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SOLUTION SOME BOUNDARY VALUE PROBLEMS OF STRETCH-PRESS OF PLATES BY METHOD OF I. VEKUA FOR APPROXIMATION N=2

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Abstract

In the present paper we consider a isotropic homogeneous plate with constant thickness. We consider stretch-press equations for second approximation using of I. Vekua method and solve some problems. Obtained results is compared to the results obtained by plane elasticity theory .

Key words and phrases: boundary value problem, stretch-press, approximation, stress tensor, general solution, analytic function, complex combinations of stresses .

AMS subject classification: 74K20, 74K10.

As well-known the displacement vector $\mathbf{u}$ of I. Vekua's plate theory for approximation $N = 2$ can be written as follows

$$\mathbf{u} = \sum_{m=0}^{2} \overset{(m)}{\mathbf{u}} P_m\left(\frac{x_3}{h}\right) = \overset{(0)}{\mathbf{u}} + \frac{x_3}{h} \overset{(1)}{\mathbf{u}} + \left(\frac{3x_3^2}{2h^2} - \frac{1}{2}\right) \overset{(2)}{\mathbf{u}},$$

where $P_m\left(\frac{x_3}{h}\right)$ is Legendre polinom, $2h$ is a thicknes of plate.

The stretch-press equations system in the components of stress tensor has the form

$$\begin{cases} \partial_\alpha \overset{(0)}{\sigma}_{\alpha\beta} + \overset{(0)}{F}_\beta = 0, \\ \partial_\alpha \overset{(1)}{\sigma}_{\alpha 3} - \frac{3}{h} \overset{(0)}{\sigma}_{33} + \overset{(1)}{F}_3 = 0, \\ \partial_\alpha \overset{(2)}{\sigma}_{\alpha\beta} - \frac{5}{h} \overset{(1)}{\sigma}_{3\beta} + \overset{(2)}{F}_\beta = 0, \end{cases} \tag{1}$$

$$\begin{aligned} \overset{(0)}{\sigma}_{\alpha\beta} &= \lambda \left(\overset{(0)}{\theta} + \frac{1}{h} \overset{(1)}{u}_3 \right) \delta_{\alpha\beta} + \mu \left(\partial_\alpha \overset{(0)}{u}_\beta + \partial_\beta \overset{(0)}{u}_\alpha \right), \\ \overset{(0)}{\sigma}_{33} &= \lambda \overset{(0)}{\theta} + \frac{\lambda + 2\mu}{h} \overset{(1)}{u}_3, \end{aligned}$$

$$\begin{aligned}
\overset{(1)}{\sigma}_{\alpha 3} &= \mu(\partial_\alpha \overset{(1)}{u}_3 + \frac{3}{h}\overset{(2)}{u}_\alpha),\\
\overset{(2)}{\sigma}_{\alpha\beta} &= \lambda \overset{(2)}{\theta}\delta_{\alpha\beta} + \mu(\partial_\alpha \overset{(2)}{u}_\beta + \partial_\beta \overset{(2)}{u}_\alpha),\\
\overset{(2)}{\sigma}_{33} &= \lambda \overset{(2)}{\theta} \quad (\alpha,\beta = 1,2),
\end{aligned}$$

where λ, μ are Lame constants,

$$\overset{(2)}{\theta} = \partial_1 \overset{(2)}{u}_2 + \partial_2 \overset{(2)}{u}_2, \quad \overset{(0)}{\sigma}_{\alpha\beta} = \frac{1}{2h}\int_{-h}^{h}\sigma_{\alpha\beta}dx_3, \quad \overset{(2)}{\sigma}_{\alpha\beta} = \frac{5}{2h}\int_{-h}^{h}\sigma_{\alpha\beta}P_2\left(\frac{x_3}{h}\right)dx_3,$$

$$\overset{(1)}{\sigma}_{\alpha 3} = \frac{3}{2h}\int_{-h}^{h}\sigma_{\alpha 3}P_1\left(\frac{x_3}{h}\right)dx_3, \quad \overset{(0)}{u}_\alpha = \frac{1}{2h}\int_{-h}^{h}u_\alpha dx_3, \quad \overset{(2)}{u}_\alpha = \frac{5}{2h}\int_{-h}^{h}u_\alpha P_2\left(\frac{x_3}{h}\right)dx_3,$$

$$\overset{(1)}{u}_3 = \frac{3}{2h}\int_{-h}^{h}u_3 P_1\left(\frac{x_3}{h}\right)dx_3.$$

The systems (1) in components of displacement vector have the following complex form

$$\begin{cases}
\mu\Delta \overset{(0)}{u}_+ + 2(\lambda+\mu)\dfrac{\partial \overset{(0)}{\theta}}{\partial \bar{z}} + \dfrac{2\lambda}{h}\dfrac{\partial \overset{(1)}{u}_3}{\partial \bar{z}} + \overset{(0)}{F}_\alpha = 0,\\
\mu\Delta \overset{(1)}{u}_3 - \dfrac{3}{h}\left(\lambda \overset{(0)}{\theta} - \mu \overset{(2)}{\theta}\right) - \dfrac{3(\lambda+2\mu)}{h^2}\overset{(1)}{u}_3 + \overset{(1)}{F}_3 = 0,\\
\mu\Delta \overset{(2)}{u}_+ + 2(\lambda+\mu)\dfrac{\partial \overset{(2)}{\theta}}{\partial \bar{z}} - \dfrac{5\mu}{h}\left(2\dfrac{\partial \overset{(1)}{u}_3}{\partial \bar{z}} + \dfrac{3}{h}\overset{(2)}{u}_+\right) + \overset{(2)}{F}_\alpha = 0,
\end{cases} \tag{2}$$

where

$$\overset{(0)}{u}_+ = \overset{(0)}{u}_1 + i\overset{(0)}{u}_2, \; \overset{(2)}{u}_+ = \overset{(2)}{u}_1 + i\overset{(2)}{u}_2, \quad \overset{(0)}{\theta} = \partial_z \overset{(0)}{u}_+ + \partial_{\bar{z}}\overline{\overset{(0)}{u}_+}, \quad \overset{(2)}{\theta} = \partial_z \overset{(2)}{u}_+ + \partial_{\bar{z}}\overline{\overset{(2)}{u}_+}.$$

