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ON A HARMONIC VIBRATION PROBLEM OF THE SINUSOIDAL CUSPED BEAMS IN THE (0,0) APPROXIMATION

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Abstract. In the present work geometrical interpretation of an elastic beam with sinusoidal cusped edge is given. The aim of the present work is to study well-posedness of initial-boundary value problems in case of (0,0) approximation of hierarchical models. The setting of boundary conditions at the beam ends depends on the geometry of sharpenings of beams ends, while the setting of initial conditions is independent of them. The problem of harmonic vibration is studied.

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AMS subject classification (2010): 74K20, 35J70.

1. Introduction

Investigation of cusped elastic prismatic shells actually takes its origin from the fifties of the last century, namely, in 1955 I.Vekua raised the problem of investigation of elastic cusped prismatic shells, whose thickness on the prismatic shell entire boundary or on its part vanishes (see [12-14]). Such bodies, considered as 3D ones, may occupy 3D domains with, in general, non-Lipschitz boundaries. In practice, such cusped prismatic shells, in particular, cusped plates, and cusped beams (i.e.,beams whose cross-sections area vanishes at least at one end of the beam) are often encountered in spatial structures with partly fixed edges, e.g., stadium ceilings, aircraft wings, submarine wings etc., in machine-tool design, as in cutting-machines, planning-machines, in astronautics, turbines, and in many other application fields of engineering. The problem mathematically leads to the question of setting and solving of boundary value problems for even order equations and systems of elliptic type with the order degeneration in the statical case and of initial boundary value problems for even order equations and systems of hyperbolic type with the order degeneration in the dynamical case. One can find the updated survey of investigations in this direction one can find in [8], see also the survey in [5,6] and I. Vekua's comments in [14, p.86]).

In the fifties of XX century I.Vekua introduced a new mathematical model for elastic prismatic shells which was based on expansions of the three-dimensional displacement vector fields and the strain and stress tensors in linear elasticity into orthogonal Fourier-Legendre series with respect to the variable plate thickness. By taking only the first N + 1 terms of the expansions, he introduced the so called N-th approximation. Each of these approximations for N = 0; 1; ... can be considered as an independent mathematical model of plates. In particular, the approximation for N = 1 corresponds to the classical Kirchhoff plate model. In the sixties of the XX century I. Vekua developed the analogous mathematical model for thin shallow shells. All his results concerning plates and shells are collected in his monograph [14]. The works of I. Babuška, D. Gordeziani, V. Guliaev, I. Khoma, A. Khvoles, T. Meunargia,C. Schwab, T. Vashakmadze, V. Zhgenti, G. Jaiani, G. Tsikarishvili, N. Khomasuridze, W. Wendland, D. Natroshvili, S. Kharibegashvili, N. Chinchaladze, R. Gilbert, and others are devoted to further analysis of I.Vekua's models (rigorous estimation of the modeling error, numerical solutions, etc.) and their generalizations (see, e.g., [1-10]. The problem mathematically leads to the question of setting and solving of boundary value problems for even order equations and systems of elliptic type with the order degeneration in the statical case and of initial boundary value problems for even order equations and systems of hyperbolic type with the order degeneration in the dynamical case.

The analogues system in the case of (N_3, N_2) approximation of hierarchical models for cusped beams, in general, beams with variable rectangular cross-sections are derived by G. Jaiani ([7]).

In the present work geometrical interpretation of an elastic beam with sinusoidal cusped edge is given. The geometry of cusps is illustrated using MatLab for different type of cusps. The aim of the present work is to study well-posedness of initial-boundary value problems in case of (0,0) approximation of hierarchical models. The setting of boundary conditions at the beam ends depends on the geometry of sharpenings of beams ends, while the setting of initial conditions is independent of them. The problem of harmonic vibration is studied.

