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GENERAL SOLUTION OF SYSTEM OF EQUATIONS OF SPHERICAL SHALLOW SHELLS IN BINARY MIXTURE

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Abstract

The theory of mixtures of elastic materials was originated in 1960. Main mechanical properties of a new model of elastic medium with complicated internal structure were first formulated in the works of C. Truesdell and R. Toupin (see [1]). Later this theory was generalized and developed in many directions. Binary and multi-component models of different type mixtures were created and studied by means of various mathematical methods. Intensively is being developed also plane theories corresponding to above noted three-dimensional models.

In this paper we consider a version of linear theory for a body composed of two isotropic homogeneous binary mixture suggested in [2],[3],[4]. The corresponding equations system is written in any curvilinear coordinate system and obtain two-dimensional system for shallow shells using I. Vekua's method [5],[6],[7]. The obtained equations written us in the complex form with respect to isometric coordinates systems. The general solutions of shallow and strongly shallow spherical shells are written by analytic functions of complex variable .

Key words and phrases: Theory of mixtures, spherical shell, general solution .

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1. Let's consider a body composed of two isotropic homogeneous mixture, which occupy three-dimensional domain $\overline{\Omega^h}$. The open domain Ω^h with respect to middle-surface ω be the symmetric shell with $2h$ thickness (in general h is smooth enough, possitive and bounded function of the points of surface ω). The statical balance equations and the response function in any curvilinear coordinate systems (x^1, x^2, x^3) have the following form ([4],[12],[13])

$$\frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}\mathbf{P}^i\right) + \mathbf{\Phi} = 0 \quad in \ \ \Omega^h, \tag{1.1}$$

$$\mathbf{P}^i = \Lambda(\mathbf{R}^j\partial_j\mathbf{U})\mathbf{R}^i + (B-\Lambda)(\mathbf{R}^i\partial_j\mathbf{U})\mathbf{R}^j + A(\mathbf{R}^i\mathbf{R}^j)\partial_j\mathbf{U}, \tag{1.2}$$

where $\partial_i := \dfrac{\partial}{\partial x^i}$; g is a metric tensor discriminate of space; $\mathbf{P}^i = (\mathbf{P}'^i, \mathbf{P}''^i)^T$ are column-matrix composed of the contravariant stress tensors $\mathbf{P}'^i$ and $\mathbf{P}''^i$; $\mathbf{\Phi} = (\mathbf{\Phi}', \mathbf{\Phi}'')^T$ are volume forces; $\mathbf{R}^i$ are contravariant basis vectors; $\mathbf{U} = (\mathbf{u}', \mathbf{u}'')^T$ - $\mathbf{u}'$, $\mathbf{u}''$ are the displacements vectors;

$$A = \begin{pmatrix} a_1 & c \\ c & a_2 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & d \\ d & b_2 \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} \lambda_1 - \dfrac{\alpha_2\rho_2}{\rho} & \lambda_3 - \dfrac{\alpha_2\rho_1}{\rho} \\ \\ \lambda_4 + \dfrac{\alpha_2\rho_2}{\rho} & \lambda_2 + \dfrac{\alpha_2\rho_1}{\rho} \end{pmatrix},$$

$$a_1 = \mu_1 - \lambda_5, \quad c = \mu_3 + \lambda_5, \quad a_2 = \mu_2 - \lambda_5, \quad b_1 = \mu_1 + \lambda_1 + \lambda_5 - \frac{\alpha_2\rho_2}{\rho},$$

$$b_2 = \mu_2 + \lambda_2 + \lambda_5 + \frac{\alpha_2\rho_1}{\rho}, \;\; d = \mu_3 + \lambda_3 - \lambda_5 - \frac{\alpha_2\rho_1}{\rho} = \mu_3 + \lambda_4 - \lambda_5 + \frac{\alpha_2\rho_2}{\rho};$$

$\mu_1, \mu_2, \mu_3, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ are the modulus of elasticity; ρ_1, ρ_2 are partial density of mixture, $\rho = \rho_1 + \rho_2$, $\alpha_2 = \lambda_3 - \lambda_4$.

Under repeating indexes we mean summation, the Latin letters are taking values 1,2,3 and Greek letters are taking the values 1,2.

If (x^1, x^2, x^3) is a normal coordinate system of surface ω then the radius vector of any point M in Ω^h have the following form

$$\mathbf{R} = \mathbf{r}(x^1, x^2) + x^3\mathbf{n}(x^1, x^2),$$

where x^1, x^2 are Gaussian parametyer of surface ω, $\mathbf{r}$ and $\mathbf{n}$ are radius vector and normal of the point $(x^1, x^2) \in \omega$. x^3 is the relative lenght from point M to the surface ω.

If a curve of surface are a coordinate's curve on ω the covariant basis vectors $\mathbf{R}_i$ have the following forms

$$\mathbf{R}_1 = (1 - k_1x^3)\mathbf{r}_1, \quad \mathbf{R}_2 = (1 - k_2x^3)\mathbf{r}_2, \quad \mathbf{R}_3 = \mathbf{n},$$

where k_1 and k_2 are main curvature of a surface; $\mathbf{r}_1$, $\mathbf{r}_2$ are covariant basis vectors of ω; g has the form

$$\sqrt{g} = \sqrt{a}(1 - k_1x^3)(1 - k_2x^3);$$

a is the discriminant of surface quadratic form.

