damateba 1. damxmare masala funqcionaluri analizidan

**gansazRvra 1**.M simravles namdvil ricxvTa simravleze *lineali* (an *wrfi­­vi*, an *veqtoruli sivrce­­*, an *wrfivi sistema, an wrfivi garsi*) ewodeba, Tu

a) -sTvis gansazRvrulia  da -sTvis (*R* namdvil ricx­vTa simravlea) gansazRvrulia 

b) sruldeba veqtoruli algebris Semdegi aqsiomebi:







g)  iseTi, rom -sTvis, mas linealis *nulovani elementi* ewodeba;

d) -sTvis,iseTi, rom. aseT v ele­ments *u*-s *Sebrune­bu­li­­* ewodeba. (is aRiniSneba (–*u*)-Ti, xolo *u+*(*–*v) aRiniSneba (*u–*v)-Ti).

cxadia, .

**gansazRvra 2**.M simravles (ara aucileblad lineals) *met­rikuli sivrce­* ewodeba, Tu -sTvis gansaz­Rvru­lia ricxvi, romelsac ewo­de­ba *manZili* (*met­ri­ka­­*) *u* da v-s Soris, ise, rom Sesruldes metrikis Sem­de­gi aqsiomebi:







 (samkuTxedis aqsioma), 

**gansazRvra 3**.M lineals ewodeba *normirebuli*, Tu -sTvis gansazRvru­lia  ricxvi, romelsac *u* ele­mentis *norma* ewodeba, ise, rom Sesruldes Semdegi Tvi­sebebi:







 (samkuTxedis utoloba), .

**SeniSvna 1**. yoveli normirebuli lineali metrikuli siv­rcea metrikiT . marTlac, . metrikis danarCeni Tvise­be­bi cxadia.

**SeniSvna 2**. metrikuli sivrce SeiZleba gavxadoT nor­mi­re­buli, Tu is li­ne­a­lia da metrika isea SemoRebuli, rom

.

aseT SemTxvevaSi norma SeiZleba ganvsazRvroT  tolobiT. marTlac, pirveli Tviseba cxadia;

meore Tvisebis damtkiceba: ;

mesame Tvisebis damtkiceba:

;

meoTxe Tvisebis damtkiceba:

.

**gansazRvra 4**.M linealze gansazRvrulia *skalaruli nam­ravli­*, Tu nebismier wyvils  raime wesiT eTa­na­deba namdvili ricxvi



ise, rom mas hqondes Semdegi Tvisebebi:

1. 

2. -ze;

3. 

4. 

**SeniSvna 3**. vTqvaT, M linealze gansazRvrulia skala­ru­li namravli, maSin



tolobiT SeiZleba SemoviRoT  elementis norma, xolo



tolobiT SeiZleba ganvsazRvroT  elementebs So­ris manZili (metrika).

***hiolderis utoloba[[1]](#footnote-1)\*):***.

*damtkiceba*. skalaruli namravlis pirveli Tvisebis Tanaxmad, -saTvis



saidanac, skalaruli namravlis meoTxe da mesame Tvisebebis Tanaxmad



es ukanaskneli samarTliania nebismieri namdvili -saTvis, rac mxolod maSin SeiZleba, roca -s mimarT kvadratuli sam­wevris diskriminanti



e.i.

.

**SeniSvna 4**. normirebul linealze SeiZleba SemoviRoT ska­la­ruli namravli. amisaTvis aucilebeli da sakmarisia, rom norma ise iyos ganmartebuli, rom sruldebodes e.w. pa­ra­lelogramis piroba (Tviseba)

. (1)

am SemTxvevaSi skalaruli namravli SeiZleba SemoviRoT

 (2)

tolobiT. marTlac, (ix. [47], gv. 152), Tu SemoviRebT aR­ni­S­vnas

 (3)

maSin (2) da (3) tolobebidan gamomdinareobs, rom

 (4)

(1)-dan cxadia, rom

. (5)

(5)-is gaTvaliswinebiT, (4)-dan vRebulobT, rom





 (6)

(4) da (6) tolobebis wevr-wevra naxevarjami gvaZlevs, rom

 (7)

.

(7) gamosaxulebidan (1)-is gaTvaliswinebiT gveqneba, rom

 (8)

(3) da (8)-is Sedarebis Semdeg vaskvniT, rom

 (9)

axla ganvixiloT

 (10)

(9) da (2)-dan gamomdinareobs, rom

 (11)

da



e.i.

 (12)

, sadac Z mTel ricxvTa simravlea, (12) da (9)-is gaTvaliswinebiT vaskvniT, rom





amdenad, nebismieri racionaluri  ricxvisaTvis gveqneba, rom



e.i.  nebismieri racionaluri *c* ricxvisTvis. ramdenadac  funqcia uwyvetia, amitom Tu , maSin  da . amrigad, (10)-dan vaskvniT, rom

 (13)

(9), (13)-dan gamomdinareobs, rom skalaruli namravlis me-4 Tviseba srul­deba. sxva Tvisebebis samarTlianoba cxa­dia.

**gansazRvra** 5. lineals (aRvniSnoT is -iT), romelzec gan­sazRvrulia skalaruli nam­rav­li da amdenad \_ SeniSvna 3-Si miTiTebuli metrika da norma, ewodeba *wi­na­re­hil­ber­tis­­­­­* an *unitaruli sivrce*, an *sivrce skalaruli namrav­liT­*, an *sivrce kvadratuli metrikiT*, an *evklides*[[2]](#footnote-2)\*)*siv­rce­*.

**gansazRvra** 6. Tu , sadac , maSin amboben, rom  krebadia -saken -Si

-Si .

**Teorema 1**. Tu  da , , maSin

.

*Ddamtkiceba* gamomdinareobs hiolderis utolobis gaT­valis­winebiT miRebuli Semdegi utolobidan





.

**Sedegi 1.** Tu  da , maSin 

**Sedegi 2.** Tu  da , maSin , e.i. 

**gansazRvra 7**.-s ewodeba *fundamenturi mimdev­ro­ba­­*-Si, Tu



**gansazRvra 8**. sivrces ewodeba *sruli*, Tu misi yove­li fundamenturi mimdevroba masSi krebadia.

**gansazRvra 9**. srul winarehilbertis sivrces *hil­ber­tis*[[3]](#footnote-3)\*)(*H*)*sivrce­­* ewodeba, xolo srul normirebul siv­rces \_ *banaxis*[[4]](#footnote-4)\*\*)*sivrce*.

