (17) and (18) in the form of

$$\epsilon_{ij} = \frac{1}{2}\Big(\overset{\circ}{\nabla}_i U_j + \overset{\circ}{\nabla}_j U_i + \overset{\circ}{\nabla}_i U_k \circ \overset{\circ}{\nabla}_j U_k\Big), \tag{19}$$

$$H_{ij} = \frac{1}{2} S\Big(\overset{\circ}{\nabla}_i U_j - \overset{\circ}{\nabla}_j U_i + \overset{\circ}{\nabla}_i U_k \circ (E\overset{\circ}{\nabla}_j U_k)\Big), \tag{20}$$

where $U^k = (u'^k, u''^k)^T$ is the column-matrix consisting of contravariant components of vectors of displacements.

The relations (8) will take the form

$$P_{ij} = \Lambda\epsilon_k^k \mathrm{g_{ij}} + 2\mathrm{M}\epsilon_{\mathrm{ij}} - 2\lambda_5 \mathrm{H_{ij}}.$$

If to consider the following equalities [16]

$$\mathrm{g}^{ij} = \vec{R}^i \vec{R}^j, \quad (\vec{R}_k \partial_j \vec{U})\vec{R}^k = \partial_j \vec{U},$$

where $\vec{R}^j$ are contravariant basis vectors, and g^{ij} are contravariant components of a metric tensor, we have the following expression for a vector $\vec{P}^i = P^{i\cdot}_{\cdot j}\vec{R}^j$ ($P^{i\cdot}_{\cdot j}$ the mixed components of stresses tensor),

$$\vec{P}^i = \Lambda(\vec{R}^j \partial_j \vec{U})\vec{R}^i + (M + \lambda_5 S)(\vec{R}^i \partial_j \vec{U})\vec{R}^j + (M - \lambda_5 S)(\vec{R}^i \vec{R}^j)\partial_j \vec{U}$$
$$+\frac{1}{2}\Lambda(\partial_k \vec{U} \circ \partial^k \vec{U})\vec{R}^i + M(\partial^i \vec{U} \circ \partial_j \vec{U})\vec{R}^j - \lambda_5 S(\partial^i \vec{U} \circ (E\partial_j \vec{U}))\vec{R}^j, \tag{21}$$

where $\partial^j = \mathrm{g^{jk}}\partial_{\mathrm{k}}$.

If we multiphly the scalar product of contravariant basis vector $\vec{R}^i$ to both sides (16), we'll obtain the following vector

$$\begin{aligned} \vec{T}^i &= T \cdot \vec{R}^i = T^{ij}\vec{R}_j := \vec{P}^i + P^{ik} \circ \partial_k \vec{U} \\ &= (P^{ij} + P^{ik} \circ \overset{\circ}{\nabla}_k U^j)\vec{R}_j = P \cdot \vec{R}^i + (P \cdot \vec{R}^i)\vec{R}^k \circ \partial_k \vec{U}, \end{aligned} \tag{22}$$

where

$$P = P^{ij}\vec{R}^i \otimes \vec{R}_j.$$

Tensors $T = (T', T'')^T$ and $P = (P', P'')^T$ are also called the first and second stresses tensors of Piola-Kirchhoff. In view of formulas (15) and (17), we will have the following basic relations for contravariant components of the first stress tensor of the Piola-Kirchhoff

$$\frac{1}{\sqrt{\mathrm{g}}}\partial_i(\sqrt{g}\vec{T}^i) + \vec{\Phi} = 0 \quad \text{in } \Omega, \tag{23}$$

$$\vec{T}^i = \Lambda(\vec{R}^j \partial_j \vec{U})\vec{R}^i + (M + \lambda_5 S)(\vec{R}^i \partial_j \vec{U})\vec{R}^j + (M - \lambda_5 S)(\vec{R}^i \vec{R}^j)\partial_j \vec{U} + \vec{N}^i, \tag{24}$$

where the nonlinear part is denoted by $\vec{N}^i$

$$\vec{N}^i = \frac{1}{2}\Lambda(\partial_k \vec{U} \circ \partial^k \vec{U})\vec{R}^i + M(\partial^i \vec{U} \circ \partial_j \vec{U})\vec{R}^j - \lambda_5 S(\partial^i \vec{U} \circ (E\partial_j \vec{U}))\vec{R}^j$$
$$+[\Lambda(\vec{R}^j \partial_j \vec{U})] \circ \partial^i \vec{U} + [(M + \lambda_5 S)(\vec{R}^i \partial_j \vec{U})] \circ \partial_j \vec{U} + [(M - \lambda_5 S)$$
$$(\partial^i \vec{U}\vec{R}^k)] \circ \partial_k \vec{U} + \frac{1}{2}[\Lambda(\partial_k \vec{U} \circ \partial^k \vec{U})] \circ \partial^i \vec{U} + [M(\partial^i \vec{U} \circ \partial_j \vec{U})]$$
$$\circ \partial^j \vec{U} - \lambda_5[S(\partial^i \vec{U} \circ (E\partial_j \vec{U}))] \circ \partial^j \vec{U}. \tag{25}$$

If we reject the nonlinear member $\vec{N}^i$ in the formula (24) then (23), (24) will be the equations of static equilibrium of the linear theory of elastic mixtures in arbitrary curvilinear system of coordinates.