2. On a harmonic vibration of the sinusoidal cusped beams

Let a domain $\bar{V}$ of R^3, occupied by an elastic beam be

$$V := \{(x_1, x_2, x_3) :\ 0 < x_1 < L,\ \overset{(-)}{h}_i(x_1) \le x_i \le \overset{(+)}{h}_i(x_1),\ i = 2,3,\ L = const\},$$

$$2h_i(x_1) := \overset{(+)}{h}_i - \overset{(-)}{h}_i,\quad h_i \in C([0,l]) \cap C^1(]0,l[),\quad i = 2,3$$

and let $2h_3$ and $2h_2$ be correspondingly the thickness and the width of the beam and their maxima be essentially less than the length L of the bar. If at least one of the conditions $2h_i(0) = 0$ and $2h_i(L) = 0,\ \ i = 2,3$, is fulfilled, a beam is called the cusped beam. The last class of beams consists of beams with rectangular cross-sections which may degenerate in segments or points at the beam ends.

In what follows X_{ij} and e_{ij} are the stress and strain tensors, respectively, u_i are the displacements, Φ_i are the volume force components, ρ is the density, λ and μ are Lamé constants, δ_{ij} is the Kroneker delta. Moreover, repeated indices imply summation, bar under one of the repeated indices means that we do not sum.

Hierarchical models of elastic prismatic shells are the mathematical models. Their constructing is based on the multiplication of the basic equations of linear elasticity:

Motion Equations

$$X_{ij,j} + \Phi_i = \rho \ddot{u}_i(x_1, x_2, x_3, t), \quad (x_1, x_2, x_3) \subset R^3, \quad t > t_0, \quad i = 1, 2, 3; \quad (1)$$

Hooke's generalized law (isotropic case):

$$X_{ij} = \lambda \theta \delta_{ij} + 2\mu e_{ij}, \quad i, j = 1, 2, 3; \quad (2)$$

Kinematic Relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i, j = 1, 2, 3, \quad (3)$$

$$\theta = e_{ii}$$

by Legendre polynomials $P_r(ax_3 - b)$ and then integration with respect to x_3 within the limits $\overset{(-)}{h}(x_1, x_2)$ and $\overset{(+)}{h}(x_1, x_2)$.

Governing equation in (0,0) approximation has the form (see [12]):

$$(h_2 h_3 v_{j,1}(x_1, t))_{,1} + Y_{j00} = \Lambda_j^{-1} \rho\, h_2 h_3 \frac{\partial^2 v_j(x_1, t)}{\partial t^2}, \qquad j = 1, 2, 3, \quad (4)$$

where

$$v_j(x_1, t) = \frac{u_{j00}(x_1, t)}{h_2(x_1) h_3(x_1)}, \quad j = 1, 2, 3,$$

$$Y_{100} = \frac{\overset{0,0}{X_1^0}}{\lambda + 2\mu}, \quad Y_{i00} = \frac{\overset{0,0}{X_i^0}}{\mu}, \quad i = 2, 3, \quad (5)$$

u_{j00} are unknown so called weighted double moments of displacements in the (0,0) approximation, $\overset{0\,0}{X_j^0}$ are expressed by external forces acting on the face surfaces $x_i = \overset{(\pm)}{h}{}_i(x_1)$, $i = 2, 3$, and double moments of volume forces, e_{ij00} are deformation tensor's components (0,0) approximation,

$$\Lambda_j := \begin{cases} \lambda + 2\mu, & j = 1, \\ \mu, & j = 2, 3. \end{cases}$$

Let π be the length (which can happen without limitation) of the beam and let the thickness $2h_3$ and the midth $2h_2$ of the beam be given correspondingly by the expressions

$$2h_3 = h_3^0 \sin^\kappa x_1 \;\text{ and }\; 2h_2 = h_2^0, \quad (6)$$

$$\kappa = const \geq 0, \quad h_3^0, h_2^0 = const > 0, \quad x_1 \in (0, \pi).$$

