ISAAC Conference, 23-27 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

THE DIRICHLET PROBLEM FOR SECOND ORDER ELLIPTIC EQUATIONS IN A HALF-PLANE

Tovmasyan n.

Let $\mathbf{R}^2_+ = \{(x,y,t)|t>0\}$ be an upper half-plane with the boundary $\mathbf{R}^2$. We consider in $\mathbf{R}^2_+$ the second degree equation:

$$u_{xx} + u_{yy} + u_{tt} - 2au_x - 2bu_y - 2cu_t + ku = 0, \quad (x,y,t) \in \mathbf{R}^2_+, \qquad (1)$$

where a, b, c, k are the real constants. The solution $u(x,y,t)$ of equation (1) is two times continuously differentiable in $\mathbf{R}^2_+$, and continuous up to the boundary function. This function on the boundary $\mathbf{R}^2$ satisfies the Dirichlet condition

$$u(x,y,t) = f(x,y), \quad (x,y) \in \mathbf{R}^2. \qquad (2)$$

The function f is a given continuous on $\mathbf{R}^2$ function.

We obtain in the paper the following results

Theorem 1. *If the function f is bounded on $\mathbf{R}^2$, then the problem (1), (2) is uniquely solvable in a class of bounded in $\mathbf{R}^2_+$ functions if and only if $k \leq 0$.*

Theorem 2. *Let f be the function of polynomial growth on $\mathbf{R}^2$. In this case, the problem (1), (2) is uniquely solvable in a class of functions of polynomial growth on $\mathbf{R}^2_+$ if and only if $k<0$ or $k=0$ and $c>0$.*