**gansazRvra 10**.*M* simravles ewodeba *mkvrivi*-Si, Tu -saTvis *M*-Si moiZebneba misken -Si krebadi mimdevroba.

**gansazRvra 11**.-s ewodeba orTogonaluri, Tu .

**Teorema 2**. Tu  orTogonaluria *H*-Si mkvrivi *M*-is yvela elementisadmi, maSin is *H*-is nulovani elementia.

*damtkiceba* cxadia, radgan  da  da­mo­kidebulebebidan, Sedegi 1-is Tanaxmad, gamomdinareobs, rom , e.i. .

**gansazRvra 12**. Tu mocemulia *X* da *Y* raime simravleebi da wesi, romelic yovel  elements uTanadebs cal­sa­xad gansazRvrul garkveul  elements, maSin vam­bobT, rom mocemulia *A operatori*X gan­saz­Rvri­sa da *Y-Si* mniSvnelobaTa simravleebiT.

im kerZo SemTxvevaSi, roca operatori iRebs ricxviT mniS­vnelobebs, mas *funqcionali* ewodeba.

**gansazRvra 13**. operators ewodeba *wrfivi*, Tu misi gan­saz­Rvris  are linealia da

a)  da ;

b) .

**gansazRvra 14**. funqcionals, sa­dac *A* dadebiTi operatoria  linealze, romelic mkvri­via *H* hilbertis sivrceSi da , ewodeba *kvad­ra­tuli funqcionali­­*, ramdenadac  adgili aqvs



tolobas da amdenad  warmoadgens F funqcionalis kvad­ratul wevrs.

**Teorema 3**. Tu sivrceSi  metrikiT , e.i. , maSin .

**Teorema 4**. Tu fundamenturi mimdevrobaa sivrceSi  metrikiT, maSin is SemosazRvrulia, e.i.  iseTi, rom

.

**gansazRvra 15**.*P* metrikul sivrces ewodeba *separa­be­lu­ri­­­*, Tu arsebobs mis elementTa qvesimravle araumetes Tvla­disa, romelic mkvrivia am sivrceSi.

**Teorema 5**. Tu *A* dadebiTi operatoria (ix. gansazRvra 29),  energetikuli sivrcis (ix. [54], gv. 117) sepa­ra­be­lurobisTvis sakmarisia *H* sivrcis separabeluroba.

**gansazRvra 16**.*H* sivrcis  elementTa mim­dev­robas ewodeba *sruli*, Tu misi wrfivi kombinaciebis sim­ravle mkvrivia am sivrceSi, e.i.  da -sTvis  da iseTi  ricxvebi, rom

.

**gansazRvra 17**. elementebs ewodebaT *wrfi­­vad damokidebuli­­*, Tu erTi maTTagani mainc aris da­nar­Cenebis wrfivi kombinacia. winaaRmdeg SemTxvevaSi maT *wrfi­vad damoukidebeli­* ewodebaT.

**Teorema 6**. elementebis simravle wrfivad damoukidebelia maSin da mxolod maSin, roca maTi *gramis*[[5]](#footnote-5)\*) determinanti

.

**gansazRvra 18**. Tu , sistemis nebis­mie­ri ori elementi orTogonaluria , ma­Sin mas -Si *orTogonaluri sistema* ewodeba, xolo Tu amis garda is normirebulicaa , maSin \_ *orTonormirebuli*.

**gansazRvra 19**.sistemas ewodeba *wrfi­­vad damoukidebeli­­*-Si, Tu misi elementebis nebis­mieri sasruli raodenobisgan Semdgari sistema wrfivad da­mou­kidebelia.

**Teorema 7**. yoveli orTonormirebuli sistema wrfivad damou­kidebelia.

**gansazRvra 20**.*H* sivrceSi srul da wrfivad damou­ki­de­bel sistemas misi *bazisi* ewodeba.

**gansazRvra 21**., mimdevrobas ewo­deba *H*-is *bazisiSauderis*[[6]](#footnote-6)\*)*azriT*, Tu  SeiZleba war­mod­genili iqnas calsaxad

,

saxiT.

**SeniSvna 5**. Tu , sistema orTo­nor­mi­re­bulia, maSin bazisis gansazRvra 20 da gansazRvra 21 erTma­neTs emTxvevian.

**gansazRvra 22**. bazisis elementTa *n* ricxvs ewodeba *H* siv­rcis ganzomileba da aRiniSneba  simboloTi.

**gansazRvra 23**. Tu , orTonor­mi­re­bu­li sistemaa -Si,  da , , ma­Sin  mwkrivs ewodeba *u*-s Sesabamisi *furies mwkri­vi­*, xolo -s \_ *furies koeficientebi*.

**Teorema 8**. Tu , masSi orTonor­mi­re­bu­li sistemaa, maSin  elementis Sesabamisi furies mwkri­vi *H*-Si krebadia da samarTliania *beselis[[7]](#footnote-7)\*)**utoloba*. amasTan aucilebeli da sakmarisi pi­roba misi *u*-sken krebadobisaTvis mdgomareobs *par­se­va­lis[[8]](#footnote-8)\*\*)­­­*



*tolobis* SesrulebaSi.

**gansazRvra 24**. orTonormirebul sistemas ewodeba *H*-Si *Ca­ketili­*, Tu sistemis yvela elementisadmi orTo­go­na­lu­ri elementi mxolod nulovani elementia.

**gansazRvra 25**. orTonormirebul sistemas ewodeba *sru­li­H*-Si, Tu yoveli -sTvis Sesabamisi furies mwkri­vi *H*-Si krebadia *u*-sken, e.i. Tu sruldeba parsevalis to­loba.

**SeniSvna 6**. orTonormirebuli sistemis SemTxvevaSi sis­ru­lis gansazRvra 16 da gansazRvra 25 erTmaneTs emTxve­va.

**Teorema 9.** orTonormirebuli sistema srulia *H*-Si ma­Sin da mxolod maSin, roca is Caketilia masSi.

**Teorema 10.** yovel separabelur *H* sivrceSi arsebobs orTo­nor­mirebuli bazisi, romlis Sesabamisi furies mwkri-vi  elementisTvis *u*-sken krebadia *H*-Si (cxa­dia, Tu *H* separabeluria, masSi bazisi arsebobs da piri­qiT, Tu bazisi arsebobs, is separabeluria, radgan bazisi *H*-Si mkvrivi sistemaa, romelic Tvladze meti araa). Tu *M*mkvri­via separabelur *H*-Si, maSin bazisi SeiZleba aigos *M*-is elementebisgan.