The general solutions of system (2) can be written as the following $(\overset{(0)}{F}_\alpha = \overset{(1)}{F}_3 = \overset{(2)}{F}_\alpha = 0)$

$$\begin{aligned}
2\mu\overset{(0)}{u}_+ &= æ^*\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} - 4\left(\frac{1}{æ_2}\partial_{\bar{z}}\chi_1(z,\bar{z}) + \frac{1}{æ_1}\partial_{\bar{z}}\chi_2(z,\bar{z})\right),\\
2\mu\overset{(2)}{u}_+ &= \frac{4\nu h^2}{3(1+\nu)}\overline{\varphi''(z)} + \frac{4æ_1 h^2(1-\nu)}{3æ_2\nu}\partial_{\bar{z}}\chi_1(z,\bar{z}) +
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{4æ_2 h^2(1-\nu)}{3æ_1\nu}\partial_{\bar z}\chi_2(z,\bar z) + i\partial_{\bar z}\chi_3(z,\bar z),\\
2\mu \overset{(1)}{u}_3 &= -\frac{\nu h}{1+\nu}\left(\varphi'(z)+\overline{\varphi'(z)}\right) + \frac{h(1-\nu)}{\nu}(\chi_1(z,\bar z)+\chi_2(z,\bar z)).
\end{aligned}
$$

where $æ^* = \dfrac{3-\nu}{1+\nu}$, $\varphi(z)$ and $\psi(z)$ are analytic functions of complex variable z; $\chi_1(z,\bar z)$, $\chi_2(z,\bar z)$ and $\chi_3(z,\bar z)$ are the general solutions of Helmholtz's equations correspondingly

$$
\Delta\chi_1 - \eta^2\chi_1 = 0,
$$

$$
\Delta\chi_2 - \gamma^2\chi_2 = 0,
$$

$$
\Delta\chi_3 - \tau^2\chi_3 = 0,
$$

$$
\left(\eta^2 = æ_2 = \frac{3(1+\sqrt{10\nu-4})}{h^2(1-\nu)},\ \gamma^2 = æ_1 = \frac{3(1-\sqrt{10\nu-4})}{h^2(1-\nu)},\ \tau^2 = \frac{15}{h^2}\right).
$$

Complex combinations of stresses are represented as follows

$$
\begin{aligned}
\overset{(0)}{\sigma}_{11} + \overset{(0)}{\sigma}_{22} &= 2\left(\varphi'(z)+\overline{\varphi'(z)}+\chi_1(z,\bar z)+\chi_2(z,\bar z)\right),\\
\overset{(0)}{\sigma}_{11} - \overset{(0)}{\sigma}_{22} + 2i\overset{(0)}{\sigma}_{12} &= -2\left(z\overline{\varphi''(z)}+\overline{\psi'(z)}+\right.\\
&+ \left.\frac{4}{æ_2}\partial^2_{\bar z\bar z}\chi_1(z,\bar z)+\frac{4}{æ_1}\partial^2_{\bar z\bar z}\chi_2(z,\bar z)\right),\\
\overset{(2)}{\sigma}_{11} + \overset{(2)}{\sigma}_{22} &= \frac{2h^2(1-\nu)}{3\nu(1-2\nu)}\left(æ_1\chi_1(z,\bar z)+æ_2\chi_2(z,\bar z)\right),\\
\overset{(2)}{\sigma}_{11} - \overset{(2)}{\sigma}_{22} + 2i\overset{(2)}{\sigma}_{12} &= 2\left(\frac{4\nu h^2}{3(1+\nu)}\overline{\varphi'''(z)}+\frac{4æ_1h^2(1-\nu)}{3æ_2\nu}\partial^2_{\bar z\bar z}\chi_1(z,\bar z)+\right.\\
&+ \left.\frac{4æ_2h^2(1-\nu)}{3æ_1\nu}\partial^2_{\bar z\bar z}\chi_2(z,\bar z)+i\partial^2_{\bar z\bar z}\chi_3(z,\bar z)\right),\\
\overset{(1)}{\sigma}_{13} + i\overset{(1)}{\sigma}_{23} &= \frac{12}{\nu hæ_2}\partial_{\bar z}\chi_1(z,\bar z)+\frac{12}{\nu hæ_1}\partial_{\bar z}\chi_2(z,\bar z)+i\frac{3}{2h}\partial_{\bar z}\chi_3(z,\bar z),\\
\overset{(0)}{\sigma}_{33} &= 2\left(\chi_1(z,\bar z)+\chi_2(z,\bar z)\right),\\
\overset{(2)}{\sigma}_{33} &= \frac{2h^2(1-\nu)}{3(1-2\nu)}\left(æ_1\chi_1(z,\bar z)+æ_2\chi_2(z,\bar z)\right).
\end{aligned}
$$

Let's consider the infinite plate with the circular hole. Assume stresses are bounded at the infinity. The boundary conditions can be written as the following

$$
\begin{cases}
\overset{(0)}{\sigma}_{rr} - i\overset{(0)}{\sigma}_{r\theta} = 0,\\
\overset{(2)}{\sigma}_{rr} - i\overset{(2)}{\sigma}_{r\theta} = 0,\\
\overset{(1)}{\sigma}_{r3} = 0,
\end{cases}
$$

at the infinite the following conditions are given

$$\overset{(0)}{\sigma}{}^{\infty}_{11} = P_1 = const, \quad \overset{(0)}{\sigma}{}^{\infty}_{22} = P_2 = const, \quad \overset{(0)}{\sigma}{}^{\infty}_{ij} = 0 \;\; i \neq j, \; \overset{(1)}{\sigma}{}^{\infty}_{ij} = 0.$$

Hence

$$\begin{cases} \varphi^{'}(z) + \overline{\varphi^{'}(z)} + \chi_1(z,\bar{z}) + \chi_2(z,\bar{z}) \\ -\left(\dfrac{4}{æ_2}\dfrac{\partial^2\chi_1(z,\bar{z})}{\partial z\partial z} + \dfrac{4}{æ_1}\dfrac{\partial^2\chi_2(z,\bar{z})}{\partial z\partial z} + \bar{z}\varphi^{''}(z) + \psi^{'}(z)\right)e^{2i\theta} = 0, \\ \dfrac{h^2(1-\nu)}{3\nu(1-2\nu)}(æ_1\chi_1(z,\bar{z}) + æ_2\chi_2(z,\bar{z})) + \left[\dfrac{4\nu h^2}{3(1+\nu)}\varphi^{'''}(z)\right. \\ +\dfrac{4æ_1 h^2(1-\nu)}{3æ_2\nu}\dfrac{\partial^2\chi_1(z,\bar{z})}{\partial z\partial z} + \dfrac{4æ_2 h^2(1-\nu)}{3æ_1\nu}\dfrac{\partial^2\chi_2(z,\bar{z})}{\partial z\partial z} \\ \left. -i\dfrac{\partial^2\chi_3(z,\bar{z})}{\partial z\partial z}\right]e^{2i\theta} = 0, \\ \left[\dfrac{12}{\nu hæ_2}\dfrac{\partial\chi_1(z,\bar{z})}{\partial\bar{z}} + \dfrac{12}{\nu hæ_1}\dfrac{\partial\chi_2(z,\bar{z})}{\partial\bar{z}} + i\dfrac{3}{2h}\dfrac{\partial\chi_3(z,\bar{z})}{\partial\bar{z}}\right]e^{-i\theta} \\ +\left[\dfrac{12}{\nu hæ_2}\dfrac{\partial\chi_1(z,\bar{z})}{\partial z} + \dfrac{12}{\nu hæ_1}\dfrac{\partial\chi_2(z,\bar{z})}{\partial z} - i\dfrac{3}{2h}\dfrac{\partial\chi_3(z,\bar{z})}{\partial z}\right]e^{i\theta} = 0. \end{cases}$$

