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ON ONE STATISTICAL PROBLEM IN THE HILBERT SPACE

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Abstract. The paper deals with the problem of estimation by the independent observations over a random variable of an unknown probability measure in Hilbert space.

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1. In this work we set and discuss the problem of estimation by the independent observations over a random variable of an unknown probability measure in Hilbert space. This is a random variable with values in the same space and probability distribution of which is the estimated measure. Approaches to such problems may be different. We discuss the evaluation related to finite-dimensional projections, having in mind that these estimates can recover the estimated value with any degree of precision. In this case, it is possible to apply appropriate methods and results for finite-dimensional case. We use certain facilities of nonparametric estimation and show the consistency of the proposed method. The limit theorems showing the degree of accuracy of the method are proved. Moreover, we pose and solve the task of evaluating various characteristics of the measure (or weak distribution) in a Hilbert space.

In the beginning properties of generalized weak distribution in infinite dimensional Hilbert space are studied. These preliminary results are not only of interest in themselves, but are used in solving the basic problem.

2. Consider real, separable Hilbert space H and σ-algebra $\mathcal{B}$ of its Borel sets. Thus, the measurable space $(H, \mathcal{B})$ in sense of [1] is considered. Denote by $\mathcal{P}$ the set of all finite-dimensional orthogonal projections on H and by $\mathcal{N}$ the set of all finite-dimensional subspaces of Hilbert space H. So, for any $P \in \mathcal{P}$ we have $PH \in \mathcal{N}$. When we need to specify the dimension of finite-dimensional space, or a projector or the range of the orthogonal projection, we specify the appropriate index putting it for space or operator respectively at the top or bottom.

The set $P_L^{-1}(A)$ is the cylindrical set in H, where $P_L \in \mathcal{P}$ is the projector on $L \in \mathcal{N}$ and A so called base of cylinder is the Borel set in L. Cylinder sets in H with the bases in L generate σ-algebra, which is denoted by $\mathcal{B}(L)$. It is clear, that $\mathcal{B}(L) \in \mathcal{B}$. Union of all σ-algebras $\mathcal{B}(L)$ is an algebra, which is denoted by $\mathcal{B}_0$. It is called the algebra of cylinder sets. It is known that the σ-closure of $\mathcal{B}_0$ is $\mathcal{B}$.

In some cases, we need the chains of growing finite-dimensional subspaces $\{L_n\}, L_n \subset L_{n+1}, L_n \in \mathcal{N}, n \in N$ (where N is the set of natural numbers) such that $\bigcup_{n=1}^{\infty} L_n$ is tight in H. Here and below, in such situations, the index n indicates the dimension of the space L. In such cases, the algebra $\bigcup_{n=1}^{\infty} \mathcal{B}(L_n) = \mathcal{B}'$, is more simple, consists of a countable set of σ-algebras and has basic, for us, the property that the σ-closure of $\mathcal{B}'$ coincides with $\mathcal{B}$.

For any finite-dimensional subspace L of the Hilbert space H, consider signed finite measure μ_L defined on the Borel σ-algebra $\mathcal{B}_L$ of the sets from L. Assume that the family of measures $\{\mu_L, L \in \mathcal{N}\}$ is adapted in the sense that for all spaces $L_1 \subset L_2, L_i \in \mathcal{N}, i = 1, 2$, and for any Borel set $A \in \mathcal{B}_L$, we have

$$\mu_{L_1}(A) = \mu_{L_2}(P_{L_1}^{-1}(A) \cap L_2) \tag{1}$$

The family of measures $\{\mu_L, L \in \mathcal{N}\}$ satisfying the condition (1) and defined for any finite dimensional spaces L for a given separable Hilbert space H is called a weak distribution. We use for this weak distribution the designation $\mu^* = \{\mu_L, L \in \mathcal{N}\}$. If any measure $\mu_L, L \in \mathcal{N}$ is nonnegative, then the weak distribution $\mu^* = \{\mu_L, L \in \mathcal{N}\}$ is called positive and we write $\mu^* \geq 0$. In case, when the chain of growing finite dimensional subspaces $\{L_n\}, L_n \subset L_{n+1}, L_n \in \mathcal{N}, n \in N$ is considered for which $\bigcup_{n=1}^{\infty} L_n$ is tight in H, then the sequence of measures $\{\mu_{L_n}, n \in N\}$ on $\mathcal{B}_{L_n}$ and satisfying the condition

$$\mu_{L_n}(A) = \mu_{L_{n+1}}(P_{L_n}^{-1}(A) \cap L_{n+1}),$$

is called the sequence of finite dimensional distributions (mainly so-called sequence of nonnegative measures).