Let the quantity in (1.1) (1.2) are smooth enough and take supposition

$$1 - k_\alpha x^3 \cong 1, \quad \alpha = 1, 2.$$

Using I. Vekua's method we get two-dimensional infinite equations systems

$$\frac{1}{\sqrt{a}}\partial_\alpha(\sqrt{a}\overset{(k)}{\mathbf{P}}{}^{\alpha}) + \partial_\alpha \ln h \underset{=}{\overset{(k)}{\mathbf{P}}}{}^{\alpha} - \frac{1}{h}\underset{-}{\overset{(k)}{\mathbf{P}}}{}^{3} + \overset{(k)}{\mathbf{F}} = 0 \quad in \quad \omega, \tag{1.3}$$

$$\begin{gathered} \overset{(k)}{\mathbf{P}}{}^{\alpha} = \Lambda(\mathbf{r}^\gamma\partial_\gamma \overset{(k)}{\mathbf{U}})\mathbf{r}^\alpha + (B - \Lambda)(\mathbf{r}^\alpha\partial_\gamma \overset{(k)}{\mathbf{U}})\mathbf{r}^\gamma + A(\mathbf{r}^\alpha\mathbf{r}^\gamma)\partial_\gamma \overset{(k)}{\mathbf{U}} - \Lambda(\nabla \ln h \overset{(k'')}{\mathbf{U}})\mathbf{r}^\alpha - \\ -(B - \Lambda)(\mathbf{r}^\alpha \overset{(k'')}{\mathbf{U}})\nabla \ln h - A(\mathbf{r}^\alpha\nabla \ln h)\overset{(k'')}{\mathbf{U}} + \\ + \frac{1}{h}\Lambda(\mathbf{n}\overset{(k')}{\mathbf{U}})\mathbf{r}^\alpha + \frac{1}{h}(B - \Lambda)(\mathbf{r}^\alpha \overset{(k')}{\mathbf{U}})\mathbf{n}, \end{gathered} \tag{1.4}$$

$$\overset{(k)}{\mathbf{P}}{}^{3} = \Lambda(\mathbf{r}^{\gamma}\partial_{\gamma}\overset{(k)}{\mathbf{U}})\mathbf{n} + (B-\Lambda)(\mathbf{n}\partial_{\gamma}\overset{(k)}{\mathbf{U}})\mathbf{r}^{\gamma} - \Lambda(\nabla \ln h\overset{(k'')}{\mathbf{U}})\mathbf{n} -$$
$$-(B-\Lambda)(\mathbf{n}\overset{(k'')}{\mathbf{U}})\nabla \ln h + \frac{1}{h}A\overset{(k')}{\mathbf{U}} + \frac{1}{h}B(\mathbf{n}\overset{(k')}{\mathbf{U}})\mathbf{n}, \quad k=0,1,2,..., \tag{1.5}$$

where

$$\left(\overset{(k)}{\mathbf{P}}{}^{j}, \overset{(k)}{\mathbf{U}}\right) = \frac{2k+1}{2h}\int_{-h}^{h}(\mathbf{P}^j, \mathbf{U})P_k\left(\frac{x^3}{h}\right);$$

$P_k\left(\frac{x^3}{h}\right)$ is the Legandre polinomial of order k;

$$\underset{=}{\overset{(k)}{\mathbf{P}}}{}^{\alpha} := (k+1)\overset{(k)}{\mathbf{P}}{}^{\alpha} + (2k+1)(\overset{(k-2)}{\mathbf{P}}{}^{\alpha} + \overset{(k-4)}{\mathbf{P}}{}^{\alpha} + ...),$$

$$\underset{-}{\overset{(k)}{\mathbf{P}}}{}^{3} := (2k+1)(\overset{(k-1)}{\mathbf{P}}{}^{3} + \overset{(k-3)}{\mathbf{P}}{}^{3} + ...), \quad \overset{(m)}{\mathbf{P}}{}^{j} = 0, \quad when \;\; m<0;$$

$\mathbf{r}^{\gamma}$ are contravariant basis vectors on the middle-surface;

$$\overset{(k')}{\mathbf{U}} = (2k+1)(\overset{(k+1)}{\mathbf{U}} + \overset{(k+3)}{\mathbf{U}} + ...), \quad \overset{(k'')}{\mathbf{U}} = k\overset{(k)}{\mathbf{U}} + (2k+1)(\overset{(k+2)}{\mathbf{U}} + \overset{(k+4)}{\mathbf{U}} + ...);$$

$$\nabla \ln h := \partial_{\gamma} \ln h \mathbf{r}^{\gamma};$$

$$\overset{(k)}{\mathbf{F}} = \left(k+\frac{1}{2}\right)\frac{1}{h}\int_{-h}^{h}\sqrt{\frac{g}{a}}\mathbf{\Phi}P_k\left(\frac{x^3}{h}\right)dx^3 + \left(k+\frac{1}{2}\right)\frac{1}{h}\times$$
$$\times\left\{\sqrt{\frac{g_+}{a}}[\mathbf{P}^3_+ - \partial_{\alpha}h\mathbf{P}^{\alpha}_+] - (-1)^k\sqrt{\frac{g_-}{a}}[\mathbf{P}^3_- + \partial_{\alpha}h\mathbf{P}^{\alpha}_-]\right\}; \tag{1.6}$$

by lower symbols " + " and " − " are denoted corresponding quantity when $x^3 = h$ and $x^3 = -h$.

The Greak letters take values 1,2. Under reapiting indexes we mean summation.

The second main supposition is

$$\mathbf{U}(x^1, x^2, x^3) \cong \sum_{k=0}^{N}\overset{(k)}{\mathbf{U}}(x^1, x^2)P_k\left(\frac{x^3}{h}\right), \quad \overset{(k)}{\mathbf{U}} = 0, \quad when \;\; k>N, \tag{1.7}$$

where N is some fixed non-negative integer number.