The unknown functions are expressed by the series

$$\varphi^{'}(z) = \sum_{n=0}^{\infty}\frac{a_n}{z^n}, \quad \psi^{'}(z) = \sum_{n=0}^{\infty}\frac{a_n^{'}}{z^n},$$

$$\chi_1(z,\bar{z}) = \sum_{-\infty}^{+\infty} K_n(\eta r) b_n e^{in\theta},$$

$$\chi_2(z,\bar{z}) = \sum_{-\infty}^{+\infty} K_n(\gamma r) b_n^{'} e^{in\theta}, \quad \chi_3(z,\bar{z}) = \sum_{-\infty}^{+\infty} K_n(\tau r) b_n^{''} e^{in\theta}.$$

The boundary conditions have the form

$$\sum_{n=0}^{\infty}\frac{1+n}{R^n}a_n e^{-in\theta} + \sum_{n=0}^{\infty}\frac{\bar{a}_n}{R^n}e^{in\theta} - \sum_{n=0}^{\infty}\frac{\bar{a}_n^{'}}{R^n}e^{-i(n-2)\theta}$$

$$+\sum_{-\infty}^{+\infty}\frac{2(n-1)}{\eta R}K_{n-1}(\eta R) b_n e^{in\theta} + \sum_{-\infty}^{+\infty}\frac{2(n-1)}{\gamma R}K_{n-1}(\gamma R) b_n^{'} e^{in\theta} = 0,$$

$$\frac{\gamma^2 h^2(1-\nu)}{3\nu(1-2\nu)}\sum_{-\infty}^{+\infty}\left(K_n(\eta R) + (1-2\nu)K_{n-2}(\eta R)\right) b_n e^{in\theta}$$

$$+\frac{\eta^2h^2(1-\nu)}{3\nu(1-2\nu)}\sum_{-\infty}^{+\infty}\left(K_n(\gamma R)+(1-2\nu)K_{n-2}(\gamma R)\right)b_n^{'}e^{in\theta}$$

$$-i\frac{15}{4h^2}\sum_{-\infty}^{+\infty}K_{n-2}(\tau R)b_n^{''}e^{in\theta}+\frac{4\nu h^2}{3(1+\nu)}\sum_{-\infty}^{+\infty}\frac{n(n+1)}{R^{n+2}}a_ne^{-in\theta}=0,$$

$$\frac{2}{\eta\nu}\sum_{-\infty}^{+\infty}\left(K_{n+1}(\eta R)+K_{n-1}(\eta R)\right)b_ne^{in\theta}$$

$$+\frac{2}{\gamma\nu}\sum_{-\infty}^{+\infty}\left(K_{n+1}(\gamma R)+K_{n-1}(\gamma R)\right)b_n^{'}e^{in\theta}+\frac{i}{2R}\sum_{-\infty}^{+\infty}nK_n(\tau R)b_n^{''}e^{in\theta}=0.$$

Introduce following symbol

$$K_n:=K_n(\eta R),\quad \widetilde{K_n}:=K_n(\gamma R),\quad \widehat{K}_n:==K_n(\tau R).$$

Taking into account simple conditions of displacement vector

$$æ^*a_1+a_1^*=0$$

we can determine all the coefficients:

$$a_0=\frac{p_1+p_2}{4},\quad a_0^{'}=-\frac{p_1-p_2}{2}$$

$$b_0=b_0^{'}=b_0^{''}=0,\quad a_2^{'}=\frac{(p_1+p_2)R^2}{2}$$

$$a_2=-\frac{(p_1-p_2)R^2}{2}\left(1-\frac{96\nu^2(1-2\nu)\widehat{K_2}}{\gamma\eta}\frac{m}{s}\right),$$

$$a_4^{'}=-\frac{3(p_1-p_2)R^4}{2}\left(1+\frac{192\nu^2(1-2\nu)\widehat{K_2}}{\gamma\eta}\right.$$

$$\left.\times\frac{\eta(K_1+K_3)(2\widehat{K_2}(\widetilde{K_2}+(1-2\nu)\widetilde{K_0})-\gamma(\widetilde{K_1}+\widetilde{K_3})(2\widehat{K_2}(K_2+(1-2\nu)K_0))}{s}\right),$$

$$b_2=-\frac{p_1-p_2}{\gamma}$$

$$\times\frac{24\eta R\nu^2(1-2\nu)\widehat{K_2}[2\widehat{K_2}(\widetilde{K_2}+(1-2\nu)\widetilde{K_0})+\gamma R(1-\nu)(\widetilde{K_1}+\widetilde{K_3})\widehat{K_0}]}{s}$$

$$b_2^{'}=\frac{p_1-p_2}{\eta}$$

$$\times\frac{24\gamma R\nu^2(1-2\nu)\widehat{K_2}[2\widehat{K_2}(K_2+(1-2\nu)K_0)+\eta R(1-\nu)(K_1+K_3)\widehat{K_0}]}{s}$$

$$b_2^{''} = i\frac{p_1 - p_2}{\eta\gamma}\frac{q}{s}$$

where

$$m = 2\eta\widehat{K_2}K_1(\widehat{K_2} + (1-2\nu)\widehat{K_0}) - 2\gamma\widehat{K_2}\widetilde{K_1}(K_2 + (1-2\nu)K_0)$$

$$+\eta\gamma R(1-\nu)\widehat{K_0}(K_1\widetilde{K_3} - K_3\widetilde{K_1})$$

$$q = 96R^2\nu(1-2\nu)\widehat{K_2}[\gamma(\widetilde{K_1} + \widetilde{K_3})(K_2 + (1-2\nu)K_0)$$

$$-\eta(K_1 + K_3)(\widetilde{K_2} + (1-2\nu)\widetilde{K_0})]$$

$$s = 4\eta\gamma R^3(1-\nu^2)(1-2\nu)\widehat{K_2}$$

$$\times\left[K_0(\widetilde{K_2} + (1-2\nu)\widetilde{K_4}) - \widetilde{K_0}(K_2+ + (1-2\nu)K_4)\right]$$