In this paper we use the terminology of a weak distribution for both cases. The full variation $|\mu^*|$ of the weak distribution $\mu^* = \{\mu_L, L \in \mathcal{N}\}$ is colled the weak distribution (sequence of finite dimensional distributions) $|\mu^*| = \{|\mu_L|, L \in \mathcal{N}\}$, where $|\mu_L|$ denotes the full variation of measures $\mu_L = \mu_L^+ - \mu_L^-$. Certainly, $|\mu_L| = \mu_L^+ + \mu_L^-$.

If we know a priori, that there exists the measure μ on H, then by related given weak distribution $\{\mu_{L_n}, n \in N\}$, it is possible to uniquely reconstruct the measure. Indeed, knowing μ_{L_n} for the sequence of subspaces $\{L_n\}$, such that $L_n \subset L_{n+1}$ and $\bigcup_{n=1}^{\infty} L_n$ is tight in H, first to define by μ_{L_n} the measure μ on $\mathcal{B}(L_n)$, so that μ is defined on the algebra $\mathcal{B}_0 = \bigcup_{n=1}^{\infty} \mathcal{B}(L_n)$. Since this algebra generates $\mathcal{B}$, then it is clear that μ is uniquely defined on $\mathcal{B}$ too.

However, in practice, for a given weak distribution it is not always known whether or not this distributions generated by some sort of measure on a Hilbert space. To establish this fact additional conditions are needed.

Different approaches to these questions and the results can be found in a vast literature (see [2-5]).

Denote by $C_L(H)$ the space of continuous and finite functions on L. $C_L(H)$ is a Banach space in uniform norm $\|\varphi\|_{C_L(H)} = \sup_{x\in L} |\varphi(x)|$. Further, let $C^*(H)$ be the space of continuous and finite cylindrical functions. Recall that a function $\varphi(x)$ is called cylindrical if there is a $\mathcal{B}_L$ measurable function φ_L, such that the following representation is true

$$\varphi(x) = \varphi_L(P_L x). \tag{2}$$

In this case, L is called the support of this function. In the definition of $C^*(H)$ in addition we request that $\varphi_L \in C_L(H)$ for some L. $C^*(H)$ is a linear normed space. In the uniform norm its closure coincides with $C(H)$- the space of continuous and finite functions on H, with the norm $\|\varphi\|_{C(H)} = \sup_{x\in H} |\varphi(x)|$. Denote by $C^L(H)$ the space of continuous and finite cylindrical functions with base in L. Then $C^*(H) = \bigcup_L C^L(H)$.

For the functions from space $C^*(H)$ an integral on weak distribution $\{\mu_L, L \in \mathcal{N}\}$ can be defined. This integral following [1] is written in the form $\int_H \varphi(x)\mu^*(dx)$ and defined as

$$\int_H \varphi(x)\mu^*(dx) = \int_L \varphi_L(x)\mu_L(dx), \tag{3}$$

where φ_L is the function from (2). The measures μ_L are adapted, so the definition (3) is correct. This can be seen as well in the usual case (see. [1]). If $L_1 \subset L_2$ and

$$\varphi(x) = \varphi_{L_1}(P_{L_1}x) = \varphi_{L_2}(P_{L_2}x).$$

Then for $x \in L_2$

$$\varphi_{L_1}(P_{L_1}x) = \varphi_{L_2}(x).$$

Therefore it follows from adapting property that

$$\begin{aligned} &\textstyle\int_{L_2} \varphi_{L_2}(x)\mu_{L_2}(dx) = \int_{L_2} \varphi_{L_1}(x)(P_{L_1}x)\mu_{L_2}(dx) \\ &= \textstyle\int_{L_2} \varphi_{L_1}(y)\mu_{L_2}(P_{L_1}^{-1}(dy)) = \int_{L_1} \varphi_{L_1}(x)\mu_{L_1}(dx). \end{aligned}$$

The integral, defined in such a way, satisfies the properties of the linear operation

$$\int_H (c_1\varphi_1(x) + c_2\varphi_2(x))\mu^*(dx) = c_1 \int_H \varphi_1(x)\mu^*(dx) + c_2 \int_H \varphi_2(x)\mu^*(dx),$$

where $c_i \in R, \varphi_i \in C^*(H), i = 1, 2$.