For completeness of relations (23), (24) we have to connect the boundary conditions. For example, in case of the first and second (the loadings are assumed dead) main boundary value problems the following conditions are set respectively on the boundary $\partial\Omega$ of the domain Ω

$$I.\, \vec{U}(x^1, x^2, x^3) = \vec{U}^0 \quad \text{on} \;\; \partial\Omega; \tag{26}$$

$$II.\, T_{(l)} := T \cdot \vec{l} = \vec{T}^i l_i = T^{ij} l_i \vec{R}_j = \vec{T}^0_{(l)} \quad \text{on} \;\; \partial\Omega, \tag{27}$$

where $\vec{U}^0, \vec{T}^0_{(l)}$ are vector functions defined on the boundary, $\vec{l} = (l_1, l_2, l_3)$ are external normals to a surface $\partial\Omega$.

3. Reduction of three-dimensional relations (23)-(27)

Suppose Ω is a shell with thickness $2h$ symmetric to its middle surface ω. h it is a positive bounded sufficiently smooth function of the point of the surface ω. ω is a sufficiently by smooth bilateral surface. Let's denote the side surfaces of a shell by Γ. Surfaces of ω and Γ are crossed at right angle in each point. We assume that thickness $2h$ is much less in comparison with other sizes of a shell.

We consider the coordinate system normally connected with a middle of surface. In this system the radius vector $\vec{R}$ of any point M of the domain Ω is expressed by means of the formula (see fig. 1)

$$\vec{R} = \vec{r}(x^1, x^2) + x^3 \vec{n}(x^1, x^2),$$

where x^1, x^2 are Gaussian parameters of the surface ω ; $\vec{r}$ and $\vec{n}$ are radius vector and unit vector normal to the surface at the point $x^1, x^2 \in \omega$. x^3 is the relative length from the point M to the surface ω.

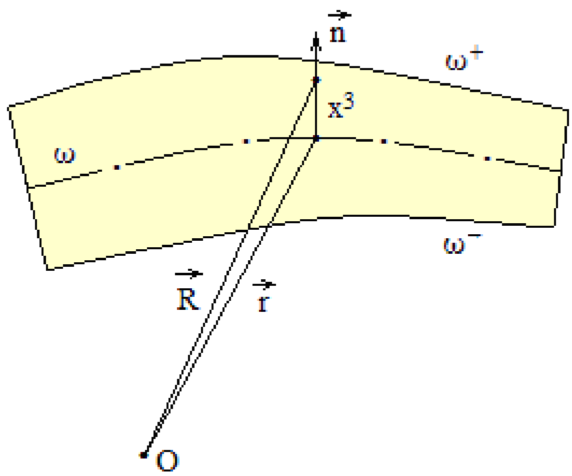

Fig. 1. The considered shell of variable thickness

Our goal is to carry out a reduction of three-dimensional relations (23)-(27) to two-dimensional system using I. N. Vekua's method and to obtain two-dimensional relations for the shallow shells consisting of a mixture of two elastic materials.

We multiply both members of equilibrium equations (19) by functions

$$\Big(k+\frac{1}{2}\Big)\frac{1}{h}P_k\Big(\frac{x^3}{h}\Big)\sqrt{\frac{\mathrm{g}}{a}},\quad k=0,1,\dots,$$

where $P_k\left(\frac{x^3}{h}\right)$ is the Legandre polynomials of order k; a is the discriminant of quadratic form of surface ω.

We integrate both parts of the obtained relations by the thickness of the coordinates x^3 from $-h$ to h. The conditions for stresses are satisfied on the surfaces $x^3=h$ and $x^3=-h(\omega^-$ and ω^+, respectively) of a shell. As a result we obtain the following infinite system of the equations for the functions of two variables x^1,x^2

$$\frac{1}{\sqrt{a}}\partial_\alpha\left(\sqrt{a}\overset{(k)}{\vec{T}^{\alpha}}\right)+\partial_\alpha\ln h\underset{=}{\overset{(k)}{\vec{T}^{\alpha}}}-\frac{1}{h}\underset{-}{\overset{(k)}{\vec{T}^{3}}}+\overset{(k)}{\vec{F}}=0,\quad k=0,1,\dots,\tag{28}$$

where

$$\overset{(k)}{\vec{T}^{j}}=\Big(k+\frac{1}{2}\Big)\frac{1}{h}\int\limits_{-h}^{h}\sqrt{\frac{\mathrm{g}}{a}}\vec{T}^{j}P_k\Big(\frac{x^3}{h}\Big)dx^3;$$

$$\underset{-}{\overset{(k)}{\vec{T}^{3}}}=(2k+1)\left(\overset{(k-1)}{\vec{T}^{3}}+\overset{(k-3)}{\vec{T}^{3}}+\dots\right),$$

$$\underset{=}{\overset{(k)}{\vec{T}^{\alpha}}}=(k+1)\overset{(k)}{\vec{T}^{\alpha}}+(2k+1)\left(\overset{(k-2)}{\vec{T}^{\alpha}}+\overset{(k-4)}{\vec{T}^{\alpha}}+\dots\right),\quad \overset{(-n)}{\vec{T}^{j}}=0,$$

