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A POWER METHOD FOR APPROXIMATION OF SINGULARITIES

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Abstract. In this paper, we modify the method suggested in an earlier paper by the author and overcome its main deficiency. The method enables the approximation of the locations of jump discontinuities of a function, one by one, by means of ratios of so called higher order Fourier-Jacobi coefficients of the function.

It is shown that the location of singularity of a piecewise constant function with one discontinuity is recovered exactly and the locations of singularities of a piecewise constant function with multiple discontinuities are recovered with exponential accuracy. Unlike the previous one, the modified method is robust, since its success is independent of whether or not a location of the discontinuity coincides with a root of Jacobi polynomial. We also give more detailed estimate for the error term.

In addition, the stability of the method is discussed and some numerical examples are presented.

Keywords and phrases: Approximating the jump discontinuities, Fourier-Jacobi coefficients

AMS subject classification (2000): 33C45.

1. Introduction

Truncated Fourier series of functions with jump discontinuous are known to exhibit the Gibbs phenomenon, which makes these partial sums a poor approximation tool. However, if the locations of the singularities and the associated jumps of the function are known, then a number of spectral methods for the reconstruction of the function are already available. Thus, it is essential to accurately recover the locations of singularities and magnitudes of jumps utilizing only Fourier coefficients of a function.

A number of authors (see [2] - [8] and the indicated references) studied the problem of approximating the singularity locations and the associated jumps of a piecewise smooth function given a finite number of its Fourier-Jacobi coefficients. The authors have applied a variety of approaches.

In the present paper, we modify the method suggested by us in [5] and overcome its main deficiency.

It was proved in Theorem 1 [5, p. 140] that the location of singularity of a piecewise constant function with one discontinuity is recovered exactly and the locations of singularities of a piecewise constant function with multiple discontinuities are recovered with exponential accuracy by means of so called higher order Fourier-Jacobi coefficients of the given piecewise constant function. Namely, the value of x_1 for a piecewise constant function with singularity locations at $|x_1| > |x_2| \geq |x_3| \geq \cdots \geq |x_M|$ is approximated to within $O(1)(x_2/x_1)^n$.

However, the method's success was heavily dependent on whether or not a location of the discontinuity coincided with a root of Jacobi polynomial. In the modified method we overcome this problem by re-defining the higher order Fourier-Jacobi coefficients. We also give more detailed estimate for the error term, and the condition $|x_1| > |x_2|$ may be also removed.

In addition, the stability of the method is discussed and some numerical examples are presented.

2. Preliminaries

Throughout this paper we use the following general notations: $\mathbb{N}$, $\mathbb{Z}_+$, and $\mathbb{R}$ are the sets of positive integers, nonnegative integers, and real numbers, respectively.

By $[f](x) \equiv f(x+) - f(x-)$ we denote the jump of a piecewise smooth function f at the point x, where $f(x+)$ and $f(x-)$ denote the right-hand and left-hand side limits of the function f at a point x.

For quantities a_n and b_n, possibly depending on some other variables as well, we write $a_n = o(b_n)$ or $a_n = O(b_n)$ if $\lim_{n\to\infty} a_n/b_n = 0$ or $\sup_{n\in\mathbb{N}} |a_n/b_n| < \infty$.

We say that $\rho^{(\alpha,\beta)}$ is a Jacobi weight if $\rho^{(\alpha,\beta)}(x) \equiv (1-x)^\alpha(1+x)^\beta$, $\alpha > -1$ and $\beta > -1$. If $\rho^{(\alpha,\beta)}$ is a Jacobi weight, then by $\sigma(\rho^{(\alpha,\beta)}) \equiv (P_n^{(\alpha,\beta)}(x))_{n=0}^{\infty}$ we denote the corresponding system of orthogonal polynomials $P_n^{(\alpha,\beta)}(x) = \gamma_n(\alpha,\beta)x^n + \textit{lower degree terms}$, $\gamma_n(\alpha,\beta) > 0$, normalized by the condition $P_n^{(\alpha,\beta)}(1) = \begin{pmatrix} n+\alpha \\ n \end{pmatrix}$, $n \in \mathbb{Z}_+$; i.e.,

$$\int_{-1}^{1} P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\beta)}(x) \rho^{(\alpha,\beta)}(x)dx = 0, \qquad n \neq m.$$

The system $\sigma(\rho^{(\alpha,\beta)})$ is defined uniquely and is called the Jacobi system of orthogonal polynomials. Some important special cases of the Jacobi system are the Chebyshev ($\alpha = \beta = -1/2$), Legendre ($\alpha = \beta = 0$), and Gegenbauer ($\alpha = \beta$) systems.