Let $M^*(H)$ be the space of weak distribution on H. This is a linear space with the operations: for $\mu_i^* = \{\mu_L^i\}, i = 1, 2$ and $c \in R$ we have $\mu_1^* + \mu_2^* = \{\mu_L^1 + \mu_L^2\}$ and $c\mu_i^* = \{c\mu_L^i\}, i = 1, 2$. It is possible to set the duality between $C^*(H)$ and $M^*(H)$ by formula $\langle\varphi,\mu^*\rangle = \int_H \varphi(x)\mu^*(dx) = \mu^*(\varphi)$. As we have seen, this duality is well defined. It is also easy to show that the following equality is valid

$$\int_H \varphi(x)(\mu_1^*(dx)+\mu_2^*(dx)) = \int_H \varphi(x)\mu_1^*(dx) + \int_H \varphi(x)\mu_2^*(dx).$$

So that

$$\langle c_1\varphi_1 + c_2\varphi_2, \mu^*\rangle = c_1\langle\varphi_1,\mu^*\rangle + c_2\langle\varphi_2,\mu^*\rangle$$

and

$$\langle\varphi, c_1\mu_1^* + c_2\mu_1^*\rangle = c_1\langle\varphi,\mu_1^*\rangle + c_2\langle\varphi,\mu_2^*\rangle.$$

Thus the weak distribution is a linear form on the space $C^*(H)$.

Note also the following properties of the integral in the weak distribution:

a) If $\varphi(x) \in C^*(H), \varphi(x) \geq 0$ and $\mu^* \geq 0$, then

$$\int_H \varphi(x)\mu^*(dx) \geq 0.$$

b)If the sequence of cylindrical functions $\varphi_n(x), n \in N$, has the joint support L, converges to cylindrical function $\varphi(x)$ on measure $|\mu_L|$, and is majorized by integrable with respect to measure $|\mu_L|$ cylindrical function $\psi(x)$ with support L, then

$$\lim_{n\to\infty} \int_H \varphi_n(x)\mu^*(dx) = \int_H \varphi(x)\mu^*(dx).$$

c) For the weak distribution $\mu^* = \{\mu_L, L \in \mathcal{N}\}$ and (N,N)-measurable function $\Phi : H \Rightarrow H$ it can be defined related to that weak distribution $\mu_\Phi^* = \{\mu_L^\Phi, L \in \mathcal{N}\}$, where $\mu_L^\Phi(A) = \mu_L(\Phi^{-1}(A)), A \in B(L)$.

d) Under the condition c) the formula of change of variable holds

$$\int_H \psi(\varphi(x))\mu^*(dx) = \int_H \psi(x)\mu_\varphi^*(dx).$$

e) If $\varphi(x)$ is integrable by μ^*, then

$$|\int_H \varphi(x)\mu^*(dx)| \leq \int_H |\varphi(x)||\mu^*|(dx).$$

Let $M_L^*(H)$ be the space of measures on L. Then the direct product $\prod_L M_L^*(H)$ represents an isomorphic (in the sense of bijections) set in $M^*(H)$. Indeed, for any fixed in both sets we obtain respectively the space $M_L^*(H)$ as direct multiplier for first case and the same for other case as interception of each one-point subset $M^*(H)$.

It is possible to introduce in $M^*(H)$ a norm. Suppose, that $\mu^* \in M^*(H)$. We define the norm μ^* as the common value of total variations of measures

$\|\mu_L\| = \mu_+^L(H)+\mu_-^L(H)$ on L. In future assume that $\|\mu_L\| = 1$. In case when weak distribution is positive, we speak about weak probability distribution.

The main problem of this theory is to find conditions for which weak distribution is generated by measure. The classical Kolmogorov type theorems give positive answer to this question, so that any measure always exists in the space of all functions (more precisely in R^∞). The question when this measure is concentrated in a Hilbert space H is solved in topological theorems under the Minlos-Sazonov type conditions. Other conditions in weak distribution terms (or pre-measures) were established by Bourbaki and Skorokhod. As Skorokhod has shown (see. [1]), for positive weak distributions these conditions can be reduced to the question of extending the integral in weak distribution to a wider class of functions.

