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ON DISCLOSING THE LOCATION OF ACCIDENTAL GAS ESCAPE FROM THE COMPLICATED MAIN PIPE-LINE WITH NON-STATIONARY FLOW AND DETERMINING THE INTENSITY OF THE ESCAPE

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Abstract

Linear mathematical model of gas flow in the main pipe-line is considered. The method of disclosing the location of accidental gas escape from the complicated main with non-stationary gas flow and determining the intensity of the escape is described and appropriate formulas are received

The efficiency of the presented method is illustrated on a test example .

Key words and phrases: boundary condition, pipe-line, gas flow, accidental escape .

AMS subject classification: 76N15, 35K20.

The solution of the problem of disclosing the location of an accidental gas escape from the main pipe-line is known not only for simple pipe-line [1] but also for the complicated one [2] with stationary flow. The problem (with determining the intensity of the escape) is solved also with non-stationary flow for simple pipe-line [3]. In the presented work off-shoots from the main pipe-line are taken into account.

Distribution of the gas pressure along the main $u = u(x,t)$ is described by the following equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + \sum_{k=1}^{m} M_k \delta(x - x_k) + q\delta(x - x^*)\sigma(t - t_0), \quad 0 < x < L \;\; t > 0 \quad (1)$$

where $a^2 = const > 0, m$ is number of off-shoots, $M_k(k = \overline{1,m})$- expenses of gas in the off-shoots, $x_k(k = \overline{1,m})$-coordinates of points of off-shoots, q-coordinate of the point of escape, t_0- the moment of the begining of the escape, $\delta(.)$-Dirac function, $\delta(.)$-Heaviside function [1], L- the length of the main x^* is considered the point of off-shoot.

With equation (1) let us consider the following initial and boundary conditions:

$$u(x,0) = Q(x), \quad 0 \leq x \leq L, \quad (2)$$

$$\frac{\partial u}{\partial x} = g_1 \quad x = 0, \quad t > 0, \tag{3}$$

$$\frac{\partial u}{\partial x} = g_2 \quad x = L, \quad t > 0, \tag{4}$$

where $Q(x)$ is the initial distribution of pressure, which can be represented in the following form:

$$Q(x) = \frac{S - R}{L} x + R.$$

Here R and S $(R \geq S)$ are gas pressures at the begining and ending of the main, respectively, at the initial moment.

Sought for values are: coordinate of the escape location x^* and intensity of the escape q. Therefore two additional conditions are needed. For this purpose we will use the values $\overline{R}$ an $\overline{S}$ received by measuring pressure at the begining and ending of the main at moment of time T:

$$u(0, T) = \overline{R}, \tag{5}$$

$$u(L, T) = \overline{S}. \tag{6}$$

It is known [4] that the solution of the problem (1)-(4) can be represented by the following formula:

$$u(x, t) = \int_0^t \int_0^L G(x, \xi, t - \tau) \omega(\xi, \tau) d\xi d\tau, \tag{7}$$

where G is Green function

$$G(x, \xi, t - \tau) = \frac{1}{L} \left[1 + 2 \sum_{n=1}^{\infty} cos \frac{n\pi x}{L} cos \frac{n\pi \xi}{L} e^{-\frac{n^2 a^2 \pi^2}{L^2}(t-\tau)} \right].$$

Here

$$\omega(x, t) = Q(x)\delta(t) - a^2\delta(x) g_1 + a^2 \delta(L - x) g_2 + \sum_{k=1}^{m} M_k \delta(x - x_k) + q\delta(x - x^*)\sigma(t - t_0).$$

Taking into account the last equality, formula (7) will have the following form:

$$u = \int_0^L G(x, \xi, t) Q(\xi) d\xi - a^2 g_1 \int_0^t G(x, 0, t - \tau) d\tau + a^2 g_2 \int_0^t G(x, L, t - \tau) d\tau$$

$$+ \sum_{k=1}^{\infty} M_k \int_0^t G(x, x_k, t - \tau) d\tau + q \int_{t_0}^t G(x, x^*, t - \tau) d\tau \tag{8}$$