**gansazRvra 26.***B* banaxis sivrceSi gansazRvrul fun­qcio­nals ewodeba *SemosazRvruli*, Tu

.

umciress *C* mudmivebs Soris ewodeba *Lfunqcionalis nor­ma­* da aRiniSneba  simboloTi.

**Teorema 11** (*risis*[[9]](#footnote-9)\*) Teorema). *H* hilbertis sivrceSi yo­­ve­li SemosazRvruli wrfivi *l* funqcionali SeiZleba war­­mo­vad­ginoT

,

skalaruli namravlis saxiT, sadac  fiqsirebuli ele­­menti calsaxadaa gansazRvruli, amasTan .

**Teorema 12** (*han*[[10]](#footnote-10)\*\*)-*banaxis* Teorema). *B* banaxis sivrcis rai­me *M* wrfiv mravalsaxeobaze (magaliTad, linealze) gan­saz­Rvruli yoveli wrfivi SemosazRvruli *l* fun­qcio­na­li SeiZ­leba normis SenarCunebiT ganvavrcoT mTel *B* siv­rce­ze, e.i. SeiZleba avagoT iseTi *L* funqcionali, rom

1); 2).

**gansazRvra 27.** wrfiv normirebul *E* sivrceze gan­saz­Rvru­li  funqcionalebi qmnian banaxis *E\** siv­rces, romelsac ewodeba *E* sivrcis *SeuRlebuli* (*dua­luri*) sivrce*­*.

**gansazRvra 28.** vTqvaT,  operatori asaxavs  wrfiv normirebul sivrces  wrfiv normirebul siv­rce­Si, xolo -ze gansazRvruli wrfivi fun­qcionalia, maSin , sadac  wrfi­vi funqcionali -zea gansazRvruli. amrigad, am to­lobiT avageT raRac  operatori

,

romelic gansazRvrulia -is SeuRlebul  sivrceze da am ukanaskneli sivcis  elementebs uTanadebs -is SeuR­lebuli  sivrcis elementebs. *A\** operators *A* ope­ratoris *SeuRlebuli operatori* ewodeba.

Tu  hilbertis sivrcea, maSin risis Teo­remis Tanaxmad moiZebneba iseTi  elementebi, rom

, ,

e.i.

 da 

toloba yovel *y\**-s uTanadebs erTaderT *x\** elements, e.i.

.

amdenad, *H* hilbertis sivrceSi SeuRlebuli operatori SeiZ­leba ganimartos ukanasknelis wina tolobiT.

kerZod, Tu *A* operatori gansazRvrulia *H*-Si mkvriv  linealze da  iseTi, rom

,

maSin vambobT, rom *y\**-is simravle qmnis SeuRlebuli *A\** operatoris  gansazRvris ares da TviT operatori gani­sazRvreba tolobiT

.

 linealia, xolo *A\** \_ wrfivi operatori. , rad­gan mas yovelTvis ekuTvnis nulovani elementi.

**gansazRvra 29.***H* sivrceSi mkvriv  linealze gan­saz­Rvrul *A* operators ewodeba *simetriuli*-ze, Tu is wrfivia da  sruldeba



toloba. simetriul operators ewodeba *dadebiTi opera­to­ri­­*, Tu



xolo Tu  rom

,

maSin mas -Si *dadebiTad gansazRvruli operatori*ewo­de­ba.

**gansazRvra 30.** Tu *A* simetriuli operatoria, cxadia,  da amdenad  *A\** operatori *A*operatoris *ga­far­Toeba­­a*. Tu , maSin *A\*=A* da mas *TviT­SeuR­le­bu­li operatori­­­­* ewodeba.

**SeniSvna 7.** SemosazRvruli operatorebisTvis simetriu­lo­bisa da TviTSeuRlebulobis ganmartebebi erTmaneTs emTxve­va.

**Teorema 13.** yoveli dadebiTad gansazRvruli operatori SeiZ­leba gavafarTooT TviTSeuRlebul operatorobamde (e.w. *fridrixsis*[[11]](#footnote-11)\*)*gafarToeba*).

**gansazRvra 31.** or *A* da *B* dadebiTad gansazRvrul ope­ra­tors ewodeba *kongruentuli*, Tu .

**gansazRvra 32.** or kongruentul operators ewodeba *mo­­na­Tesave­­­*, Tu  iseTebi, rom

,

sadac *I* erTeulovani operatoria.

**gansazRvra 33.***A* operators ewodeba *SemosazRvruli*, Tu

. (14)

**gansazRvra 34.***A* operators ewodeba *uwyveti*, Tu

.

**Teorema 14.** imisTvis, rom wrfivi operatori iyos uw­yve­­­ti, aucilebeli da sakmarisia, rom igi iyos Semo­saz­Rvru­­li.

**gansazRvra 35.** umciress *M*-ebs Soris (14)-Si ewodeba *A* ope­­ratoris norma: .

**gansazRvra 36.**-Ti aRiniSneba iseTi hilbertis siv­rce, romlis elementebi ekuTvnis-s da aqvT *k* ri­gamde CaTvliT ganzogadebuli warmoebulebi  areSi, romlebic agreTve ekuTvnis-s, da sadac skalaruli nam­ravli mocemulia



tolobiT.

**gansazRvra 37.**-Ti aRiniSneba iseT finitur fun­qcia­Ta (e.i. -Si kompaqturi sayrdenis mqone) simravle, ro­melTac aqvT  areze uwyveti yvela rigis warmoebuli. fun­qciis *sayrdeni*­ ewodeba funqciis gansazRvris wer­til­Ta im simravlis Caketvas, sadac funqcia nulis toli araa.

**Teorema 15.** vTqvaT, wrfivi operatori , sadac  misi mniSvnelobaTa area. maSin Sebrunebuli operatori arsebobs maSin da mxolod maSin, roca iqidan, rom , sadac , sadac .