when $n>0$;

$$\overset{(k)}{\vec{F}}=\Big(k+\frac{1}{2}\Big)\frac{1}{h}\int\limits_{-h}^{h}\sqrt{\frac{\mathrm{g}}{a}}\vec{\Phi}P_k\Big(\frac{x^3}{h}\Big)dx^3+\Big(k+\frac{1}{2}\Big)\frac{1}{h}$$

$$\left\{\sqrt{\frac{\mathrm{g}_+}{a}}[\vec{T}^3_+-\partial_a h\vec{T}^a_+]-(-1)^k\sqrt{\frac{\mathrm{g}_+}{a}}[\vec{T}^3_-+\partial_\alpha h\vec{T}^a_-]\right\};$$

by lower symbols "+" and "–" are denoted the values of the top and bottom surfaces of the shell.

The infinite system of equations (28) is equivalent to the system of equations (23) since Legendre's polynomials from a complete system in the interval [-1;1].

Remark. We believe that all vector functions or tensor fields in relations (23)-(25) for each point $(x^1, x^2) \in \omega$ have expansion in series to the coordinate x^3 on Legendre's polynomials $P_k\Big(\frac{x^3}{h}\Big)$.

Now we have to obtain the expressions for the so-called moments of order k $\overset{(k)}{\vec{T}}$ from the relations (24). We will literally repeat verbatim the algorithm of a reduction of Vekua for a reduction of linear part (20), and we will use T. Meunargia's works [17-19] for nonlinear part. In these works the method of Vekua was generalized for both nonlinear geometrically and physically non-shallow shell.

Thus, for obtaining the two-dimensional relations corresponding to the generalized Hooke's law, I. Vekua makes the assumption of geometrical character which is called the first main assumption and it is as follows

$$1 - k_1 x^3 \cong 1, \ \ 1 - k_2 x^3 \cong 1, \ \ -h \le x^3 \le h. \tag{29}$$

These requirements mean that either the main curvatures k_1 and k_2 of a middle surface are small (shallow shell), or the thickness of the shell is small (thin shell). It follows from the assumptions (29) that the spatial covariant and contravariant basis vectors are approximately equal to the corresponding basis vectors of a middle surface. Therefore, corresponding covariant and contravariant components and discriminants of metric tensors of space and a middle surface are also approximately equal

$$\vec{R}_\alpha \cong \vec{r}_\alpha, \ \ \vec{R}^\alpha \cong \vec{r}^{\,\alpha}, \ \ \vec{R}^3 = \vec{r}_3 = \vec{n}, \ \ \mathrm{g}_{\alpha\beta} \cong a_{\alpha\beta}, \ \mathrm{g}^{\alpha\beta} \cong a^{\alpha\beta}, \ \ \mathrm{g} \ \cong a. \tag{30}$$

According to an assumption (29) we will have

$$\overset{(k)}{\vec{T}^j} = \Big(k + \frac{1}{2}\Big)\frac{1}{h}\int\limits_{-h}^{h} \vec{T}^j P_k\Big(\frac{x^3}{h}\Big)dx^3; \tag{31}$$

Let's assume that $\vec{U} = (\vec{u}', \vec{u}'')^T$ are sufficiently smooth functions and they are placed as a uniformly convergent series at Legendre's polynomials for each fixed point (x^1, x^2) of surface of ω with respect to the argument x^3

$$\vec{U}(x^1, x^2, x^3) = \sum_{k=0}^{\infty} \overset{(k)}{\vec{U}}(x^1, x^2) P_k\Big(\frac{x^3}{h}\Big),$$

where

$$\overset{(k)}{\vec{U}}(x^1, x^2) = \Big(k + \frac{1}{2}\Big)\frac{1}{h}\int\limits_{-h}^{h} \vec{U}(x^1, x^2, x^3) P_k\Big(\frac{x^3}{h}\Big)dx^3.$$

Substituting expressions (24) in to formula (31), taking into account assumptions (30), we have

$$\overset{(k)}{\vec{T}^a} \cong \Lambda(\vec{r}^\gamma D_\gamma \overset{(k)}{\vec{U}})\vec{r}^\alpha + (M+\lambda_5 S)(\vec{r}^\alpha D_\gamma \overset{(k)}{\vec{U}})\vec{r}^\gamma + (M-\lambda_5 S)(\vec{r}^\alpha \vec{r}^\gamma) D_\gamma \overset{(k)}{\vec{U}}$$

$$+\Lambda(\vec{n} D_3 \overset{(k)}{\vec{U}})\vec{r}^a + (M+\lambda_5 S)(\vec{r}^\alpha D_3 \overset{(k)}{\vec{U}})\vec{n} + \overset{(k)}{\vec{N}^\alpha}. \tag{32}$$

$$\overset{(k)}{\vec{T}^3} \cong \Lambda(\vec{r}^\gamma D_\gamma \overset{(k)}{\vec{U}})\vec{n} + (M+\lambda_5 S)(\vec{n} D_\gamma \overset{(k)}{\vec{U}})\vec{r}^\gamma + (M-\lambda_5 S) D_3 \overset{(k)}{\vec{U}}$$