The following is the recurrence formula for the Jacobi polynomials [9, p. 71])

$$A_n^{(\alpha,\beta)} P_{n+1}^{(\alpha,\beta)}(x) + (x + B_n^{(\alpha,\beta)}) P_n^{(\alpha,\beta)}(x) + C_n^{(\alpha,\beta)} P_{n-1}^{(\alpha,\beta)}(x) = 0, \tag{1}$$

where

$$A_n^{(\alpha,\beta)} = -\frac{2(n+1)(n+\alpha+\beta+1)}{(2n+\alpha+\beta+1)(2n+\alpha+\beta+2)}, \tag{2}$$

$$B_n^{(\alpha,\beta)} = \frac{\alpha^2-\beta^2}{(2n+\alpha+\beta+2)(2n+\alpha+\beta)}, \tag{3}$$

$$C_n^{(\alpha,\beta)} = -\frac{2(n+\alpha)(n+\beta)}{(2n+\alpha+\beta+1)(2n+\alpha+\beta)} \tag{4}$$

for $n \geq 2$ and $P_0^{(\alpha,\beta)}(x) = 1$ and $P_1^{(\alpha,\beta)}(x) = (\alpha+\beta+2)x/2 + (\alpha-\beta)/2$.

If $x_{k,n}^{(\alpha,\beta)}$, $k = 1, 2, \cdots, n$, are the zeros of the polynomial $P_n^{(\alpha,\beta)}(x)$, then [9, p. 46]

$$-1 < x_{k,n+1}^{(\alpha,\beta)} < x_{k,n}^{(\alpha,\beta)} < x_{k+1,n+1}^{(\alpha,\beta)} < 1. \tag{5}$$

The estimate

$$|P_{n-1}^{(\alpha,\beta)}(x)| = O(1)n^{-1/2}((1-x)^{1/2} + (\frac{1}{n}))^{-\alpha-1/2}((1+x)^{1/2} + (\frac{1}{n}))^{-\beta-1/2} \tag{6}$$

holds for $x \in [-1,1]$ and $n \in \mathbb{N}$ (cf. [1, p. 226]), and as it was shown [6, Lemma 2.1]

$$((P_n^{(\alpha,\beta)}(x))^2 + (P_{n+1}^{(\alpha,\beta)}(x))^2)^{-\frac{1}{2}} = \pi^{\frac{1}{2}} 2^{-\frac{\alpha+\beta}{2}} n^{\frac{1}{2}} (\rho^{(-\alpha+1/2,-\beta+1/2)}(x) + O(\frac{1}{n}))^{-\frac{1}{2}} \tag{7}$$

for $x \in [c,d] \subset (-1,1)$

If $f\rho^{(\alpha,\beta)}$ is integrable on $[-1,1]$, then by

$$a_n^{(\alpha,\beta)}(f) \equiv \int_{-1}^{1} f(t) P_n^{(\alpha,\beta)}(t) \rho^{(\alpha,\beta)}(t) dt$$

we denote the nth Fourier-Jacobi coefficient of the function f.

To avoid unnecessary complication of notations, we sometimes omit dependence on parameters $\alpha > -1$ and $\beta > -1$, as they are arbitrary, but fixed.

For a given function f, the polynomial of k variables $a_n^{(k)}(t_1, t_2, \ldots, t_k) \equiv a_n^{(k)}(f, t_1, t_2, \ldots, t_k)$, $t_i \in \mathbb{R}$, $i = 1, 2, \ldots, k$, $n \geq k$, is defined as