**Teorema 16.** wrfivi operatoris Sebrunebuli opera­to­ri wrfivia.

**gansazRvra 38.** vTqvaT,  linealia. masSi Semovi­RoT *H*-is metrika, Tu *N* amasTan srulia, maSin mas ewo­de­ba *H* sivrcis *wrfivi qvesivrce*.

**gansazRvra 39**. amboben, rom  elementi  hil­ber­tis sivrcis  qvesivrcis orTogonaluria , Tu -is yvela elementis orTogonaluria.

**Teorema 17.** vTqvaT,  sivrce -is wrfivi qvesivrcea. maSin nebismieri  calsaxad SeiZleba warmovadginoT  saxiT, sadac  da . v-s ewodeba *u*-s *orTogonaluri proeqciaN*-ze.

**SeniSvna 8.** cxadia, Tu , arsebobs erTi mainc  iseTi, rom . winaaRmdeg SemTxvevaSi .

**gansazRvra 40.** amboben, rom  ares aqvs *lip­Si­cis*[[12]](#footnote-12)\*) sazRvari*­­*, Tu:

1.)  SemosazRvrulia (dasaSvebia mravladbmuloba);

2.) , *m*cali, koor­­dinatTa sistema da -ganzomilebian  ku­beb­ze

,

uwyveti , iseTi funqciebi, rom

a.) yovel  wertili SeiZleba warmodgenili iq­nes erT-erTi zemoxsenebul koordinatTa sistemaSi mainc



saxiT;

b.) *x* wertilebi, romelTaTvisac sruldeba

,

da



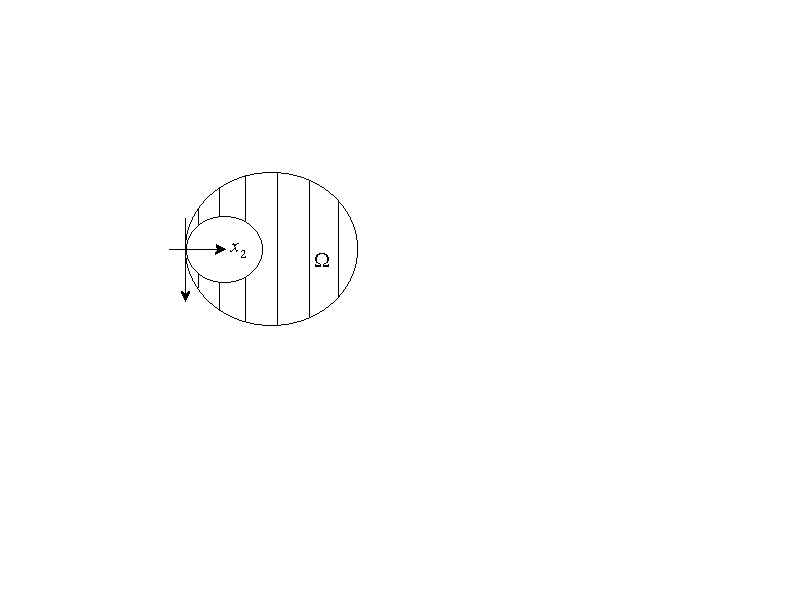
an



pirobebi, Sesabamisad mdebareoben -Si an -s gareT;

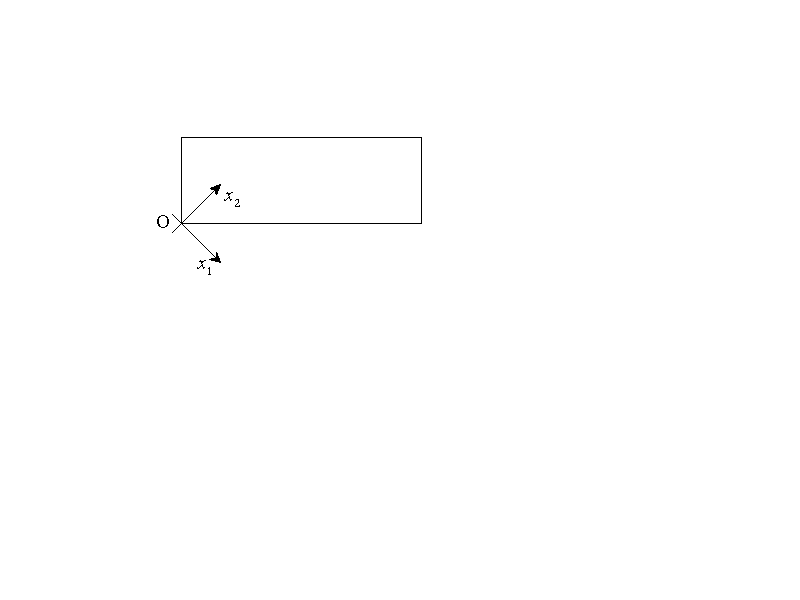
g.) yoveli , , funqcia  kub­ze akmayofilebs lipSicis pirobas, e.i. , rom



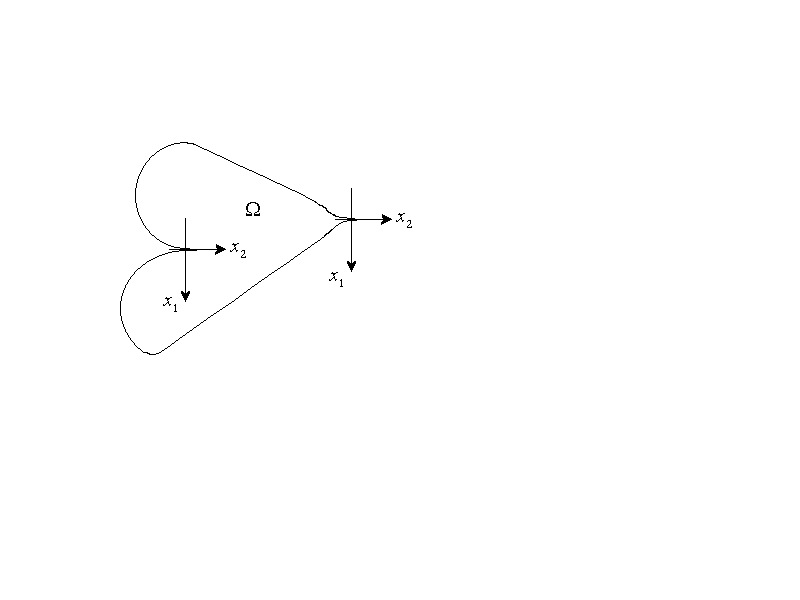
ganvixiloT ramdenime magaliTi. nax. 1-ze mocemul Sem­Txve­­vaSi

nax. 1

RerZis yvela SesaZlo mimarTulebisaTvis darRveulia b.) piroba, e.i. sazRvari ar aris lipSicis.