$$+(\Lambda+M+\lambda_5 S)(\vec{n} D_3 \overset{(k)}{\vec{U}})\vec{n} + \overset{(k)}{\vec{N}^3}, \tag{33}$$

where

$$D_j \overset{(k)}{\vec{U}} := \Big(k+\frac{1}{2}\Big)\frac{1}{h}\int_{-h}^{h} \partial_j \vec{U} P_k\Big(\frac{x^3}{h}\Big)dx^3; \quad \overset{(k)}{\vec{N}^j} = \Big(k+\frac{1}{2}\Big)\frac{1}{h}\int_{-h}^{h} \overset{(k)}{\vec{N}^j} P_k\Big(\frac{x^3}{h}\Big)dx^3.$$

The following formula is fair [16]

$$D_j \overset{(k)}{\vec{U}} = \begin{cases} \partial_\alpha \overset{(k)}{\vec{U}} - \partial_\alpha \ln h \overset{(k'')}{\vec{U}}, & j=\alpha, \\ \dfrac{1}{h}\overset{(k')}{\vec{U}}, & j=3, \end{cases}$$

where

$$\overset{(k')}{\vec{U}} := (2k+1)\Big(\overset{(k+1)}{\vec{U}} + \overset{(k+3)}{\vec{U}} + \dots\Big), \quad \overset{(k'')}{\vec{U}} := k\overset{(k)}{\vec{U}} + (2k+1)\Big(\overset{(k+2)}{\vec{U}} + \overset{(k+4)}{\vec{U}} + \dots\Big).$$

We obtain the following expression for nonlinear values $\overset{(k')}{\vec{N}}$

$$\overset{(k)}{\vec{N}^i} \cong \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r_1=0}^{min(m,n)} \alpha_{mnr_1}\Bigg\{\Bigg[\frac{1}{2}\Lambda\Big(D_j \overset{(m)}{\vec{U}} \circ D^j \overset{(n)}{\vec{U}}\Big)\vec{r}^i + M\Big(D^i \overset{(m)}{\vec{U}} \circ D_j \overset{(n)}{\vec{U}}\Big)\vec{r}^j$$

$$-\lambda_5 S\Big(D^i \overset{(m)}{\vec{U}} \circ (E D_j \overset{(n)}{\vec{U}})\Big)\vec{r}^j + \Big(\Lambda\Big(\vec{r}^j D_j \overset{(m)}{\vec{U}}\Big)\Big)\circ D^i \overset{(n)}{\vec{U}} + \Big((M+\lambda_5 S)(\vec{r}^i D_j \overset{(m)}{\vec{U}})\Big)$$

$$\circ D^j \overset{(n)}{\vec{U}}\Big((M-\lambda_5 S)(D^i \overset{(m)}{\vec{U}} \vec{r}^j)\Big)\circ D_j \overset{(n)}{\vec{U}}\Bigg]\delta_k^{m+n-2r_1} + \sum_{l=0}^{\infty}\sum_{r_2=0}^{min(l,k)} \alpha_{lkr_2}$$

$$\Bigg[\frac{1}{2}\Big(\Lambda\Big(D_j \overset{(m)}{\vec{U}} \circ D^j \overset{(n)}{\vec{U}}\Big)\circ D^i \overset{(l)}{\vec{U}} + \Big(M(D^i \overset{(m)}{\vec{U}} \circ D_j \overset{(n)}{\vec{U}})\circ D^j \overset{(l)}{\vec{U}}$$

$$-\lambda_5 \Big(S \Big(D^i \overset{(m)}{\vec{U}} \circ (ED_j \overset{(n)}{\vec{U}}) \Big) \Big) \circ D^j \overset{(l)}{\vec{U}} \Bigg] \frac{(2k+1)\delta^{l+k-2r_2}_{m+n-2r_1}}{2(l+k-2r_2)+1} \Bigg\}, \tag{34}$$

where

$$a_{mnr} = \frac{A_{m-r}A_r A_{n-r}}{A_{m+n-r}} \frac{2(m+n)-4r+1}{2(m+n)-2r+1}, \quad A_m = \frac{(2m-1)!!}{m!};$$

$$D_3 = D^3, \ \ D^\alpha = a^{\alpha\gamma} D_\gamma, \ \ \vec{r}^{\,3} = \vec{r}_3 = \vec{n}.$$