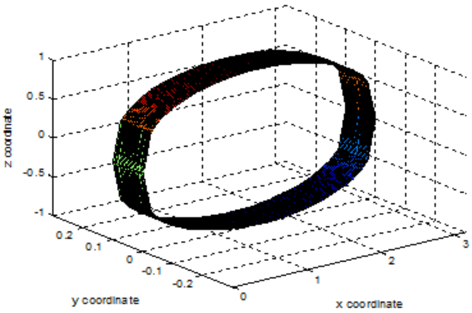

Figure 1: $0 < \kappa < 1$:

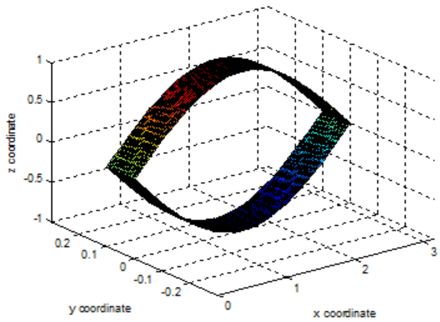

Figure 2: $\kappa = 1$

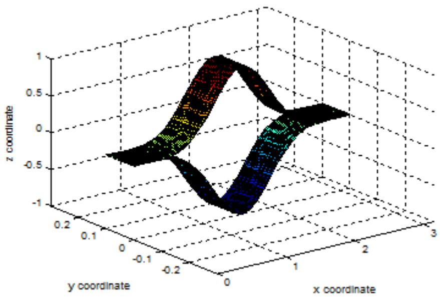

Figure 3: $\kappa > 1$

(4) can be written as follows

$$(h_2^0 h_3^0 \sin^\kappa x_1 \; v_{j,1}(x_1,t))_{,1} + Y_{j00}(x_1,t) = \Lambda_j^{-1} \rho \; h_2^0 h_3^0 \sin^\kappa x_1 \, \frac{\partial^2 v_j(x_1,t)}{\partial t^2}, \quad j = 1,2,3, \tag{7}$$

when $x_1 = 0$ and $x_1 = \pi$, cross-sectional area of the stem becomes zero. These rods are called cusped beams. In our case:

$$2h_2 \cdot 2h_3 = \begin{cases} h_2^0 h_3^0 \sin^\kappa x_1 = 0, & \text{when } x_1 = 0; \\ h_2^0 h_3^0 \sin^\kappa x_1 = 0, & \text{when } x_1 = \pi. \end{cases}$$

In Figures 1-3 geometry of the cusped beams when $0 < \kappa < 1$, $\kappa = 1$, and $\kappa > 1$, respectively, are shown. In the case $\kappa = 0$ both the thickness and width will be constant and we have to do with non-cusped beams. In case $0 < \kappa < 1$, the angle between the face surfaces will be equal to π.

Problem 1. Let $0 < k < 1$. Find the solution v_j of equation (7) in the following form

$$v_j(x_1,t) = \overset{0}{v}_j(x_1) e^{-iwt}, \quad Y_{j00}(x_1,t) = \overset{0}{Y}_{j00}(x_1) \, e^{-iwt},$$

where w=const is oscillation frequency, $\overset{0}{Y}_{j00}(x_1) \in C([0,\pi])$ are given functions, $\overset{0}{v}_j(x_1) \in C^2(]0,\pi[) \bigcap C([0,\pi])$ satisfy the following boundary conditions:

$$\overset{0}{v}_j(0) = 0 \;\; and \;\; \overset{0}{v}_j(\pi) = 0. \qquad (8)$$

Let us also consider the following problem:

Problem 2. Let $0 < k < 1$. Find the solution $v(x_1) \in C^2(]0,\pi[) \bigcap C([0,\pi])$ of the equation

$$(\overset{0}{h}_2 \overset{0}{h}_3 \sin^\kappa x_1 \overset{0}{v}_{,1}(x_1))_{,1} = -\lambda \, \overset{0}{h}_2 \overset{0}{h}_3 \sin^\kappa x_1 \cdot \overset{0}{v}(x_1) \qquad (9)$$

under the following boundary conditions:

$$\overset{0}{v}(0) = 0 \;\; and \;\; \overset{0}{v}(\pi) = 0. \qquad (10)$$

After twice integration of (9) and using boundary conditions (10) we get integral equations as follows