$$+2\eta\gamma R^4(1-\nu)(1-\nu^2)\widehat{K_2}\left[\eta(K_1 + K_3)\left[\widehat{K_0}\left(\widetilde{K_2} + (1-2\nu)\widetilde{K_4}\right)\right.\right.$$

$$\left.+\widehat{K_4}\left(\widetilde{K_2}+\left(1-2\nu)\widetilde{K_0}\right)\right] - \gamma(\widetilde{K_1} + \widetilde{K_3})$$

$$\times\left.\left[\widehat{K_0}(K_2 + (1-2\nu)K_4) + \widehat{K_4}(K_2 + (1-2\nu)K_0)\right]\right]$$

$$-96\nu^2(1-\nu^2)\widehat{K_2}\left[\eta\widetilde{K_1}(2(K_2 + (1-2\nu)K_0) - \eta R(1-\nu)\widehat{K_0})\right.$$

$$\left.-\gamma K_1(2(\widetilde{K_2} + (1-2\nu)\widetilde{K_0}) - \gamma R(1-\nu)\widehat{K_0})\right],$$

when $n \geq 3$, $a_n = a_n^{'} = b_n = b_n^{'} = 0$.

For the components of stress tensor we obtain

$$\overset{(0)}{\sigma}_{rr} = 2a_0 - \frac{a_2^{'}}{r^2} + \left[\frac{4}{r^2}a_2 - a_0^{'} - \frac{a_4^{'}}{r^4} + \right.$$

$$\left.+\frac{2}{\eta r}(K_1(\eta r) - 3K_3(\eta r))b_2 + \frac{2}{\gamma r}(K_1(\gamma r) - 3K_3(\gamma r))b_2^{'}\right]\cos 2\theta$$

$$\overset{(0)}{\sigma}_{\theta\theta} = 2a_0 + \frac{a_2^{'}}{r^2} + \left[a_0^{'} + \frac{a_4^{'}}{r^4} + \frac{2}{\eta r}(2K_2(\eta r)\right.$$

$$\left.+K_0(\eta r) + K_4(\eta r))b_2 + (2K_2(\gamma r) + K_0(\gamma r) + K_4(\gamma r))b_2^{'}\right]\cos 2\theta$$

$$\overset{(0)}{\sigma}_{r\theta} = \left[\frac{2}{r^2}a_2 + a_0^{'} - \frac{a_4^{'}}{r^4}\right.$$

$$\left.+\frac{2}{\eta r}(K_1(\eta r) + 3K_3(\eta r))b_2 + \frac{2}{\gamma r}(K_1(\gamma r) + 3K_3(\gamma r))b_2^{'}\right]\sin 2\theta$$

$$\overset{(2)}{\sigma}_{rr} = \left[\frac{8\nu h^2}{(1+\nu)r^4}a_2 + \frac{\gamma^2 h^2(1-\nu)}{3\nu(1-2\nu)}\left(2K_2(\eta r)\right.\right.$$

$$\left.+(1-2\nu)(K_0(\eta r) + K_4(\eta r))\ \right)b_2$$

$$+\frac{\eta^2h^2(1-\nu)}{3\nu(1-2\nu)}\left(2K_2(\gamma r)+(1-2\nu)(K_0(\gamma r)+K_4(\eta r))\right)b_2^{'}$$

$$\left.-i\frac{15}{4h^2}(K_0(\tau r)-K_4(\tau r))b_2^{''}\right]\cos 2\theta$$

$$\overset{(2)}{\sigma}_{\theta\theta}=\left[-\frac{8\nu h^2}{(1+\nu)r^4}a_2+\frac{\gamma^2h^2(1-\nu)}{3\nu(1-2\nu)}\left(2K_2(\eta r)\right.\right.$$

$$-(1-2\nu)(K_0(\eta r)+\ K_4(\eta r)))\,b_2$$

$$+\frac{\eta^2h^2(1-\nu)}{3\nu(1-2\nu)}\left(2K_2(\gamma r)-(1-2\nu)(K_0(\gamma r)+K_4(\eta r))\right)b_2^{'}$$

$$\left.+\ i\frac{15}{4h^2}(K_0(\tau r)-K_4(\tau r))b_2^{''}\right]\cos 2\theta$$

$$\overset{(2)}{\sigma}_{r\theta}=\left[\frac{8\nu h^2}{(1+\nu)r^4}a_2+\frac{\gamma^2h^2(1-\nu)}{3\nu}\left(K_4(\eta r)-K_0(\eta r)\right)b_2\right.$$

$$\left.+\frac{\eta^2h^2(1-\nu)}{3\nu(1-2\nu)}\left(K_4(\gamma r)-K_0(\gamma r)\right)b_2^{'}+i\frac{15}{4h^2}(K_0(\tau r)+K_4(\tau r))b_2^{''}\right]\cos 2\theta,$$

$$\overset{(1)}{\sigma}_{r3}=\left[i\frac{3K_2(\tau r)}{hr}b_2^{''}+\frac{6(K_(\eta r)+K_3(\eta r))}{\nu h\eta}b_2+\frac{6(K_1(\eta r)+K_3(\eta r))}{\nu h\gamma}b_2^{'}\right]\cos 2\theta,$$

$$\overset{(0)}{\sigma}_{33}=4\left[K_2(\eta r)b_2+K_2(\gamma r)b_2^{'}\right]\cos 2\theta,$$

$$\overset{(2)}{\sigma}_{33}=\frac{4h^2(1-\nu)}{3(1-2\nu)}\left[æ_1K_2(\eta r)b_2+æ_2K_2(\gamma r)b_2^{'}\right]\cos 2\theta.$$

It's intresting that the components of stress and displacements obtained by means of the plane theory are dependend on radiis and material, while the corresponding components obtained by I. Vekua's theory are depended such as on material as on quantity $\dfrac{R}{h}$.