****nax. 2-ze mocemul SemTxvevaSi, O-s midamoSi , ami­­tom  da sruldeba yvela pi­ro­ba, e.i. lipSicis sazRvaria.

nax. 2

nax. 3-ze mocemul kuTxovani wertilis orive Sem­Txve­va­Si dasaSvebia mxolod miTiTebuli koordinatTa sistema. wi­naaRmdeg SemTxvevaSi dairRveva b.). magram aseTi sistemis SemTxvevaSi aRniSnul wertilebSi , e.i. dairRveva g.), radgan ar iarsebebs . amrigad saz­Rvari lipSicis araa.

nax. 3

**gansazRvra41**.(miT ufro, Tu) funqciis ,  mniSvnelobebi -ze calsaxadaa gansazRvruli. funqcias ewodeba *u(x)funqciiskvali*-ze.

**Teorema 18.** Tu  lipSicis sazRvris mqone area, maSin arsebobs erTaderTi SemosazRvruli wrfivi *T* operatori, romelic  sivrces asaxavs  sivrceze:

.

Tu , maSin .

**gansazRvra 42.** wina Teoremidan-s ewodeba*u(x)* fun­qciis *kvali*­-ze, -s ki *kvalis operatori*.

**SeniSvna 9.** ramdenadac  mkvrivia -Si, Tu  ekuTvnis-s, magram arekuTvnis-s,-s kvali SeiZleba ganvixiloT, rogorc iseT funqciaTa , kvalebis zRvari -Si, romel­TaTvisac

.

**SeniSvna 10.** Tu funqcia uwyvetia -Si, ma­Sin misi kvali -ze emTxveva mis SezRudvas-ze.

**SeniSvna 11.** Tu , magram ar aris kre­badi -Si, maSin  mimdevroba SeiZleba ar iyos krebadi -Si (agebulia magaliTebi (ix. [54], gv. 341)).

**fridrixsis utoloba.** Tu  area lipSicis sazRvriT, maSin arseboben iseTi damo­ki­de­­bulni mxolod areze, rom

.

**puankares utoloba.** Tu  area lipSicis saz­RvriT, maSin arseboben iseTi  damoki­de­bul­ni mxolod areze, rom

.

**fridrixsis utoloba.** Tu  area lipSicis saz­RvriT, maSin arseboben iseTi damokide­bul­ni mxolod areze, rom

,

.

pirveli utoloba ZalaSi rCeba, Tu -s SevcvliT -iT, romelic -s Ria nawilia *lebegis*[[13]](#footnote-13)\*) dadebiTi zo­m­iT (am SemTxvevaSi  mudmivi -zecaa damokidebuli).

kerZod,



da

,

sadac

,

*mul­­tiin­deqsi­­­a*,

,

;

da simboloebi aRniSnaven ajamvas yvela iseTi -saTvis, romlebic Sesabamisad akmayofileben  da  pirobebs.

**puankares utoloba.** Tu  area lipSicis sazRvriT (am pirobis Sesusteba SesaZlebelia, ix. [30]), maSin arsebobs iseTi  damokidebuli mxolod areze, rom



**gansazRvra 43.** kompleqsuri mniSvnelobis mqo­ne funqciaTa simravles ewodeba *Svarcis*[[14]](#footnote-14)\*)*siv­rce­*, Tu nebismieri mTeli arauaryofiTi *k* ricxvisaTvis

,

sadac

,

 multiindeqsia.

**gansazRvra 44.***f* funqcionals (formas) ewodeba *anti­wrfi­vi* (semiwrfivi)*­­* raime linealze, Tu am simravlis ne­bis­mieri  da  elementebisaTvis

 da ,

sadac  nebismieri kompleqsuri ricxvia.

**gansazRvra 45.** funqcionals (formas) ewodeba *ses­qviwrfivi­* raime linealze, Tu am simravlis nebismieri , , , , ,  elementebisaTvis da nebismieri kompleqsuri ricxvisaTvis sruldeba Semdegi tolobebi:

,

,

,

.

amdenad,  asaxva fiqsirebuli -saTvis aris wrfi­vi funqcionali(forma), xolo  asaxva fiq­sirebuli -saTvis aris antiwrfivi. Tu ses­qviw­rfivi forma namdvilmniSvelobiani formaa, maSin mas *orad­wrfiviforma­* ewodeba.

**gansazRvra 46.** Tu  raime area, maSin  aris yvela iseTi  elementebis qvesivrce, ro­mel­TaTvisac supp, supp  aRniSnavs elementis *say­rden­s*, e.i. -is im qvesimravlis Caketvas, sadac  gan­sxva­vebulia nulisagan (ix. agreTve gansazRvra 37).

**Teorema 19.** sivrce mkvrivia -Si.

**gansazRvra 47.**–ze gansazRvrul SemosazRvrul anti­wrfiv *f* funqcionals ewodeba *ganawileba*-ze.  *f* ga­na­wilebis mniSvnelobas  elementze  sim­bo­loTi aRvniSnavT, xolo ganawilebaTa simravles \_ -Ti. vityviT, rom  mimdevroba -Si kre­ba­dia  elementebisaken, Tu

.

**gansazRvra 48.** vTqvaT,  da  raime mul­tiin­deqsia.  ganisazRvreba



tolobiT.

**SeniSvna 12.**gansazRvra 48-Si  SeiZleba Seicvalos -Ti.

**gansazRvra 49.**-is furies gardaqmna Semdegi to­­lobiT ganisazRvreba

,

sadac

.

**gansazRvra 50.**-isa da -is *naxvevi* ga­ni­marteba



tolobiT.

**Teorema 20.***F* homeomorfizmia -dan -Si; mi­si Sebrunebuli  gardaqmna ganisazRvreba



tolobiT.

**Teorema 21.** yoveli  akmayofilebs



*parsevalistolobas*.

**Teorema 22.** yoveli -saTvis

.