If we consider contravariant components of a vector of $\overset{(k)}{\vec{T}^i}$ in covariant basis $\overset{(k)}{\vec{T}^i} = \overset{(k)}{T}{}^{i\alpha}\vec{r}_\alpha + \overset{(k)}{T}{}^{\alpha 3}\vec{n}$, $\overset{(k)}{T}{}^{ij} = \Big(\overset{(k)}{T}{}'^{ij}, \overset{(k)}{T}{}''^{ij} \Big)^T$ and use formulas of Gauss and Weingarten the system of equilibrium equations may be rerecorded as follows

$$\begin{cases} \nabla_\alpha \overset{(k)}{T}{}^{\alpha\beta} - b^\beta_\alpha \overset{(k)}{T}{}^{\alpha 3} + \partial_\alpha \ln h \underset{=}{\overset{(k)}{T}}{}^{\alpha\beta} - \dfrac{1}{h} \underset{-}{\overset{(k)}{T}}{}^{3\beta} + \overset{(k)}{F}{}^\beta = 0, \\ \nabla_\alpha \overset{(k)}{T}{}^{\alpha 3} + b_{\alpha\beta} \overset{(k)}{T}{}^{\alpha\beta} + \partial_\alpha \ln h \underset{=}{\overset{(k)}{T}}{}^{\alpha 3} - \dfrac{1}{h} \underset{-}{\overset{(k)}{T}}{}^{33} + \overset{(k)}{F}{}^3 = 0, \quad k = 0, 1, \ldots, \end{cases} \tag{35}$$

where ∇_α is a covariant derivative on the midsurface ω; $b_{\alpha\beta}$, b^β_α are respectively, the covariant and mixed components of the tensor of curvature of the midsurface ω;

$$\underset{-}{\overset{(k)}{T}}{}^{3j} = (2k+1) \left(\overset{(k-1)}{T}{}^{3j} + \overset{(k-3)}{T}{}^{3j} + \ldots \right),$$

$$\underset{=}{\overset{(k)}{T}}{}^{\alpha j} = (k+1)\overset{(k)}{T}{}^{\alpha j} + (2k+1) \left(\overset{(k-2)}{T}{}^{\alpha j} + \overset{(k-4)}{T}{}^{\alpha j} + \ldots \right), \quad \overset{(-n)}{T}{}^{ij} = 0,$$

when $n > 0$; $\overset{(k)}{\vec{F}} = \overset{(k)}{F}{}^\alpha \vec{r}_\alpha + \overset{(k)}{F}{}^3 \vec{n}$.

We obtain the following relations from the formulas (32) and (33) for the moments of covariant a components of the first tensor of stress of Piola-Kirchhoff $\overset{(k)}{T}_{ij} = a_{im}a_{jn}\overset{(k)}{T}{}^{mn}$

$$\begin{aligned} \overset{(k)}{T}_{\alpha\beta} &= \Lambda \overset{(k)}{e}{}^m_m a_{\alpha\beta} + 2M \overset{(k)}{e}_{\alpha\beta} - 2\lambda_5 \overset{(k)}{h}_{\alpha\beta} + \overset{(k)}{N}_{\alpha\beta}, \\ \overset{(k)}{T}_{\alpha 3} &= 2M \overset{(k)}{e}_{\alpha 3} - 2\lambda_5 \overset{(k)}{h}_{\alpha 3} + \overset{(k)}{N}_{\alpha 3}, \\ \overset{(k)}{T}_{3\alpha} &= 2M \overset{(k)}{e}_{\alpha 3} + 2\lambda_5 \overset{(k)}{h}_{\alpha 3} + \overset{(k)}{N}_{3\alpha}, \\ \overset{(k)}{T}_{33} &= \Lambda \overset{(k)}{e}{}^m_m a_{\alpha\beta} + 2M \overset{(k)}{e}_{33} + \overset{(k)}{N}_{33}, \end{aligned} \tag{36}$$

where $\overset{(k)}{e}_{ij}$, $\overset{(k)}{h}_{ij}$ are the moments of the linearized components of tensors of deformation and rotation , respectively; $\overset{(k)}{e}{}^{m}_{m} = a^{mj}\overset{(k)}{e}_{mj}$; $\overset{(k)}{N}_{ij} = a_{im}a_{in}\overset{(k)}{\vec{N}}{}^{m}\vec{r}_j$.

The following formulas $(\overset{(k)}{U}_j = \overset{(k)}{\vec{U}}\vec{r}_j,\ \overset{(k)}{U}{}^{j} = \overset{(k)}{\vec{U}}\vec{r}^{\,j} = a^{jm}\overset{(k)}{U}_m)$ are fair for the values $\overset{(k)}{e}_{ij}$, $\overset{(k)}{h}_{ij}$ and $\overset{(k)}{e}{}^{m}_{m}$

$$
\begin{aligned}
&\overset{(k)}{e}_{\alpha\beta} = \frac{1}{2}(\nabla_\alpha \overset{(k)}{U}_\beta + \nabla_\beta \overset{(k)}{U}_\alpha - 2b_{\alpha\beta}\overset{(k)}{U}_3 - \overset{(k'')}{U}_\alpha \partial_\beta \ln h - \overset{(k'')}{U}_\beta \partial_\alpha \ln h),\\
&\overset{(k)}{e}_{\alpha 3} = \frac{1}{2}(\nabla_\alpha \overset{(k)}{U}_3 + b_{\alpha\beta}\overset{(k)}{U}{}^{\beta} + \frac{1}{h}\overset{(k')}{U}_\alpha - \overset{(k'')}{U}_3 \partial_\alpha \ln h),\\
&\overset{(k)}{e}_{\alpha 3} = \frac{1}{2}\overset{(k')}{U}_3,\\
&\overset{(k)}{h}_{\alpha\beta} = \frac{1}{2}S(\nabla_\alpha \overset{(k)}{U}_\beta - \nabla_\beta \overset{(k)}{U}_\alpha - \overset{(k'')}{U}_\beta \partial_\alpha \ln h + \overset{(k'')}{U}_\alpha \partial_\beta \ln h),\\
&\overset{(k)}{h}_{\alpha 3} = \frac{1}{2}S(\nabla_\alpha \overset{(k)}{U}_3 + b_{\alpha\beta}\overset{(k)}{U}{}^{\beta} - \frac{1}{h}\overset{(k')}{U}_\alpha - \overset{(k'')}{U}_3 \partial_\alpha \ln h), \quad \overset{(k)}{h}_{3\alpha} = -\overset{(k)}{h}_{\alpha 3},\\
&\overset{(k)}{e}{}^{m}_{m} = \nabla_\alpha \overset{(k)}{U}{}^{\alpha} - 2H\overset{(k)}{U}_3 - \overset{(k'')}{U}_\alpha \partial_\alpha \ln h + \frac{1}{h}\overset{(k')}{U}_3,
\end{aligned}
\tag{37}
$$

