THE FOURTH ORDER OF ACCURACY DECOMPOSITION SCHEME FOR ABSTRACT HYPERBOLIC EQUATION

Let us consider the Cauchy problem for abstract hyperbolic equation in the Hilbert space H:

$$\frac{d^2u(t)}{dt^2} + Au\left(t\right) = 0, \quad t \in \left[0, T\right], \tag{1}$$

$$u\left(0\right) = \varphi_0, \quad \frac{du\left(0\right)}{dt} = \varphi_1. \tag{2}$$

where A is a self-adjoined, positively defined (generally unbounded) operator with the definition domain $D\left(A\right)$, which is everywhere dense in H.

Let $A = A_1 + A_2$, where A_1, A_2 are self-adjoined, positively defined operators.

Let us divide the interval $[0, T]$ into $n\left(> 1\right)$ equal parts and define division points by t_k, $t_k = k\tau$, $n = 1, ..., n$, $\tau = T/n$. As it is known solution of the problem (1)-(2) satisfies the following recurrent relations:

$$u(t_{k+1}) = 2\cos\left(\tau A^{1/2}\right)u(t_k) - u\left(t_{k-1}\right).$$

Using this formula, let us construct the following decomposition scheme:

$$u_{k+1} = V\left(\tau\right)u_k - u_{k-1}, \quad k = 1, ..., n-1, \tag{3}$$

$$u_0 = \varphi_0, \quad u_1 = \frac{1}{2}\left(V\left(\tau\right)\varphi_0 + \tau V\left(\frac{\tau}{\sqrt{3}}\right)\varphi_1\right), \tag{4}$$

where

$$V\left(\tau\right) = V_0\left(\tau; A_1, A_2\right) + V_0\left(\tau; A_2, A_1\right), \tag{5}$$

$$V_0\left(\tau; A_1, A_2\right) = \left(I + \alpha\tau^2 A_1\right)^{-1}\left(I + \lambda\tau^2 A_2\right)^{-1}\left(I + \overline{\alpha}\tau^2 A_1\right)^{-1},$$

where $\lambda = \frac{1}{2} \pm \frac{1}{\sqrt{6}}$, $\quad \alpha = \frac{1-\lambda}{2} \pm i\frac{\sqrt{3-(1-\lambda)^2}}{2}$, $\overline{\alpha}$ is a conjugate of α.

We declare function u_k as an approximation of $u\left(t\right)$ in $t = t_k$ node.

It is proved that the decomposition scheme (3)-(4) is stable and the error of the solution obtained by this scheme is $\left(\tau^4\right)$.