If we retain only the first item in the expression of Green function, taking into account formula (8), equalities (5),(6) will be rewritten as follows:

$$\overline{R} = \frac{2q}{LP}\left(1 - e^{-P(T-t_0)}\right) cos\frac{\pi x^*}{L} + \frac{T-t_0}{L}q - \frac{a^2 g_1}{L}\left[T + \frac{2}{P}(1-e^{-PT})\right]$$
$$+\frac{a^2 g_2}{L}\left[T - \frac{2}{P}\left(1-e^{-PT}\right)\right] + \frac{R+S}{2} + \frac{4(R-S)}{\pi^2}e^{-PT}$$
$$+\frac{2}{LP}\left(1-e^{-PT}\right)\sum_{k=1}^{m} M_k cos\frac{\pi x_k}{L} + \frac{T}{L}\sum_{k=1}^{m} M_k, \quad (9)$$

$$\overline{S} = -\frac{2q}{LP}\left(1 - e^{-P(T-t_0)}\right) cos\frac{\pi x^*}{L} + \frac{T-t_0}{L}q - \frac{a^2 g_1}{L}\left[T + \frac{2}{P}(e^{-PT}-1)\right]$$
$$+\frac{a^2 g_2}{L}\left[T + \frac{2}{P}\left(1-e^{-PT}\right)\right] + \frac{R+S}{2} - \frac{4(R-S)}{\pi^2}e^{-PT}$$
$$-\frac{2}{LP}\left(1-e^{-PT}\right)\sum_{k=1}^{m} M_k cos\frac{\pi x_k}{L} + \frac{T}{L}\sum_{k=1}^{m} M_k, \quad (10)$$

where $P = a^2\pi^2/L^2$.

Equalities (9) and (10) form the system of equations with two unkown values:x^* and q. Summing up these equations item by item we will have:

$$\overline{R} + \overline{S} = \frac{2(T-t_0)}{L}q - \frac{2a^2T}{L}(g_1 - g_2) + R + S + \frac{2T}{L}\sum_{k=1}^{m} M_k.$$

From this

$$q = \frac{L(\overline{R} + \overline{S} - R - S) + 2a^2T(g_1-g_2) - 2T\sum_{k=1}^{m} M_k}{2(T-t_0)}. \quad (11)$$

In order to calculate x^*, equations (9) or (10) might be used. In particular, from equation (10) we will have:

$$x^* = \frac{L}{\pi}arccos\left\{\frac{1}{2q(1-e^{-P(T-t_0)})}\left[P(T-t_0)q + \frac{LP}{2}(R+S)\right.\right.$$
$$-LP\overline{S} + \frac{4LP(R-S)}{\pi^2}e^{-PT} + PT\sum_{k=1}^{m} M_k + Pa^2T(g_2-g_1)$$
$$\left.\left.+2\left(a^2 g_1 + a^2 g_2 - \sum_{k=1}^{m} M_k cos\frac{\pi x_k}{L}\right)\left(1-e^{-PT}\right)\right]\right\}, \quad (12)$$

where q is determined by formula (11).

To illustrate the presented method, let us consider a problem, the solution of which is known beforehand. Suppose, the endings of the pipe-line are closed.

$$g_1 = g_1 = 0 \tag{13}$$

and initial pressure is the same along the pipe-line:

$$S = R. \tag{14}$$

Also, let us assume,that there are two off-shoots at points located at the equal distance from the begining and ending of the main:

$$x_1 = \frac{1}{4}L, \quad x_2 = \frac{3}{4}L \tag{15}$$

and with equal intensity:

$$M_1 = M_2 = M. \tag{16}$$

Apart from the above said, suppose at a T moment if time, after measuring pressure, it was found that pressures at the begining and ending of the main are the same:

$$\overline{S} = \overline{R}. \tag{17}$$

It is obvious that under such conditions the place of gas escape will be the middle point of the main. Indeed, taking into account (13)-(17) in formulas (11),(12) we will have:

$$q = \frac{2L(\overline{R} - R) - 4TM}{2(T - t_0)}, \tag{18}$$

$$x^* = \frac{L}{\pi} arccos \left\{ \frac{T - t_0}{\left[2L(\overline{R} - R) - 4TM\right](1 - e^{-P(T-t_0)})} \left[PL(\overline{R} - R) \right.\right.$$

$$\left.\left. -2PTM + LPR - LP\overline{R} + 2PTM - 2M(cos\frac{\pi}{4} + cos\frac{3\pi}{4})\left(1 - e^{-PT}\right)\right]\right\}$$

$$= \frac{L}{\pi} arccos \left\{ \frac{T - t_0}{\left[2L(\overline{R} - R) - 4TM\right](1 - e^{-P(T-t_0)})} \cdot 0 \right\}$$

$$= \frac{L}{\pi} arccos 0 = \frac{L}{\pi} \cdot \frac{\pi}{2} = \frac{L}{2}.$$

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