If $P_1 = P, \;\; P_2 = 0$ at the boundary of hole we get

$$\overset{(0)}{\sigma}_{\theta\theta} = P - \tfrac{P}{2}\left[4+\right.$$

$$+\frac{576\nu^2(1-2\nu)K_2^2[\eta(K_1+K_3)(\widetilde{K_2}+(1-2\nu)\widetilde{K_0})-\gamma(\widetilde{K_1}+\widetilde{K_1}(K_2+(1-2\nu)K_0)]}{s}+$$

$$+\frac{(2K_2+K_0+K_4)}{\gamma}\frac{48\eta R\nu^2(1-2\nu)\widehat{K}_2[2\widehat{K}_2(\widetilde{K}_2+(1-2\nu)\widetilde{K}_0)+\gamma R(1-\nu)(\widetilde{K_1}+\widetilde{K_3})\widehat{K}_0]}{s}-$$

$$\left.-\;\frac{(2\widetilde{K_2}+\widetilde{K_0}+\widetilde{K_4})}{\eta}\frac{48\gamma R\nu^2(1-2\nu)\widehat{K}_2[2\widehat{K}_2(K_2+(1-2\nu)K_0)+\eta R(1-\nu)(K_1+K_3)\widehat{K}_0]}{s}\right]\cos 2\theta,$$

$$\overset{(2)}{\sigma}_{\theta\theta} = -\frac{Ph^2}{R^2}\left[\frac{4\nu}{1+\nu}\left(1-\frac{96\nu^2(1-2\nu)\widehat{K}_2}{\eta\gamma}\times\right.\right.$$

$$\times\frac{2\eta\widetilde{K}_2K_1(\widetilde{K_2}+(1-2\nu)\widetilde{K_0})-\gamma\widehat{K}_2\widetilde{K_1}(K_2+(1-2\nu)K_0)+\eta\gamma R(1-\nu)\widehat{K}_0(K_1\widetilde{K_3}-K_3\widetilde{K}_1)}{s}+$$

$$\left.+\;\frac{8\nu(1-\nu)\eta\gamma R^3\widehat{K}_2^2[2K_2-(1-2\nu)(K_0+K_4)][2\widehat{K}_2(\widehat{K}_2+(1-2\nu)\widetilde{K}_0)+\gamma R(1-\nu)(\widetilde{K_1}+\widetilde{K_3})\widehat{K}_0]}{s}\right]\cos 2\theta.$$

The coefficient of concentration has the form

$$K = \frac{(\sigma_{\theta\theta})_{max}}{P} = K\left(\frac{R}{h},\nu\right)\underset{\frac{R}{h}\to\infty}{\longrightarrow} 3.$$

Let's consider the infinite plat6e with the circular hole when the absolute rigid body put in. The boundary conditions has the following form

$$\begin{cases} \overset{(0)}{u}_r = 0, \\ \overset{(2)}{u}_r = 0, \\ \overset{(0)}{\sigma}_{r\theta} = 0, \\ \overset{(2)}{\sigma}_{r\theta} = 0, \\ \overset{(1)}{u}_3 = 0, \end{cases}$$

$$\overset{(0)}{\sigma}{}^{\infty}_{11} = p_1 = const, \;\; \overset{(0)}{\sigma}{}^{\infty}_{22} = p_2 = const, \;\; \overset{(0)}{\sigma}{}^{\infty}_{12} = \overset{(0)}{\sigma}{}^{\infty}_{21} = \overset{(0)}{\sigma}{}^{\infty}_{23} = \overset{(0)}{\sigma}{}^{\infty}_{33} = 0.$$

We get

$$\frac{3-\nu}{1+\nu}\left(2a_0R+\sum_{n=2}^{\infty}\frac{a_n}{(1-n)R^{n-1}}e^{-in\theta}+\sum_{n=2}^{\infty}\frac{\bar{a}_n}{(1-n)R^{n-1}}e^{in\theta}\right)-$$

$$\sum_{n=0}^{\infty}\frac{\bar{a}_n}{R^{n-1}}e^{in\theta}-\sum_{n=0}^{\infty}\frac{a_n}{R^{n-1}}e^{-in\theta}-\bar{a}_0^{'}Re^{-2i\theta}-a_0^{'}Re^{2i\theta}-$$

$$-\sum_{n=2}^{\infty}\frac{\bar{a}_n^{'}}{(1-n)R^{n-1}}e^{i(n-2)\theta}-\sum_{n=2}^{\infty}\frac{a_n^{'}}{(1-n)R^{n-1}}e^{-i(n-2)\theta}$$

$$+\frac{2}{\eta}\sum_{-\infty}^{+\infty}(K_{n-1}+K_{n+1})b_ne^{in\theta}+\frac{2}{\gamma}\sum_{-\infty}^{+\infty}(\widetilde{K}_{n+1}+\widetilde{K}_{n+1})b_n^{'}e^{in\theta}=0,$$

$$\frac{4\nu}{3(1+\nu)}\left(\sum_{n=1}^{\infty}\frac{n\bar{a}_n}{R^{n+1}}e^{in\theta}+\sum_{n=1}^{\infty}\frac{na_n}{R^{n+1}}e^{-in\theta}\right)$$

$$+\frac{2\gamma^2(1-\nu)}{3\nu\eta}\sum_{-\infty}^{+\infty}(K_{n-1}+K_{n+1})b_ne^{in\theta}$$

$$+\frac{2\eta^2(1-\nu)}{3\nu\gamma}\sum_{-\infty}^{+\infty}(\widetilde{K}_{n-1}+\widetilde{K}_{n+1})b_n^{'}e^{in\theta}+i\frac{15}{2h^4}\sum_{-\infty}^{+\infty}(\widehat{K}_{n+1}-\widehat{K}_{n-1})b_n^{''}e^{in\theta}=0,$$

$$\sum_{n=1}^{\infty}\frac{na_n}{R^n}e^{-in\theta}-\sum_{n=1}^{\infty}\frac{n\bar{a}_n}{R^n}e^{in\theta}-\sum_{n=0}^{\infty}\frac{a_n^{'}}{R^n}e^{-i(n-2)\theta}+\sum_{n=0}^{\infty}\frac{\bar{a}_n^{'}}{R^n}e^{i(n-2)\theta}$$

$$+\sum_{-\infty}^{+\infty}(K_{n+2}-K_{n-2})b_ne^{in\theta}+\sum_{-\infty}^{+\infty}(K_{n+2}-K_{n-2})b_n^{'}e^{in\theta}=0,$$

$$\frac{4\nu}{3(1+\nu)}\left(\sum_{n=1}^{\infty}\frac{n(n+1)}{R^{n+2}}a_ne^{-in\theta}-\sum_{n=1}^{\infty}\frac{n(n+1)}{R^{n+2}}\bar{a}_ne^{in\theta}\right)$$

$$+\frac{\gamma^2(1-\nu)}{3\nu}\sum_{-\infty}^{+\infty}(K_{n-2}-K_{n+2})b_ne^{in\theta}+\frac{\eta^2(1-\nu)}{3\nu}\sum_{-\infty}^{+\infty}(\widetilde{K}_{n-2}-\widetilde{K}_{n+2})b_n^{'}e^{in\theta}$$

$$-i\frac{15}{4h^4}\sum_{-\infty}^{+\infty}(\widehat{K}_{n-2}+\widehat{K}_{n+2})b_n^{''}e^{in\theta}=0,$$

$$\sum_{n=1}^{\infty}\frac{a_n}{R^n}e^{-in\theta}+\sum_{n=1}^{\infty}\frac{\bar{a}_n}{R^n}e^{in\theta}-\frac{1-\nu^2}{\nu^2}\left(\sum_{-\infty}^{+\infty}(K_nb_ne^{in\theta}+\sum_{-\infty}^{+\infty}\widetilde{K}_nb_n^{'}e^{in\theta}\right)=0.$$