**Teorema 23.** yoveli -is da  mul­tiin­deq­si­saT­­vis

,

.

**gansazRvra 51.***ganawilebas* ewodeba *regu­la­­ru­li­­­­*, Tu arsebobs iseTi *lokalurad jamebadi* (e.i. -dan) funqcia, romelsac simartivisaTvis aRvniS­navT -iT, rom

.

regularul ganawilebaTa simravles aRvniSnavT -iT. ganawilebas ewodeba *singularuli*, Tu is regularuli ar aris.

**gansazRvra 52.** ganawilebaTa simravles, ro­mel­Ta furies gardaqmna  da

,

sadac  (*R* namdvil ricxvTa simravlea), ewodeba *be­se­lispotencialTa sivrce­­*, roca . Tu amasTan  mas *sobolev-slobodeckis sivrce*, xolo Tu  natu­ra­lu­ria *sobolevissivrce*ewodeba da aRiniSneba  sim­bo­loTi.

**gansazRvra 53.** aris -is qvesivrce, ro­me­lic Sedgeba yvela iseTi -sagan, rom supp.

**gansazRvra 54.** aris yvela iseT  ele­men­tTa sivrce, romlebic SeiZleba -dan gafarTovdnen -mde, an piriqiT-is elementebis -ze Sez­Rudvebis sivrce. -s norma ganisazRvreba



to­lobiT, sadac  aRniSnavs -s mniSvnelobebs -ze. Tu , maSin*u*-s ewodeba -s *SezRudva*-ze, xolo -s \_ *u*-s *gafarToeba*-dan -ze.

**SeniSvna 13.** Tu , sadac *k* arauaryofiTi mTeli ric­xvia, maSin  (ix. gansazRvra 36).

**Teorema 24.** da  dualuri (SeuRle­bu­­­li) sivrceebia. sivrcis SeuRlebuli sivrcea .

**gansazRvra 55.** vTqvaT,  aris  klasis Caketili ze­­dapiri, romelic -s yofs Siga da gare  da  aree­­bad;  erTeulovani gare normalia -s mimarT  wertilSi. maSin (ix. [13], gv. 143 da [11], gv. 212):

1. arsebobs -is iseTi Ria areebis  simravle, rom

\_  faravs -s, e.i. ;

–  normalebi ar ikveTebian  gaerTianebaSi;

\_ yoveli -saTvis arsebobs  \_ difeo­mor­fizmi -dan -Si; mis Sebrunebuls aRvniSnavT -iT;

\_ Tu , maSin arsebobs



simravlidan



simravleSi  \_ difeomorfizmi dadebiTi iako­bia­niT, amasTan iseTi, rom .

(*difeomorfizmi* aris diferencirebadi homeomorfizmi, xo­lo *homeomorfizmi* aris uwyveti bieqciuri, e.i. urTier­Tcalsaxa asaxva).

2. arsebobs erTeulis iseTi regularuli da­­­yofa -ze, rom

\_ ;

\_ ;

\_ .

-ze gansazRvruli yoveli, magaliTad,  fun­qcii­saTvis samarTliania

,

gaSla. yoveli -saTvis movaxdinoT cvladTa Sem­­degi gardaqmna



da ganvavrcoT (gavafarTovoT)  nuliT -is gareT mTel  sivrceze. axla ganvsazRvroT , , Sem­­degnairad

.

am sivrceSi norma SeiZleba SemoviRoT



tolobiT. erTeulis sxvadasxva dayofas Seesabameba ekviva­le­n­turi normebi. hilbertis sivrcea.

**Teorema 25.** Tu  aris SemosazRvruli antiwrfivi fun­qcionali, gansazRvruli *H* hilbertis sivrceze, xolo  \_ sesqviwrfivi forma, romelic akmayofilebs Se­mo­saz­Rvrulobis



da koercitiulobis



pirobebs, sadac *M* da  dadebiTi mudmivebia da  aris nor­­ma*H*sivrceSi, maSin



amocanas aqvs erTaderTi amonaxsni  iseTi, rom

,

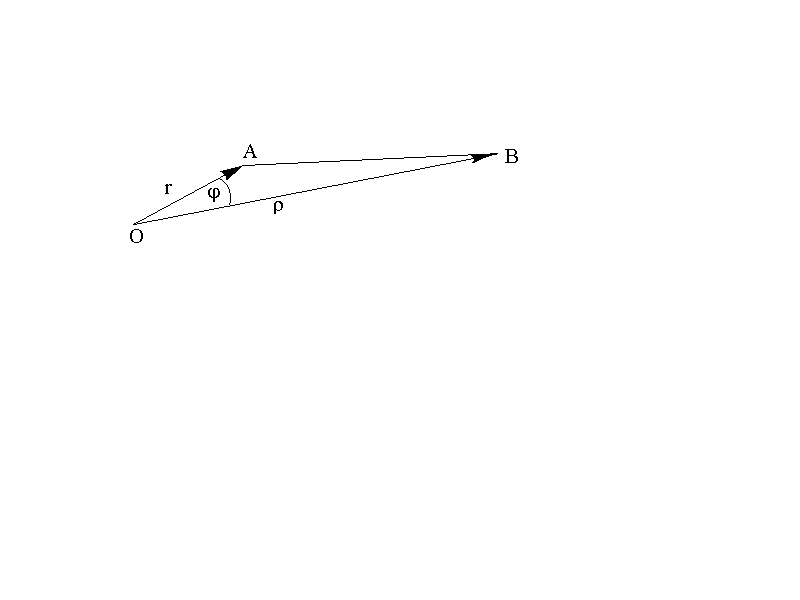
sadac  aris norma *H*-is dualur  sivrceSi, xolo *c* da­­debiTi mudmivia. kerZod, SeiZleba aviRoT .