$$a_n^{(0)} \equiv a_n^{(0)}(f) \equiv 2(n+1) a_{n+1}^{(\alpha,\beta)}(f)$$

for $n \in \mathbb{Z}_+$ and

$$\begin{aligned} a_n^{(k)}(t_1, t_2, \ldots, t_k) \equiv \; & A_n^{(\alpha+1,\beta+1)} a_{n+1}^{(k-1)}(t_1, t_2, \ldots, t_{k-1}) \\ & + (t_k + B_n^{(\alpha+1,\beta+1)}) a_n^{(k-1)}(t_1, t_2, \ldots, t_{k-1}) \\ & + C_n^{(\alpha+1,\beta+1)} a_{n-1}^{(k-1)}(t_1, t_2, \ldots, t_{k-1}) \end{aligned} \tag{8}$$

for $k \in \mathbb{N}$, where

$$a_n^{(1)}(t_1) \equiv A_n^{(\alpha+1,\beta+1)} a_{n+1}^{(0)} + (t_1 + B_n^{(\alpha+1,\beta+1)}) a_n^{(0)} + C_n^{(\alpha+1,\beta+1)} a_{n-1}^{(0)}.$$

Particular values of the polynomial $a_k^{(n)}(t_1, t_2, \ldots, t_k)$ will be called higher order Fourier-Jacobi coefficients of the function f and will be denoted by

$$a_n^{(k)} \equiv a_n^{(k)}(f) \equiv a_n^{(k)}(f, 0, 0, \cdots, 0) \tag{9}$$

for $k \in \mathbb{N}$.

We also set

$$b_n^{(k)}(t_1,t_2,\ldots,t_k) \equiv a_n^{(k)}(t_1,t_2,\ldots,t_k) + \mathbf{i}a_{n+1}^{(k)}(t_1,t_2,\ldots,t_k) \qquad (10)$$

and

$$b_n^{(k)} \equiv a_n^{(k)} + \mathbf{i}a_{n+1}^{(k)}, \qquad (11)$$

where $\mathbf{i} = (0,1)$.

It is known [5, Lemma 2.2, p. 736] that for a given piecewise constant function f, defined on $[-1,1]$, and with jump discontinuities at $x_1, x_2, \ldots, x_M$,

$$a_n^{(k)}(f,t_1,t_2,\ldots,t_k) = \sum_{m=1}^{M} D_n(x_m) \prod_{s=1}^{k}(t_s - x_m) \qquad (12)$$

for $n \geq k$, where

$$D_n(x) \equiv [f](x)\rho^{(\alpha+1,\beta+1)}(x)P_n^{(\alpha+1,\beta+1)}(x). \qquad (13)$$

Correspondingly, due to (10) and (11),

$$b_n^{(k)}(f,t_1,t_2,\ldots,t_k) = \sum_{m=1}^{M} \bar{D}_n(x_m) \prod_{s=1}^{k}(t_s - x_m) \qquad (14)$$

and

$$b_n^{(k)} = (-1)^k \sum_{m=1}^{M} \bar{D}_n(x_m)(x_m)^k, \qquad (15)$$

where

$$\bar{D}_n(x) \equiv D_n(x) + \mathbf{i}D_{n+1}(x) \equiv [f](x)\bar{\rho}(x)\bar{P}_n(x). \qquad (16)$$

3. Main result

Theorem 1. *Let f be a piecewise constant function defined on $[-1,1]$ with the discontinuities at the points*

$$|x_1| > |x_2| \geq |x_3| \geq \cdots \geq |x_M|. \qquad (17)$$

Then

$$\begin{aligned} \tilde{x}_1 &\equiv -\frac{b_n^{(n)}(f)}{b_n^{(n-1)}(f)} \\ &= x_1 + \frac{[f](x_2)\bar{\rho}(x_2)\bar{P}_n(x_2)(x_2 - x_1)}{[f](x_1)\bar{\rho}(x_1)\bar{P}_n(x_1)}\Big(\frac{x_2}{x_1}\Big)^{n-1} + o\Big(\frac{x_2}{x_1}\Big)^{n}. \qquad (18) \end{aligned}$$

Proof. We follow the proof of Theorem 1 [6, p. 140]. First let us mention that due to (5), $\bar{P}_n(x) = P_n^{(\alpha+1,\beta+1)}(x) + \mathbf{i}P_{n+1}^{(\alpha+1,\beta+1)}(x) \neq 0$ for $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

Furthermore, since $|\bar{P}_n(x)| = \sqrt{(P_n^{(\alpha,\beta)}(x))^2 + (P_{n+1}^{(\alpha,\beta)}(x))^2}$, due to (6) and (7), we have explicit estimate for the term $|\frac{\bar{P}_n(x_2)}{\bar{P}_n(x_1)}|$.