where $\overset{(k')}{U}_j = (2k+1)\sum\limits_{m=0}^{\infty}\overset{(k+2m+1)}{U_j}$, $\overset{(k'')}{U}_j = k\overset{(k)}{U}_j + (2k+1)\sum\limits_{m=1}^{\infty}\overset{(k+2m)}{U_j}$; $H = \frac{1}{2}(k_1+k_2) = \frac{1}{2}b^{\alpha}_{\alpha}$ are the mean curvatures of the surface ω.

We have the following formula for the moments of contravariant components of a nonlinear tensor $\overset{(k)}{N}{}^{ij} = \overset{(k)}{\vec{N}}{}^{i}\vec{r}^{\,j}$

$$
\begin{aligned}
\overset{(k)}{N}{}^{ij} = &\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r_1=0}^{min(m,n)}\alpha_{mnr_1}\bigg\{\bigg[\frac{1}{2}\Lambda\Big(\overset{*}{\nabla}_q\overset{(m)}{U}_p \circ \overset{*}{\nabla}{}^q\overset{(n)}{U}{}^p\Big)a^{\alpha\beta} + M\Big(\overset{*}{\nabla}{}^i\overset{(m)}{U}{}^p \circ \overset{*}{\nabla}{}^j\overset{(n)}{U}_p\Big)\\
&-\lambda_5 S\Big(\overset{*}{\nabla}{}^i\overset{(m)}{U}{}^p \circ \Big(E\overset{*}{\nabla}{}^j\overset{(n)}{U}_p\Big)\Big) + \Big(\Lambda\overset{*}{\nabla}_q\overset{(m)}{U}{}^q\Big) \circ \overset{*}{\nabla}{}^i\overset{(n)}{U}{}^j + ((M+\lambda_5 S)\overset{*}{\nabla}_q\overset{(m)}{U}{}^i) \circ \overset{*}{\nabla}{}^q\overset{(n)}{U}{}^j\\
&+((M-\lambda_5 S)\overset{*}{\nabla}{}^i\overset{(m)}{U}{}^p) \circ \overset{*}{\nabla}_p\overset{(n)}{U}{}^j\bigg]\delta_k^{m+n-2r_1} + \sum_{l=0}^{\infty}\sum_{r_2=0}^{min(l,k)} a_{lkr_2}\bigg[\frac{1}{2}\Big(\Lambda\Big(\overset{*}{\nabla}_q\overset{(m)}{U}_p\\
&\circ\overset{*}{\nabla}{}^q\overset{(n)}{U}{}^p\Big) \circ \overset{*}{\nabla}{}^i\overset{(l)}{U}{}^j + \Big(M\overset{*}{\nabla}{}^i\overset{(m)}{U}_p \circ \overset{*}{\nabla}_q\overset{(n)}{U}{}^p\Big) \circ \overset{*}{\nabla}{}^q\overset{(l)}{U}{}^j - \lambda_5(S(\overset{(*)}{\nabla}{}^i\overset{(m)}{U}_p\\
&\circ(E\overset{*}{\nabla}_q\overset{(n)}{U}{}^p))\overset{*}{\nabla}{}^q\overset{(l)}{U}{}^j\bigg]\frac{(2k+1)\delta^{l+k-2r_2}_{m+n-2r_1}}{2(l+k-2r_2)+1}\bigg\},
\end{aligned}
$$

where the following notation is introduced

$$\overset{*}{\nabla}_i \overset{(k)}{U_j} := \Big(k+\frac{1}{2}\Big)\frac{1}{h}\int\limits_{-h}^{h} \overset{\circ}{\nabla}_i U_j P_k\Big(\frac{x^3}{h}\Big)dx^3.$$