From the (1.4) (1.5) and (1.7) we get

$$\overset{(k)}{\mathbf{P}}{}^{j} = 0, \quad when \;\; k>N.$$

Hence we obtain system of $6N+6$ equations in the components of displacement vectors from with same quantity unknown functions of two variable

$$\begin{cases} A\nabla_{\alpha}(\nabla^{\alpha}\overset{(k)}{U}{}^{\beta}) + (B-\Lambda)\nabla_{\alpha}(\nabla^{\beta}\overset{(k)}{U}{}^{\alpha}) + \Lambda\nabla^{\beta}(\nabla_{\alpha}\overset{(k)}{U}{}^{\alpha}) + \overset{(k)}{M}{}^{\beta} + \overset{(k)}{F}{}^{\beta} = 0, \\ A\nabla_{\alpha}(\nabla^{\alpha}\overset{(k)}{U}{}_{3}) + \overset{(k)}{M}{}_{3} + \overset{(k)}{F}{}_{3} = 0 \quad in \;\; \omega, \end{cases} \tag{1.8}$$

$$(k = 0, 1, ..., N, \quad \beta = 1, 2),$$

where $\overset{(k)}{U}{}^{\beta} = (\overset{(k)}{u}{}'^{\beta}, \overset{(m)}{u}{}''^{\beta})^T = \overset{(k)}{\mathbf{U}}\mathbf{r}^{\beta}$, $\overset{(k)}{U}{}_3 = (\overset{(k)}{u}{}'_3, \overset{(k)}{u}{}''_3)^T = \overset{(k)}{\mathbf{U}}\mathbf{n}$; $\overset{(k)}{M}{}^{j} = (\overset{(k)}{M}{}'^{j}, \overset{(k)}{M}{}''^{j})^T$- involve the unknown functions $\overset{(k)}{u}{}'^{i}, \overset{(k)}{u}{}''^{i}$ and first order derivatives with variable x^1, x^2.

Let D be a domain on the plane Oxy which on bijectively and continuously maps the surface ω.

We can set following classical boundary value problem: Find $\overset{(k)}{\mathbf{U}} \in C^2(D) \cap C^1(\overline{D})$ which satisfy the following boundary conditions

Problem I. $\overset{(k)}{\mathbf{U}}|_{\partial\omega} = \overset{(k)}{\mathbf{f}} \quad (k = 0, 1, ..., N)$;

Problem II. $\overset{(k)}{\mathbf{P}}{}_{(l)} = \overset{(k)}{\boldsymbol{\varphi}} \quad (k = 0, 1, ..., N)$,

where

$$\overset{(k)}{P}{}_{(l)} = \Lambda(\mathbf{r}^{\gamma}\partial_{\gamma}\overset{(k)}{\mathbf{U}})\mathbf{l} + (B-\Lambda)(\mathbf{l}\partial_{\gamma}\overset{(k)}{\mathbf{U}})\mathbf{r}^{\gamma} + A(\mathbf{l}\mathbf{r}^{\gamma})\partial_{\gamma}\overset{(k)}{\mathbf{U}} - \Lambda(\nabla \ln h\overset{(k'')}{\mathbf{U}})\mathbf{l}$$

$$-(B-\Lambda)(\mathbf{l}\overset{(k'')}{\mathbf{U}})\nabla \ln h - A(\mathbf{l}\nabla \ln h)\overset{(k'')}{\mathbf{U}} + \frac{1}{h}\Lambda(\mathbf{n}\overset{(k')}{\mathbf{U}})\mathbf{l} + \frac{1}{h}(B-\Lambda)(\mathbf{l}\overset{(k')}{\mathbf{U}})\mathbf{n},$$

$\mathbf{l}$ is tangential normal of $\partial\omega$; $\mathbf{l} \times \mathbf{s} = \mathbf{n}$; $\overset{(k)}{\mathbf{f}}, \overset{(k)}{\boldsymbol{\varphi}}$ are given functions on the boundary.

Furthermore, can be given several mixed boundary conditions (see [6]).

2. **Approximation** $N = 0$. If $N = 0$, then from the (1.3) (1.4) and (1.5) we obtain ($\overset{(0')}{\mathbf{U}} = \overset{(0'')}{\mathbf{U}} = 0$)

$$\frac{1}{\sqrt{a}}\partial_{\alpha}(\sqrt{a}\overset{(0)}{\mathbf{P}}{}^{\alpha}) + \partial_{\alpha}\ln h\overset{(0)}{\mathbf{P}}{}^{\alpha} + \overset{(0)}{\mathbf{F}} = 0, \tag{2.1}$$

$$\overset{(0)}{\mathbf{P}}{}^{\alpha} = \Lambda(\mathbf{r}^{\gamma}\partial_{\gamma}\overset{(0)}{\mathbf{U}})\mathbf{r}^{\alpha} + (B-\Lambda)(\mathbf{r}^{\alpha}\partial_{\gamma}\overset{(0)}{\mathbf{U}})\mathbf{r}^{\gamma} + A(\mathbf{r}^{\alpha}\mathbf{r}^{\gamma})\partial_{\gamma}\overset{(0)}{\mathbf{U}}, \tag{2.2}$$

$$\overset{(0)}{\mathbf{P}}{}^{3} = \Lambda(\mathbf{r}^{\gamma}\partial_{\gamma}\overset{(0)}{\mathbf{U}})\mathbf{n} + (B-\Lambda)(\mathbf{n}\partial_{\gamma}\overset{(0)}{\mathbf{U}})\mathbf{r}^{\gamma}. \tag{2.3}$$