Next, by (13), (15), (16), and (17) we have:

$$\begin{aligned}
\tilde{x}_1 = -\frac{b_n^{(n)}}{b_n^{(n-1)}} &= \frac{\sum_{m=1}^M \bar{D}_n(x_m)(x_m)^n}{\sum_{m=1}^M \bar{D}_n(x_m)(x_m)^{n-1}} \\
&= x_1 \frac{\bar{D}_n(x_1) + \sum_{m=2}^M \bar{D}_n(x_m)(x_m/x_1)^n}{\bar{D}_n(x_1) + \sum_{m=2}^M \bar{D}_n(x_m)(x_m/x_1)^{n-1}} \\
&= x_1(1 + \frac{\sum_{m=2}^M \bar{D}_n(x_m)(x_m/x_1)^{n-1}(x_m/x_1 - 1)}{\bar{D}_n(x_1) + \sum_{m=2}^M \bar{D}_n(x_m)(x_m/x_1)^{n-1}}) \\
&= x_1 + \frac{\bar{D}_n(x_2)(x_2 - x_1)}{\bar{D}_n(x_1)}(\frac{x_2}{x_1})^{n-1} + o(\frac{x_2}{x_1})^{2n-2} + o(\frac{x_3}{x_1})^{n-1} \\
&= x_1 + \frac{[f](x_2)\bar{\rho}(x_2)\bar{P}_n(x_2)(x_2 - x_1)}{[f](x_1)\bar{\rho}(x_1)\bar{P}_n(x_1)}(\frac{x_2}{x_1})^{n-1} + o(\frac{x_2}{x_1})^n. \quad (19)
\end{aligned}$$

Trivially, $\tilde{x}_1 = x_1$ for a piecewise constant function with one discontinuity. □

4. Description of the algorithm and numerical examples

Now we describe the main idea of the algorithm which we propose to locate the discontinuities.

First of all, introducing modified higher order Fourier-Jacobi coefficients, $b_n^{(k)}$, we have made the method robust: its success is independent of whether or not a location of the discontinuity coincides with a root of Jacobi polynomial. We also have obtained a better and more accurate estimate for the error term.

The second, utilizing the identity (14), we do not require anymore that the locations of the jump discontinuities of a given piecewise constant function on $[-1, 1]$ satisfy the strict inequality $|x_1| > |x_2| > \cdots > |x_M|$.

Indeed, simply assuming that the locations of discontinuities are $-1 < y_1 < y_2 < \ldots < y_M < 1$, we consider the higher Fourier-Jacobi coefficients, setting $t_i = -1$ or $t_i = 1$, $i = 1, 2, \ldots, M$, in (14). As a result, we obtain:

$$b_n^{(k)}(1, 1, \ldots, 1) = \sum_{m=1}^{M} \bar{D}_n(y_m) \prod_{s=1}^{k}(1 - y_m), \quad (20)$$

where

$$0 < 1 - y_M < 1 - y_{M-1} < \ldots < 1 - y_1 < 2, \quad (21)$$

and

$$b_n^{(k)}(-1, -1, \ldots, -1) = (-1)^k \sum_{m=1}^{M} \bar{D}_n(y_m) \prod_{s=1}^{k}(1 + y_m), \quad (22)$$

where

$$0 < 1 + y_1 < 1 + y_2 < \ldots < 1 + y_M < 2. \tag{23}$$

In other words, the quantities $1 - y_1$ and $1 + y_M$ in (20) and (22), correspondingly, are the greatest by the absolute value. Hence, the estimate (19) is applicable, where $|x_1| \equiv 1 - y_1$ or $1 + y_M$, and we get

$$\lim_{n\to\infty}(1 - \frac{b_n^{(n)}(1,1,\ldots,1)}{b_n^{(n-1)}(1,1,\ldots,1)}) = y_1 \tag{24}$$

and

$$\lim_{n\to\infty}(-1 - \frac{b_n^{(n)}(-1,-1,\ldots,-1)}{b_n^{(n-1)}(-1,-1,\ldots,-1)}) = y_M. \tag{25}$$

(See (32), Tables 7 and 8.)