$$\overset{0}{v}(x_1) - \lambda \int\limits_0^\pi K(x_1,\xi) h(\xi) \overset{0}{v}(\xi) d\xi = 0, \qquad (11)$$

where

$$K(x_1,\xi) = \begin{cases} \dfrac{\int\limits_0^\xi h^{-1}(\zeta)d\zeta \int\limits_{x_1}^\pi h^{-1}(\zeta)d\zeta}{\int\limits_0^\pi h^{-1}(\zeta)d\zeta}, & 0 \le \xi \le x_1, \\[2ex] \dfrac{\int\limits_0^{x_1} h^{-1}(\zeta)d\xi \int\limits_\xi^\pi h^{-1}(\zeta)d\zeta}{\int\limits_0^\pi h^{-1}(\zeta)d\zeta}, & x_1 \le \xi \le \pi, \end{cases}$$

$$h(x_1) := \overset{0}{h}_2 \overset{0}{h}_3 \sin^\kappa x_1.$$

Proposition 1. *$K(x_1,\xi)$ is symmetric with respect to x_1 and ξ.*

Proof. For z_1 and z_2, such that $0 < z_1, \;\; z_2 < \pi$, we get:

$$K(z_1,z_2) = \begin{cases} \dfrac{\int\limits_0^{z_2} h^{-1}(\zeta)d\zeta \int\limits_{z_1}^\pi h^{-1}(\zeta)d\zeta}{\int\limits_0^\pi h^{-1}(\zeta)d\zeta}, & 0 \le z_2 \le z_1, \\[2ex] \dfrac{\int\limits_0^{z_1} h^{-1}(\zeta)d\zeta \int\limits_{z_2}^\pi h^{-1}(\zeta)d\zeta}{\int\limits_0^\pi h^{-1}(\zeta)d\zeta}, & z_1 \le z_2 \le \pi, \end{cases}$$

$$K(z_2, z_1) = \begin{cases} \dfrac{\int\limits_0^{z_1} h^{-1}(\zeta)d\zeta \int\limits_{z_2}^{\pi} h^{-1}(\zeta)d\zeta}{\int\limits_0^{\pi} h^{-1}(\zeta)d\zeta}, \ z_1 \leq z_2 \leq \pi, \\ \\ \dfrac{\int\limits_0^{z_2} h^{-1}(\zeta)d\zeta \int\limits_{z_1}^{\pi} h^{-1}(\zeta)d\zeta}{\int\limits_0^{\pi} h^{-1}(\zeta)d\zeta}, \ 0 \leq z_2 \leq z_1, \end{cases}$$

i.e,

$$K(z_1, z_2) = K(z_2, z_1), \quad for \ \ any \quad z_1, z_2 \in [0, \pi].$$

■

(9) can be rewritten as follows:

$$u(x_1) - \lambda \int\limits_0^{\pi} R(x_1, \xi)u(\xi)d\xi = 0, \tag{12}$$

where

$$\begin{aligned} u(x_1) &:= \overset{0}{v}(x_1)\sqrt{h(x_1)}, \quad j = 1, 2, 3, \\ R(x_1, \xi) &:= \sqrt{h(x_1)}K(x_1, \xi)\sqrt{h(\xi)}. \end{aligned} \tag{13}$$

(12) is an integral equation with a symmetric kernel.

Let us denote eigenfunctions and eigenvalues of equation (12) respectively, by $u^n(x_1)$ and λ^n.