From the above definitions it is easy to see that the integral in the weak distribution can also be defined not only for continuous, but for cylindrical bounded measurable functions, or direct determination from the beginning, or as the limit of continuous cylindrical functions. But it turns out that this is not all. The integral can be extended further to some non-cylindrical functions.

The notion of weak distribution is in fact, equivalent to the notion of a quasi-measure (see [3]). Indeed, as we have outlined above, the family of $\{\mathcal{B}_L, L \in N\}$ forms a limit structure, indexed by L, with a limit of $\mathcal{B}_0 = \lim_L \mathcal{B}(L)$.

Furthermore, μ^* is also a finitely additive set function on $\mathcal{B}_0$. So triple $(H, \mathcal{B}_0, \mu^*)$ is a space with quasi-measure. Using this fact, we can define the characteristic functional of a weak distribution, as a characteristic functional of quasi-measure.

For each $y \in H$, we define a linear mapping $f_y(x) = (x, y)_H$ of the space $(H, \mathcal{B}_0)$ on R. Since the function $f_y(x) = (x, y)$ is a cylindrical function, we can determine the measure μ_y in R using the relationship

$$\mu_y(A) = \mu^*(f_y^{-1}(A)), \quad A \in \mathcal{B}(R).$$

Then the characteristic functional of quasi-measure can be determined from

$$\chi_y^*(\varphi) = \int_R e^{it} d\mu_y(t).$$

It is clear that the concept of weak distribution and its characteristic function are equivalent. In the general case, the problem of determining the conditions on the characteristic functional for which the measure in the Hilbert space is determined by the weak distribution has not yet been solved, but some interesting questions in these terms are solved efficiently. In particular, by one-dimensional measures $\mu_y(A)$ the whole weak distribution can be restored (but not the measure generated by this distribution. This is possible only under the extra conditions). This makes it possible to define different functionals of weak distribution, such as the moment functions. For example, the first two moments can be calculated using the following formulas:

$$m_1(y) = \int_{-\infty}^{\infty} t\mu_y(dt) = \int_H (y,x)\mu^*(dx) = (\chi_y^*(0), y)$$

$$m_2(y_1,y_2) = \int_{R^2}\int ts\mu_{y_1y_2}(dtds) = \int_h (y_1,x)(y_2,x)\mu^*(dx) = (\chi^*(0)y_1, y_2).$$

The second of these form is called the correlation form. In terms of the correlation we can give simple conditions for the discussed problem in the case of weak positive distributions: if the correlation form $m_2(y_1,y_2)$ is continuous in Sazonov topology $\tau_S(H)$, then the weak distribution is generated by measure in H. Gaussian weak distribution with a identity correlation operator is not generated by a measure in the initial Hilbert space, but is generated by measure in the kernel topology, i.e. in the extension of the given Hilbert space using a Hilbert-Schmidt operator (see [3]).

In the general case, i.e. for a sign-changing weak distributions, such a criterion is known

Theorem 1 ([3]). *Let μ^* be the weak distribution such that for any $\varepsilon >$ 0 and for some $\rho > 0$, there exists an environment U of zero in $(H, \tau_S(H))$, such that*

$$|\mu^*(A \cap \{y \in H : |(x,y) > \rho\})| < \varepsilon \tag{4}$$

for any $A \in \mathcal{B}_0$ and $x \in U$. Then μ^* is generated by measure from H and conversely. And if the initial weak distribution is positive, then for the validity of the statement it is necessary and sufficient to find for any $\varepsilon > 0$ a compact K such that, $\mu_L(L - P_L(K)) \le \varepsilon$ for any $L \in N$.

Consider a bounded measurable function $f(x) = f(x_1, x_2, ..., x_n)$ in R^n. Then, for any $y_i \in H, i = 1, 2, ..., n$ we can consider the cylindrical function

$$f(x) = f((x,y_1),(x,y_2),...,(x,y_n)) \tag{5}$$

and the integral makes sense

$$\int_H f(x)\mu^*(dx) = \int_{L_n} f(t_1,t_2,...,t_n)\mu_{y_1,y_2,...,y_n}(dt_1dt_2...dt_n).$$

Where L_n is the finite dimensional subspace of H and unitarily isomorphic to R^n,, $\mu_{y_1,y_2,...,y_n}$ the image of the weak distribution μ^* in L_n. If $f(x)$ is the uniform limit of cylindrical functions $f_n(x)$, then we assume that

$$\int_H f(x)\mu^*(dx) = \lim_{n\to\infty}\int_H f_n(x)\mu^*(dx)$$

when this limit exists.