The recovery of the remaining singularities follows the old scheme: once x_1 is located approximately as $\tilde{x}_1$, we consider a new sequence $\frac{b_n^{(n)}(\tilde{x}_1,\tilde{x}_1,\ldots,\tilde{x}_1)}{b_n^{(n-1)}(\tilde{x}_1,\tilde{x}_1,\ldots,\tilde{x}_1)}$, which approximates the point of discontinuity $\tilde{y}_1$ with the maximum distance from x_1.

Next, we continue the recovery of the remaining singularities, one by one, as follows: the sequence $\frac{b_n^{(n)}(\tilde{x}_1,\tilde{x}_1,\tilde{y}_1,\tilde{y}_1,0,0\ldots,0)}{b_n^{(n-1)}(\tilde{x}_1,\tilde{x}_1,\tilde{y}_1,\tilde{y}_1,0,0,\ldots,0)}$ which approximates the next singularity location the largest by the absolute value, etc.

Once all the singularity locations are recovered, in order to approximate the associated jumps, we solve the system of linear equations (see (15))

$$(-1)^k \sum_{m=1}^{M} f_m \bar{\rho}(\tilde{x}_m)\bar{P}_n(\tilde{x}_m)(\tilde{x}_m)^k = b_n^{(k)} \tag{26}$$

for f_m, where $k = 1, 2, \cdots, M$. As it is known [6, Theorem 4.1], the coefficient matrix of the linear system (26) is never singular.

Now let us discuss the stability of the suggested method.

If evaluating $b_n^{(k)}$ we encounter roundoff error $e_n^{(k)}$, then our computed value $\tilde{b}_n^{(k)}$ by the formula (8) yields

$$\begin{aligned}
|e_n^{(k)}| &= |\tilde{b}_n^{(k)} - b_n^{(k)}| \\
&= |A_n^{(\alpha+1,\beta+1)} e_{n+1}^{(k-1)} + B_n^{(\alpha+1,\beta+1)} e_n^{(k-1)} + C_n^{(\alpha+1,\beta+1)} e_{n-1}^{(k-1)}| \\
&\le (|A_n^{(\alpha+1,\beta+1)}| + |B_n^{(\alpha+1,\beta+1)}| + |C_n^{(\alpha+1,\beta+1)}|) \\
&\quad \times \max(|e_{n+1}^{(k-1)}|, |e_n^{(k-1)}|, |e_{n-1}^{(k-1)}|) \\
&\equiv (|\bar{A}_n| + |\bar{B}_n| + |\bar{C}_n|) \max(|e_{n+1}^{(k-1)}|, |e_n^{(k-1)}|, |e_{n-1}^{(k-1)}|).
\end{aligned} \tag{27}$$

In particular, for Gegenbauer polynomials, (see (2) - (4)) $|\bar{A}_n| + |\bar{B}_n| + |\bar{C}_n| < 1$ for all $n \in \mathbb{N}$. If we assume that the roundoff errors $e_n^{(0)}$ are bounded by some constant $\epsilon > 0$, then (27) leads to

$$|b_n^{(k)} - \tilde{b}_n^{(k)}| \le \epsilon, \tag{28}$$

thus the method is stable for Gegenbauer polynomials.

In order to illustrate the numerical results obtained by the described algorithm, we will consider the piecewise continuous function studied in [5].

$$f_1(x) = \begin{cases} 5 & \text{if } -1 < x < -1/3, \\ 1 & \text{if } -1/3 < x < 1/2, \\ 1/3 & \text{if } 1/2 < x < 4/5, \\ 0 & \text{if } 4/5 < x < 1. \end{cases} \tag{29}$$

The results of calculations are summarized in Tables 1 and 2.