Proposition 2. *The number of eigenvalues of* (12) *is not finite .*

Proof. Let it be finite, and $n = \overline{1, m}$. Then we can express $R(x_1, \xi)$as follows (see [11])

$$R(x_1, \xi) = \sum_{n=1}^{m} \frac{u^n(x_1)\, u^n(\xi)}{\lambda^n},$$

where $u^n \in C^2(]0, \pi[)$, i.e.,

$$R(x_1, \mu) \in C^2(]0, \pi[\times]0, \pi[).$$

On the other hand,

$$K'_{x_1}(x_1, \xi)\big|_{\xi \to x_1 -} - K'_{x_1}(x_1, \xi)\big|_{\xi \to x_1 +}$$

$$= \frac{h^{-1}(x_1) \int\limits_0^{\xi} h^{-1}(\xi)d\xi}{\int\limits_0^{\pi} h^{-1}(\xi)d\xi} + \frac{h^{-1}(x_1) \int\limits_{\xi}^{\pi} h^{-1}(\xi)d\xi}{\int\limits_0^{\pi} h^{-1}(\xi)d\xi} = h^{-1}(x_1) \neq 0.$$

but, we have got contradiction, thus the number of λ^n is not finite. ■

Proposition 3. *The number of eigenvalues of* (12) *is positive.*

Proof. Obviously, if we denote by orthonormalized eigenfunctions of (12), then

$$\overset{0}{v}{}^n(x_1) = \frac{u^n(x_1)}{\sqrt{h(x_1)}},$$

are eigenfunctions of

$$\overset{0}{v}(x_1) - \lambda \int\limits_0^\pi K(x_1, \xi) h(\xi) \overset{0}{v}(\xi) d\xi = 0. \tag{14}$$

Let us multiply both sides of the following equation

$$\left(h(x_1) \left(\overset{0}{v}{}^n(x_1)\right)_{,1}\right)_{,1} = -\lambda^n h(x_1) \overset{0}{v}{}^n(x_1),$$

by $\overset{0}{v}{}^n(x_1)$ and integrate it from 0 to π. Taking into account of (13) we obtain

$$\int\limits_0^\pi \overset{0}{v}{}^n(x_1)(h(x_1)(\overset{0}{v}{}^n(x_1))_{,1})_{,1} dx_1 = -\int\limits_0^\pi \lambda^n h(x_1) \overset{0}{v}{}^n(x_1) \overset{0}{v}{}^n(x_1) dx_1. \tag{15}$$

Further,

$$\lambda^n \int\limits_0^\pi h(x_1)(\overset{0}{v}{}^n(x_1))^2 dx_1 = \int\limits_0^\pi h(x_1)((\overset{0}{v}{}^n(x_1))_{,1})^2 dx_1 \geq 0.$$

Hence $\lambda^n \geq 0$. $\lambda^n > 0$ since we consider the non trivial case $\overset{0}{v}{}^n(x_1) \neq 0$. ■

Let us consider the system of nonhomogeneous equations

$$(h(x_1) \overset{0}{v}_{j,1}(x_1))_{,1} + \overset{0}{Y}_{j00} = -\Lambda_j^{-1} \omega^2 \rho \overset{0}{v}_j(x_1) h(x_1), \quad j = 1, 2, 3. \tag{16}$$

In this case our problem can be reduced to the following integral equation:

$$\overset{0}{v}_j - \Lambda_j^{-1} \omega^2 \rho \int\limits_0^\pi K(x_1, \xi) h(\xi) \overset{0}{v}_j(\xi) d\xi = \int\limits_0^\pi K(x_1, \xi) \overset{0}{Y}_{j00}(\xi) d\xi$$

$$=: \Phi_j(x_1), \quad j = 1, 2, 3,$$

which can be rewritten as follows

$$\overset{0}{u}_j - \Lambda_j^{-1} \omega^2 \rho \int\limits_0^\pi R(x_1, \xi) \overset{0}{u}_j(\xi) d\xi = \Phi_j(x_1) \sqrt{h(x_1)}, \quad j = 1, 2, 3,$$

where $\overset{0}{u}_j := \overset{0}{u}_j(x_1)\sqrt{h(x_1)}, \quad j = 1,2,3$. If $\Lambda_j^{-1}\rho\,\omega^2 \neq \lambda^n$, the unique solution of the last system can be written as follows (such ω exist, e.g., if we denote by $\Lambda_0 := \max\{\Lambda_j\}$ in virtue of Statement 3 all $\lambda^n > 0$, which means that there exist $\min\{\lambda^n\} =: \lambda^0$ and ω can be chosen as $\omega^2 < \frac{\lambda^0}{\rho\Lambda_0}$ (see, e.g [11])

