ISAAC Conference, 23-27 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I. Vekua

EFFICIENT BOUNDARY ELEMENT METHOD IN SOLID MECHANICS

Wendland W.[1]***, Hsiao G.***[2]***, Of G.***[3]***, Steinbach O.***[3]

[1] Univ. Stuttgart and Babes–Bolyai Univ.Cluj–Napoca
[2]Univ. of Delaware, [3]TU Graz,

Boundary integral methods for existence and regularity analysis of elliptic boundary value problems have a long history which goes back to the basic ideas of Green, Gauss and Poincaré in the 19th century. Till these days they serve as decisive tools for intrinsic analysis of elliptic problems. Their numerical employment began during the 50s of the last century when electronic computation opened the opportunity of numerical simulation of larger complexity.

The reformulation of three–dimensional elliptic boundary value problems in terms of the nonlocal boundary integral equations reduces interior as well as exterior problems to equations on the two–dimensional boundary. A finite element approximation of the boundary charges requires the triangulation of the boundary surface only which is a great advantage in practice. The Galerkin (as well as collocation) discretization of the boundary integral equations results in large systems of equations involving fully populated matrices which was rather disadvantageous up to about 30 years ago. More recently, however, the combination of degenerate kernel and low rank approximation with a hierarchical structure of the boundary triangulation has lead to a drastic improvement of the performance of boundary element methods. For boundary charges with N degrees of freedom now the matrix times vector product can be executed with $N \log^2 N$ complexity. With appropriate preconditioning, nowadays large, enormous systems can be solved very efficiently. In combination with nonoverlapping domain decomposition, boundary element methods became a very efficient simulation instrument with a wide variety of industrial applications.