Table 1: The errors in the approximation to the discontinuity locations of the function f_1 using its Fourier-Legendre coefficients.

n	20	40	80
$x_1 = 4/5$	4.5(−5)	5.0(−9)	1.1(−17)
$x_2 = 1/2$	1.3(−5)	5.0(−9)	4.6(−18)
$x_3 = -1/3$	4.3(−14)	4.1(−25)	1.0(−20)

Table 2: The errors in the approximation to the jump magnitudes of the function f_1 using its Fourier-Legendre coefficients.

n	20	40	80
$[f](x_1)$	2.5(−5)	1.0(−8)	4.7(−18)
$[f](x_2)$	1.3(−4)	6.8(−9)	3.4(−17)
$[f](x_3)$	5.1(−5)	6.4(−10)	1.4(−17)

f_2 is a piecewise constant function with three jump discontinuities.

$$f_2(x) = \begin{cases} 5 & \text{if } -1 < x < 1/3, \\ -1 & \text{if } 1/3 < x < 1/2, \\ 1/3 & \text{if } 1/2 < x < 4/5, \\ 0 & \text{if } 4/5 < x < 1. \end{cases} \tag{30}$$

Below we present the absolute values of the error in the estimation of the points of discontinuity of the function f_2 obtained by applying the suggested method and summarized in Tables 3 and 4.

Table 3: The errors in the approximation to the discontinuity locations for the function f_2 using its Fourier-Legendre coefficients.

n	20	40	80
$x_1 = 4/5$	9.3(−5)	1.0(−8)	2.2(−17)
$x_2 = 1/2$	1.1(−4)	4.2(−8)	3.9(−17)
$x_3 = 1/3$	3.9(−7)	7.1(−11)	9.2(−18)

Table 4: The errors in the approximation to the jump magnitudes of the function f_2 using its Fourier-Legendre coefficients.

n	20	40	80
$[f](x_1)$	2.5(−4)	3.5(−8)	1.6(−16)
$[f](x_2)$	5.1(−4)	3.8(−7)	4.9(−17)
$[f](x_3)$	8.6(−4)	4.6(−7)	1.5(−16)

The function f_3 has four jump discontinuities, all within $[1/5, 1/2]$:

$$f_3(x) = \begin{cases} 0 & \text{if } -1 < x < 1/5, \\ 1 & \text{if } 1/5 < x < 1/4, \\ -1/2 & \text{if } 1/4 < x < 1/3, \\ 3/2 & \text{if } 1/3 < x < 1/2, \\ 1 & \text{if } 1/2 < x < 1. \end{cases} \tag{31}$$

Table 5: The errors in the approximation to the discontinuity locations of the function f_3 using its Fourier-Legendre coefficients.

n	20	40	80
$x_1 = 1/2$	3.0(−4)	2.5(−8)	5.7(−15)
$x_2 = 1/3$	6.9(−5)	1.1(−7)	1.1(−12)
$x_3 = 1/4$	1.7(−5)	1.4(−8)	1.5(−13)
$x_4 = 1/5$	1.3(−4)	2.1(−5)	7.3(−9)

Table 6: The errors in the approximation to the jump magnitudes of function f_3 using its Fourier-Legendre coefficients.

n	20	40	80
$[f](x_1)$	$2.7(-3)$	$7.0(-7)$	$1.8(-10)$
$[f](x_2)$	$1.6(-2)$	$6.5(-5)$	$2.0(-8)$
$[f](x_3)$	$2.8(-2)$	$5.5(-4)$	$1.8(-7)$
$[f](x_4)$	$1.6(-2)$	$4.7(-4)$	$1.6(-7)$

Finally, f_4 represents an example of function with a pair of equidistant singularities, i.e., $|x_1| = |x_2|$. Tables 7 and 8 show the approximation results.

$$f_4(x) = \begin{cases} -1 & \text{if } -1 < x < -2/3, \\ 0 & \text{if } -2/3 < x < 2/3, \\ 1/2 & \text{if } 2/3 < x < 1. \end{cases} \tag{32}$$

Table 7: The errors in the approximation to the discontinuity locations for function f_4 using its Fourier-Legendre coefficients.

n	20	40
$x_1 = 2/3$	$6.0(-14)$	$8.2(-29)$
$x_2 = -2/3$	$1.0(-29)$	$1.0(-29)$

Table 8: The errors in the approximation to the jump magnitudes of function f_4 using its Fourier-Legendre coefficients.

n	20	40
$[f](x_1)$	$1.9(-12)$	$1.0(-27)$
$[f](x_2)$	$4.0(-15)$	$1.0(-27)$

In conclusion let us mention that although there already exists a method producing better numerical results (see [6]), we still find it interesting to explore this particular approach. It is conceptually simple and resembles the Power Method used to approximate eigenvalues of a given matrix.

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