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ON DISCLOSING THE LOCATION OF ON ACCIDENTAL GAS ESCAPE FROM THE MAIN PIPE-LINE WITH NON-STATIONARY FLOW

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Abstract

Under the stationary gas flow process in the pipe-line, which is described by the second order linear parabolic equation with pertial derivatives, by analyze of boundary conditions, is described a method and abtained a formula to establish a place of gas accidental escape. Effectivenes of obtained formula is shown on the test escample.

Key words and phrases: Parabolic equation, boundary condition, pipe-line, gas flow, accidental escape.

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It is well-known that with stationary gas consumption the flow of gas is stationary as well. From the moment of accidental gas escape a non-stationary process begins. After a certain period of time a new stationary situation is established. Under stationary situation the problem of disclosing the location of accidental gas escape is solved not only fer a simple pipe-line [1], but also for a one with branches [2]. In order to reduce the loss of gas, as well as for ecological reason, it is important to disclose the location of accidental gas escape while the process is still non-stationary.

Distribution of gas preassure $u = u(x,t)$ along the gas main is described by the following equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \delta(x - x_0) \cdot q, \quad 0 < x < L, \ t > 0, \tag{1}$$

where $a^2 = const$ is a known value, L-the length of the gas main, $\delta(.)$-the Dirac function, x_0 is a coordinate of the location of gas escape which is to be found, q-the intensity of the escape $(q < 0)$ [1].

For equation (1) let us consider the following initial and boundary conditions.

$$u(x,0) = Q(x), \quad 0 \leq x \leq L, \tag{2}$$

$$\frac{\partial u}{\partial x} = g_1, \quad x = 0, \quad t > 0, \tag{3}$$

$$\frac{\partial u}{\partial x} = g_2, \quad x = L, \quad t > 0, \tag{4}$$

where $Q(x)$ is the initial distribution of preassure which can be represented in the following form:

$$Q(x) = \frac{S - R}{L} x + R.$$

Here R and S are gas preassures respectively the begining and ending of the main pipe-line before the moment of gas accidental escape ($R > S$), g_1 and g_2 are known constant values.

Note that because of the consumption of the gas accumulated in the pipe-line, the intensity of escape q can not be represented as the difference betwecn gas supply at the begining of the main-g_1 and gas consumption at the end of the main-g_2. As a result, we have to search not only for coordinatc of the location of the gas escape, but also for escape intensity q. Becanse of the above said, two more additional conditions are needed. For this purpose we will use the values obtained by measuring preassure at the begining and ending of the main. In particular, assuming that from the moment of escape for T period of time at the begining of the gas main the preassure fell from R to $\bar{R}$ and at the end of the main- from S to $\bar{S}$, we will receive the following additional conditions :

$$u(0, T) = \bar{R} \tag{5}$$

$$u(L, T) = \bar{S} \tag{6}$$

It is known [3] that the solution of problem (1)-(4) may be represented by the following formula:

$$u(x, t) = \int_0^t \int_0^L G(x, \xi, t - \tau) \omega(\xi, \tau) d\xi d\tau, \tag{7}$$

where G is Green's Function

$$G(x, \xi, t - \tau) = \frac{1}{L}[1 + 2 \sum_{n=1}^{\infty} cos \frac{n\pi x}{L} \cdot cos \frac{n\pi \xi}{L} \cdot \exp^{-\frac{n^2 a^2 \pi^2}{L^2} \cdot (t - \tau)}].$$

Here

$$\omega(x, t) = \delta(x - x_0) q + Q(x) \delta(t) - a^2 \delta(x) g_1 + a^2 \delta(L - x) g_2.$$

Taking into account the latter equation, formula (7) will be transformed in this way

$$u = q \int_0^t G(x, x_0, t - \tau) d\tau + \int_0^L G(x, \xi, t) d\xi -$$

$$-a^2 g_1 \int_0^t G(x,0,t-\tau)d\tau + a^2 g_2 \int_0^t G(x,L,t-\tau)d\tau. \tag{8}$$

In the representation of Green's Function if we retain just the first component then, in view of formula (8), equations (5),(6) can be rewritten in the following form:

$$\bar{R} = \frac{2q}{Lp}(1-\exp^{-pT})cos\frac{\pi x_0}{L} + \frac{qT}{L} - \frac{a^2 g_1}{L}[T + \frac{2}{p}(1-\exp^{-pT})] +$$

$$+\frac{a^2 g_2}{L}[T - \frac{2}{p}(1-\exp^{-pT})] + \frac{R+S}{2} + \frac{4(R-S)}{\pi^2}\cdot \exp^{-pT}, \tag{9}$$

$$\bar{S} = -\frac{2q}{Lp}(1-\exp^{-pT})cos\frac{\pi x_0}{L} + \frac{qT}{L} - -\frac{a^2 g_1}{L}[T + \frac{2}{p}(\exp^{-pT} - 1)] +$$

$$+\frac{a^2 g_2}{L}[T + \frac{2}{p}(1-\exp^{-pT})] + \frac{R+S}{2} - \frac{4(R-S)}{\pi^2}\exp^{-pT}, \tag{10}$$

where $p = a^2\pi^2/L^2$. Equations (9) and (10) form the system of equations with two unknown values x_0 and q.

If we sum equations (9) and (10) member by member we will receive:

$$\bar{R} + \bar{S} = \frac{2qT}{L} - \frac{2a^2 T}{L}(g_1 - g_2) + R + S.$$

From this equation

$$q = \frac{L(\bar{R}+\bar{S}) + 2a^2 T(g_1 - g_2) - L(R+S)}{2T}. \tag{11}$$

To calculate the sought for x_0 we can use equation (9) or (10). In particular, from equation (10) we have:

$$x_0 = \frac{L}{\pi}arccos\{\frac{a^2(g_1+g_2)}{q} + \frac{1}{2q(1-\exp^{-pT})}[\frac{pL(R+S)}{2} +$$

$$+pT(q - a^2 g_1 + a^2 g_2) - pL\bar{S} - \frac{4pL(R-S)}{\pi^2}\exp^{-pT}]\}, \tag{12}$$

where q is determined by formula (11).

Note that the presented method can be used not only for new pipe-lines, but also to discover the escape location while testing old pipe-lines after restoration. For this purpose it is enough to assume $g_1 = g_2 = 0$ in formulae (11) and (12).

To demonstrate the presented method let us consider one example, solution of which is known beforehand. In particular, suppose the ends of the main are closed

$$g_1 = g_2 = 0 \tag{13}$$

and initial preassure is constant along the pipe-line

$$S = R. \tag{14}$$

Besides, suppose after certain period of time T (from the moment of gas escape) we found that at the ends of the main the preassures fell to the same values:

$$\bar{S} = \bar{R} \tag{15}$$

If is obvious that under such conditions the location of gas escape will be the middle point of the main. In fact, taking into account formulae (11),(12) in (13)-(15), we will receive;

$$q = \frac{L(\bar{R} - R)}{T}, \tag{16}$$

$$x_0 = \frac{L}{\pi} arccos[\frac{1}{2q(1 - \exp^{-pT})}(pLR + pTq - pL\bar{R})].$$

In view of equation (16), the last formula will take the form

$$x_0 = \frac{L}{\pi} arccos[\frac{T}{2L(\bar{R} - R)(1 - \exp^{-pT})}(pLR + pL(\bar{R} - R) - pL\bar{R})],$$

$$x_0 = \frac{L}{\pi} arccos 0 = \frac{L}{\pi} \cdot \frac{\pi}{2} = \frac{L}{2}.$$

R E F E R E N C E S

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