The operator $\overset{\circ}{\nabla}_i$ implies its approximate value when the following relations are fair

$$\overset{*}{\nabla}_\alpha \overset{(k)}{U_\beta} = \nabla_\alpha \overset{(k)}{U_\beta} - b_{\alpha\beta}\overset{(k)}{U_3} - \partial_\alpha \ln h \overset{(k'')}{U_\beta},$$

$$\overset{*}{\nabla}_\alpha \overset{(k)}{U_3} = \partial_\alpha \overset{(k)}{U_3} + b_\alpha^\beta \overset{(k)}{U_\beta} - \partial_\alpha \ln h \overset{(k'')}{U_3},$$

$$\overset{*}{\nabla}_3 \overset{(k)}{U_\alpha} = \frac{1}{h}\overset{(k')}{U_\alpha} + b_\alpha^\beta \overset{(k)}{U_\beta},$$

$$\overset{*}{\nabla}_3 \overset{(k)}{U_3} = \frac{1}{h}\overset{(k')}{U_3}.$$

Substituting formulas (37) into relations (32) and introducing the obtained expressions into the system of the equations (35), we will receive the following two-dimensional system of equations concerning the moments of the displacement vector

$$\begin{cases} (M-\lambda_5 S)\nabla_\alpha(\nabla^\alpha \overset{(k)}{U^\beta}) + (M+\lambda_5 S)\nabla_\alpha(\nabla^\beta \overset{(k)}{U^\alpha}) \\ \qquad +\Lambda\nabla^\beta(\nabla_\alpha \overset{(k)}{U^\alpha}) + \overset{(k)}{M^\beta} + \overset{(k)}{M^\beta_{NL}} + \overset{(k)}{F^\beta} = 0, \\ (M-\lambda_5 S)\nabla_\alpha(\nabla^\alpha \overset{(k)}{U_3}) + \overset{(k)}{M^3} + \overset{(k)}{M^3_{NL}} + \overset{(k)}{F_3} = 0, \quad k=1,\dots, \end{cases} \tag{38}$$

where $\overset{(k)}{M^j} = (\overset{(k)}{M'^j}, \overset{(k)}{M''^j})^T$ are homogeneous linear differential expressions containing the functions $\overset{(k)}{u'_i}$ and $\overset{(k)}{u''_i}$ and the first order derivatives with variables x^1, x^2;

$$\overset{(k)}{M^\beta_{NL}} = \nabla_\alpha \overset{(k)}{N^{\alpha\beta}} - b_\alpha^\beta \overset{(k)}{N^{\alpha 3}} + \partial_\alpha \ln h \underset{=}{\overset{(k)}{N^{\alpha\beta}}} - \frac{1}{h}\underset{-}{\overset{(k)}{N^{3\beta}}},$$

$$\overset{(k)}{M^3_{NL}} = \nabla_\alpha \overset{(k)}{N^{\alpha 3}} + b_{\alpha\beta} \overset{(k)}{N^{\alpha\beta}} + \partial_\alpha \ln h \underset{=}{\overset{(k)}{N^{\alpha 3}}} - \frac{1}{h}\underset{-}{\overset{(k)}{N^{33}}},$$

$$\underset{-}{\overset{(k)}{N^{3j}}} = (2k+1)\left(\overset{(k-1)}{N^{3j}} + \overset{(k-3)}{N^{3j}} + \dots\right),$$

$$\underset{=}{\overset{(k)}{N^{\alpha j}}} = (k+1)\overset{(k)}{N^{\alpha j}} + (2k+1)\left(\overset{(k-2)}{N^{\alpha j}} + \overset{(k-4)}{N^{\alpha j}} + \dots\right).$$

Using the same method of reduction we will obtain boundary conditions from the boundary conditions (26), (27) for the moments of displacements and stresses on the boundary of a middle surface

I. $\overset{(k)}{\vec{U}}(x^1,x^2) = \overset{(k)}{\vec{U}^0}, \quad (x^1,x^2) \in \partial\omega$

II. $\overset{(k)}{\vec{T}}_{(l)}(x^1,x^2) = \overset{(k)}{\vec{T}}{}^0_{(l)}, \quad (x^1,x^2) \in \partial\omega, \quad k = 0,1,\dots,$

where $\vec{l}$ is a tangential normal to the lateral boundary surface of the shell

$$\left(\overset{(k)}{\vec{U}^0}, \overset{(k)}{\vec{T}}{}^0_{(l)}\right) = \left(k+\frac{1}{2}\right)\frac{1}{h}\int\limits_{-h}^{h}(\vec{U}^0, \vec{T}^0_{(l)})P_k\left(\frac{x^3}{h}\right)dx^3.$$

The obtained system of the equations (38) contains an infinite number of equations and an infinite number of unknown functions. To pass to the finite system Vekua makes an assumption which he calls the second main assumption. The second main assumption is that the vector of displacement is taken equal to the polynomial of finite order with respect to the coordinate x^3

$$\vec{U}(x^1,x^2,x^3) = \sum_{k=0}^{N}\overset{(k)}{\vec{U}}(x^1,x^2)P_k\Big(\frac{x^3}{h}\Big),$$

where N is some fixed non-negative integer number.