SevniSnoT, rom am damatebaSi gadmocemuli masalis Sed­genisas ZiriTadad gamoyenebuli iyo [54], [13] da [11].

damateba 2. leJandris polinomebi

1. leJandris polinomebis warmomqmneli funqcia

klasikur orTogonalur polinomTa Soris yvelaze adrin­delia leJandris polinomebi, romelTa warmoSoba da­kav­Sirebulia maTematikuri fizikis amocanebTan, kerZod, po­tencialis TeoriasTan. sivrcis ori *A* da *B* wertilis mi­zidulobis velis potenciali gamoisaxeba maT Soris man­Zi­lis Sebrunebuli (ix. nax. 1)



sididiT.

nax. 1

Tu aq , maSin gveqneba, rom

,

sadac . rogorc qvemoT davinaxavT, Tu



wi­lads gavSliT xarisxovan mwkrivad *t*-s mimarT saTavis mi­damoSi, maSin mwkrivis koeficientebi *x*-is polinomebia, ro­melTac *leJandris polinomebi* ewodebaT (ix. qvemoT (1) da (2)).

gavixsenoT binomialuri mwkrivi



sadac  da .

Tu , maSin gveqneba, rom



es mwkrivi absoluturad krebadi iqneba, Tu . am garemoebis gamo ufleba gvaqvs, frCxilebis gaxsnis Sem­deg mwkrivi davalagoT *t*-s xarisxebad:

.

rogorc vxedavT, pirveli sami koeficienti warmoadgens Se­sabamisad nulovani, pirveli da meore xarisxis po­li­noms. danarCenic rom polinomebia, amaSi davrwmundebiT, ro­desac davadgenT rekurentul formulas, romelic er­Tma­neTTan akavSirebs yovel urTierTmomdevno sam po­li­noms. aqve unda SevniSnoT, rom leJandris polinomebis wa­r­modgena bevrnairad SeiZleba, razedac saubari qvemoT gveq­neba. axla ki davubrundeT

 (1)

tolobas, sadac, rogorc ukve aRvniSneT,  war­moad­gens leJandris *n*-uri rigis polinoms. tolobis marcxena mxa­reSi mdgom gamosaxulebas ewodeba *leJandris poli­nom­Ta warmomqmneli funqcia*, ramdenadac igi warmoSobs zemo­xse­nebuli xarisxovani mwkrivis koeficientTa  mim­devrobas. rogorc vxedavT,

. (2)

(1) mwkrividan advili dasadgenia, rom

.

marTlac, Tu CavsvamT , maSin, radganac roca , mar­cxena mxareSi gveqneba usasrulod klebadi geomet­riu­li progresiis jami da miviRebT, rom

.

aqedan uSualod Cans, rom . analogiurad, Tu , maSin

,

rac niSnavs, rom . warmomqmneli fun­qciis (1) warmodgenidan gamomdinareobs, rom Tu , maSin kenti xa­risxis leJandris polinomi nulis tolia, e.i., . marTlac, Tu , warmomqmneli funqcia binomialuri mwkrivis saxiT ase warmoidgineba

,

saidanac (1) warmodgenasTan Sedarebis Semdeg davaskvniT, rom , .

2. rekurentuli formulebi

pirvel rigSi davamtkicoT, rom leJandris polino­me­bi­saT­vis samarTliania Semdegi rekurentuli formula

. (3)

amisaTvis gavawarmooT (1) toloba *t*-s mimarT:

.

Tu orive mxares gavamravlebT -ze, miviRebT, rom

,

an, (1)-is Tanaxmad,

.

Tu orive mxareSi gamravlebas SevasrulebT da  xarisxTan mdgom koeficientebs erTmaneTs gavutolebT, miviRebT, rom



saidanac gamomdinareobs saZiebeli (3) toloba. misi arsi isaa, rom SegviZlia TanmimdevrobiT avagoT gansaxilavi polinomebi. radgan ukve dadgenilia, rom , , amis Semdeg (3) tolobis gamoyenebiT avagebT da­narCenebsac. ase, magaliTad,

, , .

(2)-is gaTvaliswinebiT, (3) rekurentuli formulebidan, cxa­dia, rom luwi rigis polinomi luwi funqciaa, xolo ken­ti rigis ki kenti funqciaa. garda (3) formulisa, arse­bobs kidev ramdenime rekurentuli Tanafardoba, rom­le­bic polinomebis warmoebulebsac Seicaven. advili gamo­say­vania Semdegi formula

. (4)

marTlac, gavawarmooT (1) toloba *x*-iT:

, (5)

saidanac

.

Tu marcxena mxareSi CavsvamT (1) mwkrivs, xolo mar­jvena mxareze SevasrulebT gamravlebas da amis Semdeg  xarisxebis win mdgom koeficientebs erTmaneTs gavu­to­lebT, miviRebT (4)-s.

davubrundeT isev



tolobas.gavamravloTorivemxare *t*-zedaSevadaroT(5)-s, miviRebT, rom

,

an sxvanairad

.

gavutoloT orive mxareSi  xarisxebis win mdgom koe­fi­cien­tebi

. (6)

axla am tolobidan ganvsazRvroT  da SevitanoT (4) tolobaSi, e.i. (4) da (6) tolobidan gamovricxoT . es mogvcems kidev erT rekurentul formulas

. (7)

Tu (7) tolobas gamovaklebT (6)-s, miviRebT, rom

. (8)

marTalia (8) davamtkiceT, roca , magram is samar­Tlia­nia maSinac, roca , radgan Tu -s, cxadia, is sruldeba, rac uSualod mowmdeba. da bolos, davam­tki­coT

, (9)

formula. CavsvaT (8)-Si *n*-is nacvlad , maSin

.

am ukanasknelidan da (6)-dan gamovricxoT , ris Se­degad swored (9)-s miviRebT.

(5)-dan gamomdinareobs, rom

.

marcxena mxareSi CavsvaT (1), maSin



gavutoloT , , xarisxebis koeficientebi

. (10)

(10) da (6)-dan gamovricxoT  (amisaTvis (6)-Si *n* Sev­cvaloT -iT, gavamravloT 2-ze da miRebul ga­mo­sa­xulebas wevr-wevrad davamatoT (10))

. (11)

Tu (11) formulis marjvena mxareSi CavsvamT (11) for­mu­li­dan *n*–is (*n*–2)–iT SecvliT miRebul –is ga­mo­sa­xu­lebas da aseT operaciebs gavagrZelebT vidre marjvena mxa­reSi gveqneba leJandris polinomis warmoebulis Sem­cve­li wevri, miviRebT, rom



 (12)



induqciiT davamtkicoT (12) formulis samarTlianoba. ro­ca , (12) formula samarTliania. davuSvaT, rom igi samarTlia­nia -sTvis da davamtkicoT *n*-sTvis. (11)-isa da Cveni daSvebis Tanaxmad,





sadac ukanasknelis wina jamSi .