The vectors $\overset{(0)}{\mathbf{P}}{}^{j}$, $\overset{(0)}{\mathbf{U}}$ and $\overset{(0)}{\mathbf{F}}$ in the covariant basis have the following forms

$$\overset{(0)}{\mathbf{P}}{}^{j} = \overset{(0)}{P}{}^{j\beta}\mathbf{r}_{\beta} + \overset{(0)}{P}{}^{j3}\mathbf{n}, \quad \overset{(0)}{\mathbf{U}} = \overset{(0)}{U}{}^{\beta}\mathbf{r}_{\beta} + \overset{(0)}{U}{}_{3}\mathbf{n}, \quad \overset{(0)}{\mathbf{F}} = \overset{(0)}{F}{}^{\beta}\mathbf{r}_{\beta} + \overset{(0)}{F}{}_{3}\mathbf{n},$$

where $\overset{(0)}{P}{}^{ij} = (\overset{(0)}{P}{}'^{ij}, \overset{(0)}{P}{}''^{ij})^T$ are the contravariant components of stress tensors, we get the following system

$$\begin{aligned} &\nabla_{\alpha}\overset{(0)}{P}{}^{\alpha\beta} - b_{\alpha}^{\beta}\overset{(0)}{P}{}^{\alpha 3} + \partial_{\alpha}\ln h\overset{(0)}{P}{}^{\alpha\beta} + \overset{(0)}{F}{}^{\beta} = 0, \quad \beta = 1, 2, \\ &\nabla_{\alpha}\overset{(0)}{P}{}^{\alpha 3} + b_{\alpha\beta}\overset{(0)}{P}{}^{\alpha\beta} + \partial_{\alpha}\ln h\overset{(0)}{P}{}^{\alpha 3} + \overset{(0)}{F}{}_{3} = 0 \quad in \ \omega, \end{aligned} \tag{2.4}$$

$$\begin{aligned}
&\overset{(0)}{P}{}^{\alpha\beta} = \Lambda \overset{(0)}{\theta} a^{\alpha\beta} + 2M \overset{(0)}{\in}{}^{\alpha\beta} - 2\lambda_5 \overset{(0)}{\hbar}{}^{\alpha\beta},\\
&\overset{(0)}{P}{}^{\alpha 3} = 2M \overset{(0)}{\in}{}^{\alpha 3} - 2\lambda_5 \overset{(0)}{\hbar}{}^{\alpha 3},\\
&\overset{(0)}{P}{}^{\alpha 3} = 2M \overset{(0)}{\in}{}^{\alpha 3} - 2\lambda_5 \overset{(0)}{\hbar}{}^{\alpha 3},\\
&\overset{(0)}{P}{}^{3\alpha} = 2M \overset{(0)}{\in}{}^{\alpha 3} + 2\lambda_5 \overset{(0)}{\hbar}{}^{\alpha 3},\\
&\overset{(0)}{P}{}^{33} = \lambda \overset{(0)}{\theta};
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
&\overset{(0)}{\in}{}^{\alpha\beta} := (\overset{(0)}{e}{}'^{\alpha\beta}, \overset{(0)}{e}{}''^{\alpha\beta})^T = \frac{1}{2}(\nabla^\alpha \overset{(0)}{U}{}^\beta + \nabla^\beta \overset{(0)}{U}{}^\alpha - 2b^{\alpha\beta} \overset{(0)}{U}{}_3),\\
&\overset{(0)}{\in}{}^{\alpha 3} := (\overset{(0)}{e}{}'^{\alpha 3}, \overset{(0)}{e}{}''^{\alpha 3})^T = \frac{1}{2}(\nabla^\alpha \overset{(0)}{U}{}_3 + b^{\alpha\beta} \overset{(0)}{U}{}_\beta),\\
&\overset{(0)}{\hbar}{}^{\alpha\beta} := (\overset{(0)}{h}{}^{\alpha\beta}, \overset{(0)}{h}{}^{\beta\alpha})^T = \frac{1}{2}S(\nabla^\alpha \overset{(0)}{U}{}^\beta - \nabla^\beta \overset{(0)}{U}{}^\alpha),\\
&\overset{(0)}{\hbar}{}^{\alpha 3} := (\overset{(0)}{h}{}^{\alpha 3}, \overset{(0)}{h}{}^{3\alpha})^T = \frac{1}{2}S(\nabla^\alpha \overset{(0)}{U}{}_3 + b^{\alpha\beta} \overset{(0)}{U}{}_\beta),\\
&\overset{(0)}{\theta} := \in^\gamma_\gamma = \nabla_\gamma \overset{(0)}{U}{}^\gamma - 2H \overset{(0)}{U}{}_3,
\end{aligned} \tag{2.6}$$

where $\overset{(0)}{e}{}'^{\alpha j}$, $\overset{(0)}{e}{}''^{\alpha j}$ are zero approximation of the contravariant components of srain tensors; $\overset{(0)}{\hbar}{}^{\alpha j}$ are zero approximation of the contravariant components of rotation tensor ($\overset{(0)}{h}{}^{\alpha j} = -\overset{(0)}{h}{}^{j\alpha}$). $b_{\alpha\beta}, b^{\alpha\beta}, b^\alpha_\beta$ are covariant, contravariant and mixed components of curvature surface tensors accordingly; $H = \frac{1}{2}(k_1 + k_2) = \frac{1}{2}b^\gamma_\gamma$ is mean curvature of surface ω; ∇_α, ∇^α are covariant and contravariant derivative;

$$M := \begin{pmatrix} \mu_1 & \mu_3 \\ \mu_3 & \mu_2 \end{pmatrix}, \quad S := \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