Thus, we have in all expressions obtained above that

$$\overset{(k)}{\vec{U}} = 0, \quad \text{when} \quad k > N, \quad \text{i.e.} \quad \overset{(k)}{U}_j = \overset{(k)}{U}{}^j = 0, \quad \text{when} \quad k > N.$$

Besides we keep the first $6N+6$ of the equation in the system of the equations (38). Thus we obtain the system of the $6N+6$ equations with partial derivatives for of $6N+6$ unknown functions from two independent variables

$$\begin{cases} (M-\lambda_5 S)\nabla_\alpha(\nabla^\alpha \overset{(k)}{U}{}^\beta) + (M+\lambda_5 S)\nabla_\alpha(\nabla^\beta \overset{(k)}{U}{}^\alpha) + \Lambda\nabla^\beta(\nabla_\alpha \overset{(k)}{U}{}^\alpha) \\ +\overset{(k)}{M}{}^\beta + \overset{(k)}{M}{}^\beta_{NL} + \overset{(k)}{F}{}^\beta = 0, \\ (M-\lambda_5 S)\nabla_\alpha(\nabla^\alpha \overset{(k)}{U}_3) + \overset{(k)}{M}{}^3 + \overset{(k)}{M}{}^3_{NL} + \overset{(k)}{F}_3 = 0, \quad k = 0,1,\dots,N. \end{cases} \tag{39}$$

The system of equations (39) is recorded in any curvilinear system of coordinates on a middle surface of a shell.

4. Conclusion

In the paper we consider a geometrically nonlinear model of binary mixture of two isotropic materials. The main two-dimensional relations for the shallow shells consisting of binary mixture are obtained from the main three-dimensional equations of the considered model. The further interest is to use the method of Vekua-Meunargia for obtaining the equations for non-shallow shells having similar internal structure. And, of course, the great interest is also the consideration of concrete approximations for specific boundary value problems.

Acknowledgement. The designated project has been fulfilled by a financial support of Shota Rustaveli National Science Foundation (Grant SRNSF/FR/358/5-109/14). Any idea in this publication is possessed by the author and may not represent the opinion of Shota Rustaveli National Science Foundation itself.

R E F E R E N C E S

1. Truesdell C. ToupinR. “he classical field theories” in Handbuch der physic. *III/1, Berlin, Springer*, 1960.

2. Truesdell C. Mechanical basis of diffusion. *J. Chem. Phys.*, **37** (1962), 2336-2344.

3. Green A.E., Naghdi P.M. A dynamical theory of two interacting continua. *Int. J. Eng. Sci.*, **3**, 2 (1965), 231-241.

4. Green A.E., Steel T.R. Constitutive equations for interacting continua. *Int. J. Eng. Sci.*, **4**, 4 (1966), 483-500.

5. Steel T.R. Applications of a theory of interacting continua. *Q. J. Mech. Appl. Math.*, **20**, 1 (1967), 57-72.

6. Green A.E., Naghdi P.M. On basic equations for mixture. *Q. J. Mech. Appl. Math.*, **22** (1969), 427-438.

7. Green A.E., Naghdi P.M. On thermodynamics and the nature of the second law for mixtures of interacting continua. *Q. J. Mech. Appl. Math.*, **31**, 3 (1978), 265-293.

8. Aron M. On existence and uniqueness of solutions in the linear theory of mixtures of two elastic solids. *Arch. Of Mech.*, **26**, 4 (1974), 717-728.

9. Natroshvili D.G., Jagmaidze A.Ya., Svanadze M.Zh. Some problems of the linear theory of elastic mixtures (Russian) *TSU press*, 1986.

10. Basheleishvili M. Two-dimensional boundary value problems of statics in the theory of elastic mixtures. *Mem. Differential Equations Math. Phys.*, **6** (1995), 59-105.

11. Basheleishvili M. Application of analogues of general Kolosov-Muskhelishvili representations in the theory of elastic mixtures. *Georgian Math. J.*, **6**, 1 (1999), 1-18.

12. Gordeziani D., Avalishvili G. On the investigation of the boundary value problem for non-linear model in the theory of elastic mixtures. *Bull. Georgian Acad. Sci.*, **164**, 3 (2001), 451-453.

13. Gordeziani D., Avalishvili G. Investigation of non-linear models in the theory of elastic mixtures. *AMI*, **7** , 2 (2002), 41-65.

14. Janjgava R. On I. Vekua’s method in the theory of elastic Mixtures. *Bulletin of TICMI*, **4** (2000), 47-51.

15. Janjgava R. Derivation of two-dimensional equation for shallow shells by means of the method of I.Vekua in the case of linear theory of elastic mixtures. *Journal o Mathematical Sciences. Spr. New York*, **157**, 1 (2009), 70-78.

16. Vekua I.N. Some general methods of construction of various variants of the shell theory (Russian) *Nauka, Moscow*, 1982.

17. Meunargia T.V. On non-linear and non-shallow shells of variable thickness. *Reports of enlarged sessions of the seminar of VIAM*, **10**, 2 (1995), 26-30.

18. Meunargia T.V. On two-dimensional equations of the non-linear theory of shells. *AMI, TSU*, **3** (1998), 84-111.

19. Meunargia T.V. On one method of construction of geometrically and physically non-linear theory of non-shallow shells. *Proceedings of A. Razmadze Math. Institute*, **119** (1999), 131-134.

Received 12.10.2015; accepted 02.12.2015.

Author's address:

R. Janjgava
I. Vekua Institute of Applied Mathematics of
Iv. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0186
Georgia
E-mail: roman.janjgava@gmail.com