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L_p-CONNECTION FOR HOLOMORPHIC PRINCIPAL BUNDLE ON RIEMANN SURFACES AND ITS APPLICATION

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Consider holomorphic principal bundles on the compact Riemann surface with L_p-connection associated from the G_C-value elliptic system $\partial_{\bar{z}}\Phi(z) = A(z)\Phi(z)$, where $A(z)$ is meromorphic G_C-value 1-form on the Riemann surface X and $\Phi : X \to G_C$ unknown function. Here G_C is complexsification of the compact Lie group G. In this assumption we prove

Proposition 1. *The cohomology group* $H^i(CP^1, O(P))$ *and* $H^i(CP^1, A(P))$ *are isomorphic for* $i = 0,1$, *where* $O(P)$ *and* $A(P)$ *respectively are sheaves of holomorphic and generalized analytic sections of principal bundle* $P \to CP^1$ *on the Riemann sphere.*

Proposition 2. *There exists a one-to-one correspondence between the spaces of gauge equivalent* G_C*-value elliptic system and the space of holomorphic structures on the bundle* $P \to X$.

Proposition 3. *The* G_C*-value elliptic system* $\partial_{\bar{z}}\Phi(z) = A(z)\Phi(z)$ *defines a monodromy representation of the fundamental group* $\rho : \pi_1(X - S, z_0) \to G_C$ *and monodromy are given by Chen's iterated integrals* $\rho(\gamma_j) = 1 + \int_{\gamma_j}\Omega + \int_{\gamma_j}\Omega\Omega + \int_{\gamma_j}\Omega\Omega\Omega + \ldots + \ldots$, *where S is set of singular points of the 1-form* $\Omega = \partial_{\bar{z}}\Phi(z)\Phi(z)^{-1}$ *and* γ_j *are generators of* $\pi_1(X - S, z_0)$.

Above results we use for the construction the quantum gates for monodromic quantum computation. In particular, is true the following

Proposition 4. *For the collection of the unitary operators* U_1, $U_2, \ldots U_q$ *which realize the quantum algorithm, there exist a location at the points* $s_1, s_2, \ldots, s_q$ *on the compact Riemann surface* X, *and The* G_C*-value elliptic system* $\partial_{\bar{z}}\Phi(z) = A(z)\Phi(z)$ *with monodromies* $M_1, M_2, \ldots, M_q$ *such that* $U_j = F(M_1, \ldots, M_q)$.