If we substitute (2.6) into (2.5) and obtained results substitute into (2.4) we get the following system of equations in the components of displacement vector

$$A\nabla_\alpha \nabla^\alpha \overset{(0)}{U}{}^\beta + B\nabla^\beta \nabla_\alpha \overset{(0)}{U}{}^\alpha + [(B-\Lambda)K a^\beta_\gamma - A b^\beta_\alpha b^\alpha_\gamma] \overset{(0)}{U}{}^\gamma -$$

$$-[2\Lambda H a^{\alpha\beta} + (2M + A)b^{\alpha\beta}]\nabla_\alpha \overset{(0)}{U}{}_3 - 2\nabla_\alpha(\Lambda H a^{\alpha\beta} + M b^{\alpha\beta}) \overset{(0)}{U}{}_3 +$$

$$+\partial_\alpha lnh[(\Lambda \nabla_\gamma \overset{(0)}{U}{}^\gamma - 2\Lambda H \overset{(0)}{U}{}_3)a^{\alpha\beta} + A\nabla^\alpha \overset{(0)}{U}{}^\beta +$$

$$+(B-\Lambda)\nabla^\beta \overset{(0)}{U}{}^{\alpha} - 2Mb^{\alpha\beta} \overset{(0)}{U}{}_3] + \overset{(0)}{F}{}^{\beta} = 0,$$

$$A\nabla_\alpha\nabla^\alpha \overset{(0)}{U}{}_3 + A\nabla_\alpha(b^\alpha_\gamma \overset{(0)}{U}{}^{\gamma}) - 2[2\Lambda H^2 + Mb^\alpha_\gamma b^\gamma_\alpha] \overset{(0)}{U}{}_3 + \tag{2.7}$$

$$+2\Lambda H\nabla_\alpha \overset{(0)}{U}{}^{\alpha} + 2Mb^\gamma_\alpha\nabla^\alpha \overset{(0)}{U}{}_{\gamma} + \partial_\alpha \ln h[A\nabla^\alpha \overset{(0)}{U}{}_3 + Ab^{\alpha\gamma} \overset{(0)}{U}{}_{\gamma}] + \overset{(0)}{F}{}_3 = 0,$$

where $K = k_1k_2 = b^1_1b^2_2 - b^2_1b^1_2$ are Gaussian curvature of surface ω.

The obtained equations for constant thickness shell in the complex form will be written on an isometric coordinate system of middle-surface. The first main quadratic form has the following form

$$ds^2 = \dot\Lambda(dx^2 + dy^2), \quad \dot\Lambda > 0.$$

If $h = const$ the system (2.4) can be written as follows (for simplicity we don't write „ $(0)''$)

$$\dot\Lambda^{-1}\partial_z(P_{11} - P_{22} + i(P_{12} + P_{21})) + \partial_{\bar z}(P^1_1 + P^2_2 + i(P^{1\cdot}_{\cdot 2} - P^{2\cdot}_{\cdot 1}))-$$

$$-HP_+ + Q\overline{P_+} + X_+ = 0,$$

$$\dot\Lambda^{-1}\partial_z(P_+ + \partial_{\bar z}\overline{P_+}) + H(P^1_1 + P^2_2) + \tag{2.8}$$

$$Re[\overline{Q}(P^1_1 - P^2_2 + i(P^{1\cdot}_{\cdot 2} + P^{2\cdot}_{\cdot 1}))] + X_3 = 0,$$

where

$$z = x + iy, \quad P_+ = P_{13} + iP_{23}, \quad X_+ = X_1 + iX_2,$$

$$\partial_{\bar z} = \frac{1}{2}(\partial_x + i\partial_y), \;\; \partial_z = \frac{1}{2}(\partial_x - i\partial_y), \;\; Q = \frac{1}{2}(b^1_1 - b^2_2 + 2ib^1_2).$$

$P_{ij}, P^{i\cdot}_{\cdot j}$ are covariant and mixed components of stress tensor accordingly. From the (2.5), (2.6) we get

$$P_{11} - P_{22} + i(P_{12} + P_{21}) = 4M(\dot\Lambda\,\partial_{\bar z}U^+ - \dot\Lambda\,QU_3),$$

$$P^1_1 + P^2_2 + i(P^{1\cdot}_{\cdot 2} - P^{2\cdot}_{\cdot 1}) = 2B\widetilde\theta - 4(B - \lambda_5 S)HU_3 - 4\lambda_5 S\,\dot\Lambda^{-1}\partial_z U_+,$$

$$P_+ = 2A(\partial_{\bar z}U_3 + \frac{1}{2}HU_+ + \frac{1}{2}Q\overline{U_+}), \tag{2.9}$$

$${}_+P = 2(B - A)(\partial_{\bar z}U_3 + \frac{1}{2}HU_+ + \frac{1}{2}Q\overline{U_+}),$$

where $U^+ = \dot\Lambda^{-1}U_+ = U^1 + iU^2$, $\widetilde\theta = \nabla_\alpha U^\alpha = \dot\Lambda^{-1}(\partial_z U_+ + \partial_{\bar z}\overline{U_+})$, ${}_+P =$ $= P_{31} + iP_{32}$.

3. **Spherical shell.** Let's consider the case of constant thickness spherical shell. The equation of sphere with radius R and center in the spring of coordinate system has the form

$$\mathbf{r} = R\mathbf{n}, \tag{3.1}$$

where $\mathbf{n}$ is an unit vector of sphere. By derivation oh both-side of the (3.1) we get

$$\mathbf{r}_\alpha = R\mathbf{n}_\alpha = -Rb_\alpha^\beta \mathbf{r}_\beta = -Rb_{\alpha\beta}\mathbf{r}^\beta .$$

We obtain

$$a_{\alpha\beta} = -Rb_{\alpha\beta}, \quad a_\beta^\alpha = -Rb_\beta^\alpha, \tag{3.2}$$

i.e.,

$$k_1 = k_2 = -\frac{1}{R}, \quad H = -\frac{1}{R}, \quad K = \frac{1}{R^2}. \tag{3.3}$$