We can specify the classes of non-cylindrical functions for which the integral $\int_H f(x)\mu^*(dx)$ can be extended. We describe one of these classes.

Let $f(x)$ be continuous on $(-\infty, +\infty)$, real, bounded by limited unit, positive, decreasing, vanishing at infinity function $\lim\limits_{x\to\infty} f(x) = 0$. The set of such functions is denoted by $\mathcal{F}$. We show that for any function from $\mathcal{F}$ there exists the integral $\int_H f(-\|x\|^2)\mu^*(dx)$. Indeed, the function $f_n(x) = f(-(P_n x, P_n x))$ is cylindrical and uniformly converges to $f(x)$. Here P_n is the orthogonal projection on the n-dimensional subspace $L_n \subset H$. At the same time, L_n are chosen so that they increase and $\bigcup\limits_{n=1}^{\infty} L_n$ is tight in H. For the numerical sequence $\alpha_n = \int_{L_n} f_n(x)\mu_n^*(dx), n = 1, 2, ...,$ when $n > m$ then

$$|\alpha_n - \alpha_m| = |\int_{L_n} f_n(x)\mu_n(dx) - \int_{L_m} f_m(x)\mu_m(dx)|$$
$$= |\int_{L_n} f_n(x)\mu_n(dx) - \int_{L_n} f_m(x)\mu_n(dx)| \le \int_{L_n} |f_n(x) - f_m(x)||\mu_n|(dx)$$
$$\le |\mu_n|(L_n) \sup_x |f_n(x) - f_m(x)|.$$

Because of the uniform convergence, the last expression can be made arbitrarily small at $m, n \to \infty$. Therefore, the number sequence $\{\alpha_n\}$ converges to some α.

Denote by $\mathcal{W}(\mu^*)$ the set of integrable by μ^* functions. It is clear that $\mathcal{F} \subset \mathcal{W}(\mu^*)$. Since the total mass is the same for every L, then we will assume that $|\mu_L(L)| = 1$ and write $|\mu^*(H)| = 1$.

Theorem 2. *To induce by measure the weak distribution μ^8 it is necessary and sufficient that*

$$\lim_{\varepsilon\downarrow 0} \int_H f(-\varepsilon\|x\|^2)\mu^*(dx) = f(0), \quad f \in \mathcal{F} \tag{6}$$

Proof. If μ^* is induced by measure μ, then using Fatou-Lebesque theorem

$$\lim_{\varepsilon\downarrow 0} \int_H f(-\varepsilon\|x\|^2)\mu^*(dx) = \lim_{\varepsilon\downarrow 0} \int_H f(-\varepsilon\|x\|^2)\mu(dx)$$
$$= \int_H \lim_{\varepsilon\downarrow 0} f(-\varepsilon\|x\|^2)\mu^*(dx) = f(0)\mu(H) = f(0).$$

Conversely suppose we have (6) and check the validity of (4). For any set $A \in \mathcal{B}_0$ there exists $L \in \mathcal{N}$, such that $A \in \mathcal{B}_L$. Note, that since

$$|\mu^*(A \cap \{y \in H : |(x, y)| > \rho\})| \le |\mu^*(L \cap \{y \in H : |(x, y)| > \rho\})|$$

then (4) it is sufficient to check for $A = L$.

Introduce the notation $A_x = \{y \in H : |(x, y)| > \rho\}$. By the condition (6) for any $0 < \delta < 1$, there is $\varepsilon_0 > 0$, such that

$$\int_H f(-\overline{\varepsilon}\|x\|^2)\mu^*(dx) > 1 - \delta, \quad when \;\; 0 < \overline{\varepsilon} < \varepsilon_0.$$

So we write

$$1-\delta < \int_L f(-\overline{\varepsilon}\|x\|^2)\mu_L(dx) \le \int_{L\cap A_x} f(-\overline{\varepsilon}\|y\|^2)|\mu_L(dy)|$$
$$+\int_{L-(L\cap A_x)} f(-\overline{\varepsilon}\|y\|^2)|\mu_L(dy)| \le$$
$$|\mu_L(L\cap A_x)| + \sup_{y\in L:|(x,y)|\le\rho} f(-\overline{\varepsilon}\|y\|^2)(1-|\mu_L(L\cap A_x)|)$$
$$= \sup_{y\in L:|(x,y)|\le\rho} f(-\overline{\varepsilon}\|y\|^2) + (1-\sup_{y\in L:|(x,y)|\le\rho} f(-\overline{\varepsilon}\|y\|^2))|\mu_L(L\cap A_x)|.$$