$$\overset{0}{u}_j(x_1) = \Phi_j(x_1)\sqrt{h(x_1)}$$

$$+\Lambda_j^{-1}\rho\omega^2\sum_{n=1}^{\infty}\frac{1}{\lambda^n - \Lambda_j^{-1}\rho\omega^2}u^n\,(x_1)\int_0^{\pi}\Phi_j(\xi)\sqrt{h(\xi)}u^n(\xi)d\xi, \quad j = 1,2,3.$$

Solution of Problem 1 has the form (see, e.g [4]):

$$\overset{0}{v}_j(x_1) = \Phi_j(x_1)$$

$$+\Lambda_j^{-1}\rho\omega^2\sum_{n=1}^{\infty}\frac{1}{\lambda^n - \Lambda_j^{-1}\rho\omega^2}v^n(x_1)\int_0^{\pi}\Phi_j(\xi)\sqrt{h(\xi)}u^n(\xi)d\xi, \; j = 1,2,3. \tag{17}$$

Proposition 4. *The series on the right hand side of* (17) *is absolutely and uniformly convergent on* $]0,\pi[$.

Proof. Let us denote by

$$S_j^n := \frac{1}{\lambda^n - \Lambda_j^{-1}\rho\omega^2}v^n\,(x_1)\int_0^{\pi}\Phi_j(\xi)\sqrt{h(\xi)}u^n(\xi)d\xi, \quad j = 1,2,3,$$

which can be rewritten as follows

$$S_j^n := \frac{\lambda^n}{\lambda^n - \Lambda_j^{-1}\rho\omega^2}\frac{v^n(x_1)\int_0^{\pi}\Phi_j(\xi)\sqrt{h(\xi)}u^n(\xi)d\xi}{\lambda^n}, \quad j = 1,2,3,$$

According to Proposition 2, the number of eigenvalues is not finite, that means $\lambda^n \to \infty$, when $n \to \infty$, and further

$$\frac{\lambda^n}{\lambda^n - \Lambda_j^{-1}\rho\omega^2} = \frac{1}{1 - \frac{\Lambda_j^{-1}\rho\omega^2}{\lambda^n}} \to 1, \quad j = 1,2,3. \tag{18}$$

In view of

$$\Phi_j := \int_0^{\pi}K(x_1,\xi)\overset{0}{Y}_{j00}(\xi)d\xi$$

and

$$K(x_1,\xi) \in C([0,\pi]\times[0,\pi]), \;\; \overset{0}{Y}_{j00}(x_1) \in C([0,\pi]),$$

we obtain, that the following series

$$\sum_{n=1}^{\infty} \frac{u^n(x_1) \int\limits_0^\pi \Phi_j(\xi)\sqrt{h(\xi)}u^n(\xi)d\xi}{\sqrt{h(x_1)}\lambda^n} = \sum_{n=1}^{\infty} \frac{(u^n, \Phi_j\sqrt{h})}{\sqrt{h(x_1)}\lambda^n} u^n =$$

$$= \frac{1}{\sqrt{h(x_1)}} \int\limits_0^\pi R(x_1,\xi)\sqrt{h(\xi)}\Phi_j(\xi)d\xi = \int\limits_0^\pi K(x_1,\xi)h(\xi)\Phi_j(\xi)d\xi, \quad j = 1,2,3, \tag{19}$$

is absolutely and uniformly convergent (see, e.g., [11]) . ■

Remark. In view of $K(x_1,\xi) \in C([0,\pi]\times[0,\pi]) \bigcap C^2(]0,\pi[\times]0,\pi[)$ from (19) we obtain $\overset{0}{v}_j(x_1) \in C([0,\pi]) \bigcap C^2(]0,\pi[)$. So, solution of the Problem 1 can be written as an absolutely and uniformly convergence series (17).

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