We can determine all the coefficients

$$a_0 = \frac{p_1 + p_2}{4}, \quad a_0^{'} = -\frac{p_1 - p_2}{2},$$

$$b_0 = \frac{p_1 + p_2}{2(1-\nu^2)} \frac{\nu^2 \eta^3 \widetilde{K}_1}{\gamma^3 K_1 \widetilde{K}_0 - \eta^3 \widetilde{K}_1 K_0}, \quad b_0^{'} = -\frac{p_1 + p_2}{2(1-\nu^2)} \frac{\nu^2 \gamma^3 K_1}{\gamma^3 K_1 \widetilde{K}_0 - \eta^3 \widetilde{K}_1 K_0},$$

$$a_2^{'} = -\frac{p_1 + p_2}{2(1-\nu^2)} \frac{(1-\nu^2)(\gamma^3 K_1 \widetilde{K}_0 - \eta^3 \widetilde{K}_1 K_0) - 2K_1 \widetilde{K}_1 \nu^2 (\gamma^2 - \eta^2)}{\gamma^3 K_1 \widetilde{K}_0 - \eta^3 \widetilde{K}_1 K_0},$$

$$\begin{aligned} a_2 = -\frac{(p_1 - p_2)(1-\nu^2)R^3}{2S} & \Big[\gamma \widetilde{K}_2 \Big\{ R(\widehat{K}_0 + \widehat{K}_4)(4\eta K_2 + \gamma^2 R(K_1 + K_3)) \\ & -\eta(\widehat{K}_3 - \widehat{K}_1)(24K_2 - -\gamma^2 R^2 (K_0 - K_4)) \Big\} \\ & -\eta K_2 \Big\{ R(\widehat{K}_0 + \widehat{K}_4)(4\gamma \widetilde{K}_2 + \eta^2 R(\widetilde{K}_1 + \widetilde{K}_3)) \\ & -\gamma(\widehat{K}_3 - \widehat{K}_1)(24\widetilde{K}_2 - \eta^2 R^2 (\widetilde{K}_0 - \widetilde{K}_4)) \Big\} \Big]. \end{aligned}$$

$$\begin{aligned} b_2 &= \frac{(p_1 - p_2)\nu^2 \eta R}{2} \\ &\times \left\{ \frac{R(\widehat{K}_0 + \widehat{K}_4)(4\gamma \widetilde{K}_2 + \eta^2 R(\widetilde{K}_1 + \widetilde{K}_3))}{S} \right. \\ &- \left. \frac{\gamma(\widehat{K}_3 - \widehat{K}_1)(24\widetilde{K}_2 - \eta^2 R^2 (\widetilde{K}_0 - \widetilde{K}_4))}{S} \right\}, \\ b_2^{'} &= \frac{(p_1 - p_2)\nu^2 \eta R}{2} \\ &\times \left\{ \frac{R(\widehat{K}_0 + \widehat{K}_4)(4\eta K_2 + \gamma^2 R(K_1 + K_3))}{S} \right. \\ &- \left. \frac{\eta(\widehat{K}_3 - \widehat{K}_1)(24K_2 - \gamma^2 R^2 (K_0 - K_4))}{S} \right\}. \\ a_4^{'} &= 2R^4 a_0^{'} + 2R^2 a_2 - (K_4 - K_0) R_4 b_2 - (\widetilde{K}_4 - \widetilde{K}_0) R^4 b_2^{'}, \end{aligned}$$

where

$$\begin{aligned} S = & \Big[\gamma(1-\nu)(\nu - 5)\widehat{K}_2 + 2\widetilde{K}_1 \nu^2\Big] \Big[R(\widehat{K}_0 + \widehat{K}_4)(4\eta K_2 + \gamma^2 R(K_1 + K_3)) \\ & -\eta(\widehat{K}_3 - \widehat{K}_1)(24K_2 - \gamma^2 R^2 (K_0 - K_4))\Big] - \big[\eta(1-\nu)(\nu - 5)K_2 + 2K_1 \nu^2\big] \\ & \times \Big[R(\widehat{K}_0 + \widehat{K}_4)(4\gamma \widetilde{K}_2 + \eta R(K_1 + K_3)) \\ & -\gamma(\widehat{K}_3 - \widehat{K}_1)(24\widetilde{K}_2 - \eta^2 R^2 \, (\widetilde{K}_0 - \widetilde{K}_4))\Big], \end{aligned}$$

$$b_2^{''} = i\frac{2h^4}{15}\left(\frac{8\nu}{3(1+\nu)R^3}a_2 + \frac{2\gamma(1-\nu)(K_1+K_3)}{3\eta\nu}b_2 + \frac{2\eta^2(1-\nu)(\widetilde{K}_1+\widetilde{K}_3)}{3\gamma\nu}b_2^{'}\right).$$

For components of displacement vector we get

$$\begin{aligned}
2\mu \overset{(0)}{u}_r &= \frac{2(1-\nu)r}{1+\nu}a_0 + \frac{1}{r}a_2^{'} + \frac{2K_1(\eta r)}{\eta}b_0 + \frac{2K_1(\gamma r)}{\gamma}b_0^{'} \\
&- \left[\frac{4}{(1+\nu)r}a_2 + ra_0^{'} - \frac{1}{3r^3}a_4^{'}\right. \\
&- \left.\frac{2(K_1(\eta r)+K_3(\eta r))}{\eta}b_2 - \frac{2(K_1(\gamma r)+K_3(\gamma r))}{\gamma}b_2^{'}\right]\cos 2\theta, \\
2\mu \overset{(0)}{u}_\theta &= \left[\frac{2(1-\nu)}{(1+\nu)r}a_0 + ra_0^{'} + \frac{1}{3r^3}a_4^{'}\right. \\
&- \left.\frac{2(K_1(\eta r)-K_3(\eta r))}{\eta}b_2 - \frac{2(K_1(\gamma r)-K_3(\gamma r))}{\gamma}b_2^{'}\right] sin2\theta, \\
2\mu \overset{(1)}{u}_3 &= -\frac{2\nu h}{1+\nu}a_0 + \frac{h(1-\nu)}{\nu}(K_0(\eta r)b_0 + K_0(\gamma r)b_0^{'}) \\
&- \left[\frac{\nu h}{(1+\nu)r^2}a_2 - \frac{4h(1-\nu)}{\nu}(K_2(\eta r)b_2 + K_2(\gamma r)b_2^{'})\right]\cos 2\theta, \\
2\mu \overset{(2)}{u}_r &= \frac{2h^2\gamma^2(1-\nu)K_1(\eta r)}{3\eta\nu}b_0 + \frac{2h^2\eta^2(1-\nu)K_1(\gamma r)}{3\gamma\nu}b_0^{'} \\
&+ \left[\frac{8\nu h^2}{3(1+\nu)r^2}a_2 + \frac{2h^2\gamma^2(1-\nu)(K_1(\eta r)+K_3(\eta r))}{3\eta\nu}b_2\right. \\
&+ \frac{2h^2\eta^2(1-\nu)(K_1(\gamma r)+K_3(\gamma r))}{3\gamma\nu}b_2^{'} \\
&+ \left. i\frac{15}{2h^2}(K_3(\tau r) - K_1(\tau r))b_2^{''}\right]\cos 2\theta, \\
2\mu \overset{(2)}{u}_\theta &= \left[\frac{8\nu h^2}{3(1+\nu)r^3}a_2 + \frac{2h^2\gamma^2(1-\nu)}{3\eta\nu}(K_3(\eta r) - K_1(\eta r))b_2\right. \\
&+ \frac{2h^2\eta^2(1-\nu)}{3\gamma\nu}(K_3(\gamma r) - K_1(\gamma r))b_2^{'} \\
&+ \left. i\frac{15}{2h^2}(K_1(\tau r) + K_3(\tau r))b_2^{''}\right]\sin 2\theta.
\end{aligned}$$