From (3.2) and (3.3) we obtain $Q = 0$ and the system of equations (2.8), (2.9) is taking the following form

$$\dot{\Lambda}^{-1}\partial_z(P_{11} - P_{22} + i(P_{12} + P_{21})) + \partial_{\bar z}(P_1^1 + P_2^2 + i(P_{.2}^{1.} - P_{.1}^{2.})) + \frac{1}{R}P_+ + X_+ = 0, \tag{3.4}$$

$$\dot{\Lambda}^{-1}(\partial_z P_+ + \partial_{\bar z}\overline{P}_+) - \frac{1}{R}(P_1^1 + P_2^2) + X_3 = 0, \tag{3.5}$$

$$P_{11} - P_{22} + i(P_{12} + P_{21}) = 4M(\dot{\Lambda}\,\partial_{\bar z}U^+) = 4M\,\dot{\Lambda}\,\partial_{\bar z}(\dot{\Lambda}^{-1}U_+), \tag{3.6}$$

$$P_1^1 + P_2^2 + i(P_{.2}^{1.} - P_{.1}^{2.}) = 2B\theta + \frac{4}{R}(B - \lambda_5 S)U_3 - 4\lambda_5 S\,\dot{\Lambda}^{-1}\partial_z U_+, \tag{3.7}$$

$$P_+ = A\left(2\partial_{\bar z}U_3 - \frac{1}{R}U_+\right), \tag{3.8}$$

$$_+P = (B - \Lambda)\left(2\partial_{\bar z}U_3 - \frac{1}{R}U_+\right), \tag{3.9}$$

where

$$\theta = \dot{\Lambda}^{-1}(\partial_z U_+ + \partial_{\bar z}\overline{U_+}).$$

In this case the system (2.7) has the following form

$$\begin{cases} A\nabla_\alpha\nabla^\alpha U^\beta + B\nabla^\beta\nabla_\alpha U^\alpha + \dfrac{2\lambda_5}{R^2}SU^\beta + \dfrac{1}{R}\widehat{A}\nabla^\beta U_3 + F^\beta = 0, \\[2ex] A\nabla_\alpha\nabla^\alpha U_3 - \dfrac{4}{R^2}(\Lambda + M)U_3 - \dfrac{1}{R}\widehat{A}\nabla_\alpha U^\alpha + F^3 = 0, \end{cases} \tag{a}$$

where

$$\widehat{A} := A + 2M + 2\Lambda = \begin{pmatrix} 2\lambda_1 + 3\mu_1 - \lambda_5 - \dfrac{2\alpha_2\rho_2}{\rho} & 2\lambda_3 + 2\mu_3 + \lambda_5 - \dfrac{2\alpha_2\rho_1}{\rho} \\ 2\lambda_3 + 2\mu_3 + \lambda_5 - \dfrac{2\alpha_2\rho_1}{\rho} & 2\lambda_2 + 3\mu_2 - \lambda_5 + \dfrac{2\alpha_2\rho_1}{\rho} \end{pmatrix}.$$

Let's take the isometric coordinate system on the sphere as follows

$$x = tg\frac{\vartheta}{2}cos\varphi, \qquad y = tg\frac{\vartheta}{2}sin\varphi, \tag{3.*}$$

where φ, θ are geographical coordinate of the point of sphere.

$$\dot{\Lambda} = \frac{4R^2}{(1+x^2+y^2)^2} = \frac{4R^2}{(1+z\bar{z})^2} = 4R^2 cos^4 \frac{\vartheta}{2},$$

where

$$z = tg\frac{\vartheta}{2}e^{i\varphi} = x + iy.$$

Consider the homogeneous case $X_+ = X_3 = 0$.

Let action on the equation (3.4) the operator $\dot{\Lambda}^{-1}\partial_z$. After considering the real part of obtained expression, and taking into account (3.6) we get

$$4M\dot{\Lambda}^{-1}\partial_z\dot{\Lambda}^{-1}\partial_z\dot{\Lambda}\partial_{\bar{z}}\dot{\Lambda}^{-1}U_+ + 4M\dot{\Lambda}^{-1}\partial_{\bar{z}}\dot{\Lambda}^{-1}\partial_{\bar{z}}\dot{\Lambda}\partial_z\dot{\Lambda}^{-1}\overline{U}_+$$
$$+2\dot{\Lambda}^{-1}\partial^2_{z\bar{z}}(P^1_1 + P^2_2) + \frac{1}{R}\dot{\Lambda}^{-1}(\partial_z P_+ + \partial_{\bar{z}}\overline{P}_+) = 0. \qquad (3.10)$$

Here we use the following well-know formula (see [6])

$$\dot{\Lambda}^{-1}\partial_z\dot{\Lambda}^{-1}\partial_z\dot{\Lambda}\partial_{\bar{z}}\dot{\Lambda}^{-1}U_+ = \frac{1}{4}\left(\nabla^2 + \frac{2}{R^2}\right)\dot{\Lambda}^{-1}\partial_z U_+, \qquad (3.11)$$

where ∇^2-is Laplacian on the sphere

$$\nabla^2 = \dot{\Lambda}^{-1}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 4\dot{\Lambda}^{-1}\partial^2_{z\bar{z}}.$$

Taking into account of (3.5) and (3.11) we can rewrite (3.10) in the following form

$$\left(\nabla^2 + \frac{2}{R^2}\right)\left(M\theta + \frac{1}{2}(P^1_1 + P^2_2)\right) = 0. \qquad (3.12)$$

From (3.7) we get

$$P^1_1 + P^2_2 = 2(B - \lambda_5 S)\left(\theta + \frac{2}{R}U_3\right). \qquad (3.13)$$