From that we obtain

$$|\mu_L(L\cap A_x)| < 1-\frac{\delta}{1-\sup\limits_{y\in L:|(x,y)|\le\rho} f(-\overline{\varepsilon}\|y\|^2)}. \tag{7}$$

For an arbitrary $\rho > 0$, consider the open sphere: $U = \{x \in H : \|x\| < \rho^2\}$. Using the inequality $|(x,y)|^2 \le \|x\|\|y\| < \rho^2\|y\|$, we obtain the following estimation

$$\min_{x\in U} \sup_{y\in L:|(x,y)|\le\rho} f(-\overline{\varepsilon}\|y\|^2) \ge f(0)$$

and from (7)

$$|\mu_L(L\cap A_x)| < 1-\frac{\delta}{1-f(0)}. \tag{8}$$

Fix first $\rho > 0$, then for the given $\varepsilon > 0$ choose δ, such that $f(0) > 1-\frac{\delta}{1-\varepsilon}$. δ determines ε_0. Finally from (8) we obtain $|\mu_L(L\cap A_x)| < \varepsilon$ for any $x \in U$.

Remark 1. This theorem is a generalization of the Skorokhod result (see [1], Lemma 2) in the case of sign changing weak distribution. For positive measures in [1] the function $f(x) = e^x$ is used on $[0,\infty)$.

Remark 2. Further, when considering the weak probability distributions, we need criteria only for positive quasi-measures.

A good example of the construction described above is the case, when the weak distribution is positive and is given by adapted system of probability distributions:

$$\mu^* = \{F_n(x_1, x_2, ..., x_n), n = 1, 2, ...\}$$

with the adapting conditions

$$F_n(x_1, x_2, ..., x_{m-1}, -\infty, x_{m+1}, ..., x_n) = 0,$$
$$F_{n+m}(x_1, x_2, ..., x_n, +\infty, ..., +\infty) = F_n(x_1, x_2, ..., x_n),$$
$$F_n(x_1, x_2, ..., x_n) = F_n(x_{i_1}, x_{i_2}, ..., x_{i_n}),$$

where $i_1, i_2, ..., i_n$ any permutation of the indexes. Here we have an increasing system of finite dimensional spaces (without loss of generality we can assume $\{L_n = R^n, n = 1, 2, ...\})$) and μ_{R^n} are the Lebesgue-Stieltjes measures generated by distribution $F_n(x_1, x_2, ..., x_n)$.

In such cases the integral by weak distribution

$$\int_H f(x)\mu^*(dx) = \int_H f(x)F(d^*x).$$

The situation becomes more evident when the positive weak distribution is smooth. We call such a weak distribution $\mu^* = \{\mu_L, L \in \mathcal{N}\}$ that for each L measure μ_L is absolutely continuous with respect to Lebesgue measure m_L, with density $\frac{d\mu_L}{dm_L}(x) = f_L(x_1, x_2, ..., x_{\dim L})$. In this case the adapting condition of measures is equivalent to following: if $L_1 \subset L_2$, then

$$f_{L_1}(x_1, x_2, ..., x_{\dim L})$$
$$= \int_R \int_R \cdots \int_R f_{L_2}(x_1, ..., x_{\dim L_1}, x_{\dim L_1+1}, x_{\dim L_1+2}, ..., x_{\dim L_2})$$
$$dx_{\dim L_1+1} dx_{\dim L_1+2} ... dx_{\dim L_2}.$$

And if $\dim L = 1$, then $\int_R f_L(x)dx = 1$.

The system of functions $f(x) = \{f_L(x_1, x_2, ..., x_{\dim L}), L \in \mathcal{N}\}$ satisfying these conditions is called an adapted system of densities.

Smooth weak distribution is always given by adapted system of the densities and we write integral by the smooth weak distribution as

$$\int_H \varphi(x)\mu^*(dx) = \int_H \varphi(x)F(d^*x) = \int_H \varphi(x)f(x)(d^*x).$$

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