Now consider the infinite plate with the circular hole, when the rigid body put in and soldered. In infinite stress are limitary. The boundary conditions has the following form

$$\begin{cases} \overset{(0)}{u}_r + i\overset{(0)}{u}_\theta = \left[\dfrac{3-\nu}{1+\nu}\varphi(z) - z\,\overline{\varphi'(z)} - \overline{\psi'(z)}\right. \\ \left. -4\left(\dfrac{1}{\eta^2}\dfrac{\partial\chi_1.(z,\bar{z})}{\partial\bar{z}} + \dfrac{1}{\gamma^2}\ \dfrac{\partial\chi_2(z,\bar{z})}{\partial\bar{z}}\right)\right] e^{-\theta} = 0, \\ \overset{(2)}{u}_r + i\overset{(2)}{u}_\theta = \left[\dfrac{4\nu h^2}{3(1+\nu)}\overline{\varphi''(z)} + 4\dfrac{\gamma^2 h^2(1-\nu)}{3\eta^2\nu}\dfrac{\partial\chi_1(z,\bar{z})}{\partial\bar{z}}\right. \\ \left. +\,4\dfrac{\eta^2 h^2(1-\nu)}{3\gamma^2\nu}\dfrac{\partial\chi_2(z,\bar{z})}{\partial\bar{z}} + i\dfrac{\partial\chi_3(z,\bar{z})}{\partial\bar{z}}\right] e^{-\theta} = 0, \\ \overset{(1)}{u}_3 = -\dfrac{2\nu h}{E}\left(\varphi'(z) + \overline{\varphi'(z)}\right) + \dfrac{2h(1-\nu)}{E\nu}(\partial\chi_1(z,\bar{z}) + \partial\chi_2(z,\bar{z})) = 0, \end{cases}$$

$$\overset{(0)}{\sigma}{}^{\infty}_{11} = p_1 = const, \quad \overset{(0)}{\sigma}{}^{\infty}_{22} = p_2 = const.$$

The boundary condition will have following form

$$\begin{cases} \dfrac{3-\nu}{1+\nu}\left(a_0 R + \displaystyle\sum_{n=2}^{\infty}\dfrac{a_n}{(1-n)R^{n-1}}e^{-in\theta}\right) - \displaystyle\sum_{n=0}^{\infty}\dfrac{\bar{a}_n}{R^{n-1}}e^{in\theta} - \bar{a}_0' r e^{-2i\theta} \\ -\displaystyle\sum_{n=2}^{\infty}\dfrac{\bar{a}_n'}{(1-n)R^{n-1}}e^{i(n-2)\theta} + \dfrac{2}{\eta}\displaystyle\sum_{-\infty}^{+\infty}K_{n+1}(\eta R)b_n e^{in\theta} \\ +\dfrac{2}{\gamma}\displaystyle\sum_{-\infty}^{+\infty}\widetilde{K}_{n+1}(\eta R)b_n' e^{in\theta} = 0, \\ \dfrac{4\nu}{3(1+\nu)}\displaystyle\sum_{n=1}^{\infty}\dfrac{n\bar{a}_n}{R^{n+1}}e^{in\theta} + \dfrac{2\gamma^2(1-\nu)}{3\nu\eta}\displaystyle\sum_{-\infty}^{+\infty}K_{n+1}(\eta R)b_n e^{in\theta} \\ +\dfrac{2\eta^2(1-\nu)}{3\nu\gamma}\displaystyle\sum_{-\infty}^{+\infty}\widetilde{K}_{n+1}b_n' e^{in\theta} + i\dfrac{15}{2h^4}\displaystyle\sum_{-\infty}^{+\infty}\widehat{K}_{n+1}b_n'' e^{in\theta} = 0, \\ \displaystyle\sum_{n=1}^{\infty}\dfrac{a_n}{R^n}e^{-in\theta} + \displaystyle\sum_{n=1}^{\infty}\dfrac{\bar{a}_n}{R^n}e^{in\theta} - \dfrac{1-\nu^2}{\nu^2}\left(\displaystyle\sum_{-\infty}^{+\infty}K_n(\eta R)b_n e^{in\theta} + \displaystyle\sum_{-\infty}^{+\infty}\widetilde{K}_n b_n' e^{in\theta}\right) = 0. \end{cases}$$

We can determine all the coefficients

$$a_0 = \frac{p_1+p_2}{4}, \quad a_0' = -\frac{p_1-p_2}{2},$$

$$b_0 = -\frac{p_1+p_2}{2(1-\nu^2)}\frac{\nu^2\eta^3\widetilde{K}_1}{\gamma^3 K_1\widetilde{K}_0 - \eta^3\widetilde{K}_1 K_0}, \quad b_0' = \frac{p_1+p_2}{2(1-\nu^2)}\frac{\nu^2\gamma^3 K_1}{\gamma^3 K_1\widetilde{K}_0 - \eta^3\widetilde{K}_1 K_0},$$

$$a_2^{'}=\frac{(p_1+p_2)R}{2(1-\nu^2)}\frac{(1-\nu^2)(\gamma^3K_1\widetilde{K}_0-\eta^3\widetilde{K}_1K_0)+2K_1\widetilde{K}_1\nu^2(\gamma^2-\eta^2)}{\gamma^3K_1\widetilde{K}_0-\eta^3\widetilde{K}_1K_0},$$