After substituting (3.13) into (3.12) we obtain

$$\left(\nabla^2 + \frac{2}{R^2}\right)\left((A+B)\theta + \frac{2}{R}(B - \lambda_5 S)U_3\right) = 0. \qquad (3.14)$$

The general solution of (3.14) has the following form

$$(A+B)\theta + \frac{2}{R}(B - \lambda_5 S)U_3 = D\omega_1, \qquad (3.15)$$

where $\omega_1 = (\omega_1^{'},\ \omega_1^{''})^T$ is the general solution of corresponding homogeneous equation, D is any non-degenerate matrix 2×2,

$$\omega_1 = [\varphi(z) + \overline{\varphi(z)}]\frac{1-|z|^2}{1+|z|^2} + z\varphi^{'}(z) + \bar{z}\overline{\varphi^{'}(z)},$$

$\varphi(z) = (\varphi_1(z), \varphi_2(z))^T$ is the column-matrix composed analytic function of variable z.

If we substitute (3.8) and (3.13) into (3.5) we get

$$A\nabla^2 U_3 - \frac{4}{R^2}(\Lambda + M)U_3 - \frac{1}{R}\widehat{A}\theta = 0. \tag{3.16}$$

$det(A + B) > 0$, hence from (3.15) we obtain

$$\theta = -\frac{2}{R}(A + B)^{-1}(\Lambda + M)U_3 + (A + B)^{-1}D\omega_1. \tag{3.17}$$

After substituting (3.17) into (3.16) we have

$$\nabla^2 U_3 - \frac{2}{R^2}A^{-1}(2I - \widehat{A}(A+B)^{-1})(\Lambda+M)U_3 = \frac{1}{R}A^{-1}\widehat{A}(A+B)^{-1}D\omega_1. \tag{3.18}$$

We imply that matrix $\widetilde{A} := A^{-1}(2I - \widehat{A}(A+B)^{-1})(\Lambda+M)$ has simple proper numbers æ$_1$ and æ$_2$. Then the general solution of (3.18) can be written as follows

$$U_3 = \omega_1 + \omega_2, \tag{3.19}$$

where $\omega_2 = L\chi$, $\chi = (\chi_1, \chi_2)^T$, χ_1 and χ_2 are the general solutions of following Helmholtz's equations

$$\nabla^2\chi_1 - \frac{2æ_1}{R^2}\chi_1 = 0, \quad \nabla^2\chi_2 - \frac{2æ_2}{R^2}\chi_2 = 0,$$

L is matriz 2×2, which column are coresponding proper vector of proper numbers æ$_1$ and æ$_2$,

$$D = -\frac{2}{R}(A + B)\widehat{A}^{-1}A[A^{-1}(2I - \widehat{A}(A + B)^{-1})(\Lambda + M) + I].$$

By substituting (3.19) into (3.17) we get

$$\theta = A_1\omega_1 + A_2\omega_2, \tag{3.20}$$

where

$$A_1 = (A + B)^{-1}\left(D - \frac{2}{R}(\Lambda + M)\right), \quad A_2 = -\frac{2}{R}(A + B)^{-1}(\Lambda + M).$$

From (3.20) we have

$$\partial_z U_+ + \partial_{\bar z}\overline{U_+} = -2R^2\partial^2_{z\bar z}(A_1\omega_1 - A_2\widetilde{A}^{-1}\omega_2). \tag{3.21}$$

From (3.21) the function U_+ have the following form

$$U_+ = -R^2\partial_{\bar z}\left\{A_1\omega_1 - A_2\widetilde{A}^{-1}\omega_2 + iv\right\},$$

where $v = (v_1,\ v_2)^T$ are column matrix composed the unknown real functions,

$$U_+ = \partial_{\bar z} V; \tag{3.22}$$

$$V = (V', V'')^T = -R^2\left\{A_1\omega_1 - A_2\widetilde{A}^{-1}\omega_2 + iv\right\}. \tag{3.23}$$

The function v must choose such that it satisfys equations (3.4). Let's Substitute (3.22) into (3.6)-(3.8) and then let substitute obtained relations into (3.4). Taking into account the following well-known formula

$$\dot{\Lambda}^{\ -1}\partial_z\,\dot{\Lambda}\ \partial_{\bar z}\,\dot{\Lambda}^{\ -1}\partial_{\bar z} u = \frac{1}{4}\partial_{\bar z}\left(\nabla^2 + \frac{2}{R^2}\right)u,$$

we get

$$\partial_{\bar z}\left[A\nabla^2 V + \frac{B-\Lambda}{R^2}V + \frac{B}{2}\nabla^2(V+\overline{V}) + \frac{2\widehat{A}}{R^2}U_3\right] = 0.$$

From which we have

$$A\nabla^2 V + \frac{B-\Lambda}{R^2}V + \frac{B}{2}\nabla^2(V+\overline{V}) + \frac{2\widehat{A}}{R^2}U_3 = \psi(z), \tag{3.24}$$

where $\psi(z) = (\psi_1(z),\ \psi_2(z))^T$ is the column-matrix composed with the analytic function of variable z. If we substitute (3.23) into (3.24) we get that the left-hand side of the last expression is imaginary i.e. $\psi(z) = i(c_1,\,c_2)^T = = iC$ and we get

$$A\nabla^2 v + \frac{1}{R^2}(B-\Lambda)v = C.$$

But v_1 and v_2 are define with exachness any constant hence we can write $C = 0$ and we have

$$\nabla^2 v + \frac{1}{R^2}A^{-1}(B-\Lambda)v = 0.$$

The general solution of last equation has the following form

$$v = L_0\chi_0,$$

where $\chi_0 = (\chi_0^1,\ \chi_0^2)^T$, χ_0^α is the general solution of the following Helmholtz's equation

$$\nabla^2\chi_0^\alpha + \frac{æ_0^\alpha}{R^2}\chi_0^\alpha = 0,$$

$æ_0^1,\ æ_0^2$ are the proper numbers of the matrix $A^{-1}(B-\Lambda)$, the column of matrix L_0 are the corresponding proper-vector of values $æ_0^1,\ æ_0^2$.