$$b_0^{''}=0.$$

$$a_2=-\frac{(p_1-p_2)(1-\nu^2)R^3}{2}\left(\frac{\eta K_2(4\gamma\widetilde{K}_1\widehat{K}_1+\eta^2R(\widehat{K}_1\widetilde{K}_3+\widetilde{K}_1\widehat{K}_3))}{s}\right.$$

$$\left.-\frac{\gamma\widetilde{K}_2(4\gamma K_2\widehat{K}_1+\gamma^2R(\widehat{K}_1K_3+\widehat{K}_3K_1))}{S}\right),$$

$$a_4^{'}=3R^2a_2+\frac{6R^3}{\gamma}$$

$$\times\frac{K_3[4\gamma\widetilde{K}_2\widehat{K}_1+\eta^2R(\widehat{K}_1K_3+\widetilde{K}_1\widehat{K}_3))-\widetilde{K}_3(4\eta K_2\widehat{K}_1+\gamma^2R(\widehat{K}_1K_3+\widehat{K}_3K_1))]}{4\eta\widehat{K}_1K_2+\gamma^2R(\widehat{K}_1K_3+K_1\widehat{K}_3)}b_2^{'},$$

$$b_2=-\frac{p_1-p_2}{2}\frac{\eta\nu^2R[4\gamma\widetilde{K}_2\widehat{K}_1+\eta^2R(\widehat{K}_1\widetilde{K}_3+\widetilde{K}_1\widehat{K}_3)]}{S},$$

$$b_2^{'}=\frac{p_1-p_2}{2}\frac{\gamma\nu^2R[4\eta K_2\widehat{K}_1+\gamma^2R(\widehat{K}_1K_3+K_1\widehat{K}_3)]}{S},$$

$$b_2^{''}=-i\frac{2(p_1-p_2)\nu Rh^4(1-\nu)}{45\widehat{K}_1}$$

$$\times\frac{[\eta^2K_2\widetilde{K}_1(4K_2\widehat{K}_1+\gamma^2R(\widehat{K}_1K_3+K_1\widehat{K}_3))}{S}$$

$$-\frac{\gamma^2K_1(4\gamma\widetilde{K}_2\widehat{K}_1+\eta^2R(\widehat{K}_1\widetilde{K}_3+\widetilde{K}_3\widehat{K}_1))]}{S},$$

where

$$S=8\nu^2\widehat{K}_1(\gamma K_1\widetilde{K}_2-\eta\widetilde{K}_1K_2)+2\nu^2R(\eta^2K_1(\widehat{K}_1\widetilde{K}_3+\widetilde{K}_1\widehat{K}_3)-\gamma^2\widetilde{K}_1$$

$$\times(\widehat{K}_1K_3+K_1\widehat{K}_3))-R^2(1-\nu)(3-\nu)(\eta^3K_2(\widehat{K}_1\widetilde{K}_3+\widetilde{K}_1\widehat{K}_3)-\gamma^3\widetilde{K}_2(\widehat{K}_1K_3+K_1\widehat{K}_3)).$$

When $n\geq 3$ $\quad a_n=b_n=b_n^{'}=b_n^{''}=0.$

For the components of displacement vactor we get

$$2\mu\overset{(0)}{u}_r=\frac{2(1-\nu)r}{1+\nu}a_0+\frac{1}{r}a_2^{'}+\frac{2K_1(\eta r)}{\eta}b_0+\frac{K_1(\gamma r)}{\gamma}b_0^{'}-\left[\frac{4}{(1+\nu)r}a_2+ra_0^{'}\right.$$

$$\left.-\frac{1}{3r^3}a_4^{'}-\frac{2(K_1(\eta r)+K_3(\eta r))}{\eta}b_2-\frac{2(K_1(\gamma r)+K_3(\gamma r))}{\gamma}b_2^{'}\right]cos2\theta,$$

$$2\mu\overset{(0)}{u}_\theta=\left[\frac{2(1-\nu)}{(1+\nu)r}a_2+ra_0^{'}+\frac{1}{3r^3}a_4^{'}-\frac{2(K_1(\eta r)-K_3(\eta r))}{\eta}b_2\right.$$

$$- \quad \frac{2(K_1(\gamma r) - K_3(\gamma r))}{\gamma} b_2^{'} \Big] \, sin2\theta,$$

$$2\mu \overset{(1)}{u}_{3} = -\frac{4\nu h}{1+\nu} a_0 + \frac{2h(1-\nu)}{\nu}(K_0(\eta r)b_0 + K_0(\gamma r)b_0^{'})$$

$$- \left[\frac{2\nu h}{(1+\nu)r^2} a_2 - \frac{2h(1-\nu)}{\nu}(K_2(\eta r)b_2 + K_2(\gamma r)b_2^{'})\right] cos2\theta,$$

$$2\mu \overset{(2)}{u}_{r} = \frac{2h^2\gamma^2(1-\nu)K_1(\eta r)}{3\eta\nu} b_0 + \frac{2h^2\eta^2(1-\nu)K_1(\gamma r)}{3\gamma\nu} b_0^{'}$$

$$+ \Big[\frac{8\nu h^2}{3(1+\nu)r^3} a_2 + \frac{2h^2\gamma^2(1-\nu)(K_1(\eta r) + K_3(\eta r))}{3\eta\nu} b_2$$

$$+ \frac{2h^2\eta^2(1-\nu)(K_1(\gamma r) + K_3(\gamma r))}{3\gamma\nu} b_2^{'}$$

$$+ i\frac{15}{2h^2}(K_3(\tau r) - K_1(\tau r))b_2^{''} \Big] \, cos2\theta,$$

$$2\mu \overset{(2)}{u}_{\theta} = \Big[\frac{8\nu h^2}{3(1+\nu)r^3} a_2 + \frac{2h^2\gamma^2(1-\nu)}{3\eta\nu}(K_3(\eta r) - K_1(\eta r))b_2$$

$$+ \frac{2h^2\eta^2(1-\nu)}{3\gamma\nu}(K_3(\gamma r) - K_1(\gamma r))b_2^{'}$$

$$+ i\frac{15}{2h^2}(K_1(\tau r) + K_3(\tau r))b_2^{''} \Big] \sin 2\theta.$$

We have the components of stress tensor $\overset{(2)}{\sigma}_{rr}$, $\overset{(2)}{\sigma}_{r\theta}$, $\overset{(2)}{\sigma}_{\theta\theta}$, $\overset{(2)}{\sigma}_{33}$ and the components of displacement vector $\overset{(2)}{u}_{r}$, $\overset{(2)}{u}_{\theta}$ different from approximation $N = 1$. If $\frac{R}{h} \to \infty$ obtained results is coincide to the results obtained by plane elasticity theory.

R E F E R E N C E S

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