Hence the general solutions of the homogeneous equations systems spherical shell composed the binary mixture in case of approximation $N = 0$ have the form

$$U_3 = \omega_1 + \omega_2 = \omega_1 + L\chi,$$

$$U_+ = -R^2\partial_{\bar z}(A_1\omega_1 - A_2LK\chi - iL_0\chi_0),$$

where

$$K := \begin{pmatrix} \frac{1}{æ_1} & 0 \\ 0 & \frac{1}{æ_2} \end{pmatrix}.$$

If substitute (3.5) (3.26) into (3.6)-(3.9) we get the complex combination of stress tensor.

Thus the solution may be written by six analytic functions of complex variables $z = tg\frac{\theta}{2}e^{i\varphi}$, (the integral representation by analytic function for metaharmonic equation see I. Vekua [8]). Hence we can satisfy six independent boundary conditions.

4. Strongly shallow spherical shell. Let's consider the case when in the (3.∗) formula the angle ϑ changes in the small interval beside of zero, we mean

$$\cos\vartheta \cong 1, \quad \sin\vartheta \cong 0,$$

from which we get

$$\dot\Lambda \cong R^2,$$

then we can assume that x and y are Cartesian coordinate, besides in the equation system (a) we can neglect the terms which contain R^{-2} with item. As a result we get simplity system of equations.

The system of equations in the complex form when right-hand side is 0 has the following form

$$\begin{cases} A\Delta U_+ + 2B\partial_{\bar z}\theta + \dfrac{2}{R}\widehat{A}\partial_{\bar z}U_3 = 0, \\ A\Delta U_3 - \dfrac{1}{R}\widehat{A}\theta = 0. \end{cases} \qquad (b)$$

After integration the first equation of (b) we get

$$2A\partial_z U_+ + B\theta + \frac{\widehat{A}}{R}U_3 = 2A\varphi'(z), \qquad (4.1)$$

where $\varphi(z) = (\varphi_1(z), \varphi_2(z))^T$ is the column-matrix composed with any analytic functions. If we look for the real part of both side of (4.1) we get ($\theta = \partial_z U_+ + \partial_{\bar z}\overline{U_+}$)

$$(A+B)\theta + \frac{\widehat{A}}{R}U_3 = A(\varphi'(z) + \overline{\varphi'(z)}).$$

From the second equation of (b) we have

$$\theta = R\widehat{A}^{-1}A\Delta U_3. \qquad (4.2)$$

By substituting (4.2) into (4.1) we obtain

$$\Delta U_3 + \frac{1}{R^2}A^{-1}\widehat{A}(A+B)^{-1}\widehat{A}U_3 = \frac{1}{R}A^{-1}\widehat{A}(A+B)^{-1}A(\varphi'(z) + \overline{\varphi'(z)}). \quad (4.3)$$

We take the following notation

$$\widetilde{A} := A^{-1}\widehat{A}(A+B)^{-1}\widehat{A}.$$

The general solution of (4.3) has the form

$$U_3 = R\widehat{A}^{-1}A(\varphi^{'}(z) + \overline{\varphi'(z)}) + \mathcal{L}\chi(z,\bar{z}), \tag{4.4}$$

where $\chi = (\chi_1, \chi_2)^T$,

$$\Delta\chi_\alpha + \frac{æ_\alpha}{R^2}\chi_\alpha = 0,$$

the column of matrix $\mathcal{L}$ are the proper numbers $æ_1$ and $æ_2$ of $\widetilde{A}$ coresponding proper vector.

From (4.4) and (4.2) we get

$$\theta = -\frac{1}{R}(A+B)^{-1}\widehat{A}\mathcal{L}\chi(z,\bar{z}). \tag{4.5}$$

By substituting (4.4), (4.5) into (4.1) and by integrating obtained formula we get

$$2U_+ = \varphi(z) - z\overline{\varphi'(z)} - \psi(z) + 4R\breve{A}L\partial_{\bar{z}}\chi(z,\bar{z}),$$

where $\psi(z) = (\psi_1(z), \psi_2(z))^T$ are any analytic functions,

$$\breve{A} := A^{-1}(I - B(A+B)^{-1})\widehat{A}\widetilde{A}^{-1}.$$

Thus the general solution of system (b) may be written by four any analytic functions and two Helmholtz's equations solutions.

Let the radius R is sufficiently large number and the term partial differential of U_3 throw we obtain

$$\begin{cases} A\Delta U_+ + 2B\partial_{\bar{z}}\theta = 0, \\ A\Delta U_3 - \dfrac{1}{R}\widehat{A}\theta = 0. \end{cases} \tag{c}$$

The first equation of system (c) independent from second equation and coincide to the plane strain equations. The general solution of (c) have the form as follows (see [9],[10],[11])

$$2U_+ = A^*\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)},$$

$$U_3 = \frac{1}{4R}A^{-1}\widehat{A}B^{-1}A(\bar{z}\varphi(z) + z\overline{\varphi(z)}) + g(z) + \overline{g(z)},$$

where $A^* := I + 2B^{-1}A$; $\varphi(z) = (\varphi_1(z), \varphi_2(z))^T$, $\psi(z) = (\psi_1(z), \psi_2(z))^T$, $g(z) = (g_1(z),\ g_2(z))^T$ are the column-matrixs composed any analytic functions.

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