(6)-Si SevitanoT (12), miviRebT, rom

 (13)

sadac .

Tu (12)-Si CavsvamT  da gaviTvaliswinebT, rom



miviRebT, rom

 (14)

radgan -is Sesabamisi wevri yovelTvis nulia, xolo ro­ca  da *n* luwia, maSin -is Sesabamisi wevric nu­lia.

(14)-is gamoyenebiT (13) SeiZleba Semdegnairad gadavweroT

 (15)

**SeniSvna**. cxadia (ix. $2.6 da $3.2),

 (16)

SevcvaloT *x* (15)-sa da (14)-Si -Ti da miRebuli ga­­mosaxulebebi CavsvaT (16)-Si, maSin martivi gardaqmnebis Sem­­deg miviRebT, rom



,

sadac .

3. diferencialuri gantoleba, romelsac akmayofilebs  polinomi

wina paragrafSi miRebuli rekurentuli formulebis ga­moyenebiT advilad davadgenT, rom leJandris polinomebi akmayofileben Semdeg meore rigis wrfiv erTgvarovan di­fe­rencialur gantolebas, romelic *leJandris gantolebis* saxels atarebs

. (17)

amisaTvis gadavweroT (9) formula



saxiT. am ukanasknelis gawarmoeba mogvcems, rom

 (18)

(6) tolobis *n*-ze gamravlebis Semdeg yvela wevri tolo­bis marcxena mxareSi gadavitanoT

.

Tu (18) tolobas am ukanasknels gamovaklebT, miviRebT, rom

.

es ki swored imas niSnavs, rom **** polinomi akma­yo­fi­lebs (17) gantolebas.

4. rodrigis[[15]](#footnote-15)\*) formula

davamtkicoT, rom leJandris **** polinomi warmoid­gi­neba Semdegi saxiT

, (19)

romelsac *rodrigis formula* ewodeba. sxvaTa Soris, aqe­dan Cans, rom Tu *n* luwia, **** Seicavs *x*-is mxolod luw xarisxebs, xolo Tu *n* kentia, \_ kent xarisxebs. (19) to­lobissamarTlianoba davamtkicoT (17) gantolebis gaT­valiswinebiT. saxeldobr, pirvel rigSi vaCvenoT, rom

 (20)

funqcia, sadac  mudmivia, akmayofilebs (17) gantolebas. mar­Tlac, Tu

,

maSin gawarmoebis Semdeg miviRebT, rom

.

es toloba Tavis mxriv -jer gavawarmooT. laibnicis for­mulis gamoyenebiT gveqneba, rom

.

rogorc vxedavT, marcxena mxareSi mxolod sami wevris ja­mi dagvrCeba, radgan -is 2-ze maRali rigis yvela war­moebuli nulis tolia, xolo marjvena mxareSi ori wevris jami gveqneba, vinaidan *x*-is erTze maRali rigis yve­la warmoebuli nulia. amdenad miviRebT, rom



.

Tu SemoviRebT



aRniSvnas, maSin wina toloba miiRebs



saxes. radgan es gantoleba erTgvarovania, amitom masve ak­ma­yofilebs (20) gamosaxuleba, rogoric ar unda iyos  mud­mivi. vinaidan (20) gamosaxuleba warmoadgens *n* xaris­xis polinoms, romelic imave (17) gantolebas akmayo­fi­lebs, romelsac akmayofilebs leJandris **** polinomi, ami­tom (20) funqcia ****-sgan SeiZleba gansxvavdebodes mxo­lod mudmivi[[16]](#footnote-16)\*) mamravliT. maSasadame, Cven dagvrCa -s ise SerCeva, rom (19) toloba samarTliani iyos. amisaTvis jer gavarkvioT, rogoria **** polinomis ufrosi ko­e­fi­cienti. rogorc es ZiriTadi rekurentuli (3) for­mu­li­dan Cans, Tu  aRniSnavs **** polinomis ufros koe­ficients da mxedvelobaSi miviRebT, rom (3) tolobaSi Se­mavali mxolod Semdegi ori wevri  da  Seicavs -s, maSin am wevrebis ufrosi koe­ficientebi toli unda iyos. amgvarad, gveqneba

,

saidanac

, e.i., . (21)

(2)-dan, cxadia, rom , . maSasadame, (21) to­lo­bis gamoyenebiT miviRebT, rom

. (22)

Tu davubrundebiT (20) gamosaxulebas, davinaxavT, rom -is koeficienti iqneba . amgvarad, rod­rigis (19) formulis dasadgenad saWiroa, rom

,

saidanac



aqedan ki

.

amiT (19) tolobis samarTlianoba damtkicebulia.

5. leJandris polinomTa sistemis orTogonaloba  segmentze

rogorc (17) diferencialuri gantolebidan Cans, mas SeiZ­leba Semdegi saxe mieces

. (23)

advili dasamtkicebelia, rom

. (24)

marTlac, erTis mxriv, (23)-dan gvaqvs, rom

.

Tu am ukanasknels gavamravlebT -ze, miviRebT, rom

.

saidanac

 (25)

meores mxriv, Tu *m*-isa da *n*-is urTierTSenacvlebas movaxdenT, miviRebT, rom

 (26)

nawilobiTi integreba mogvcems, rom



.

analogiurad(an *m*-isa da *n*-isurTierTSenacvlebiT) da­va­d­genT, rom

.

Tu am integralebis mniSvnelobebs SevitanT (25)-sa da (26)-Si, miviRebT, rom

,

.

maTi sxvaoba ki mogvcems, rom



da radgan , amitom integrali iqneba nulis toli, riTac damtkicda (24).

axladavadginoT,rom*n*xarisxis****polinomi or­Togonaluria nebismieri polinomis, romlis xarisxi *n*-ze naklebia. amisTvis gaviTvaliswinoT, rom  warmoidgineba rogorc Semdegi wrfivi kombinacia



da, maSasadame, yoveli *n*-ze dabali xarisxis **** po­li­no­mi, cxadia, warmoadgens , ,...,  po­li­no­mebis wrfiv kombinacias mudmivi koeficientebiT. ami­tom, (24)-is Tanaxmad, vaskvniT, rom

.

imisaTvis, rom leJandris polinomTa sistema nor­mi­re­bu­li gavxadoT, saWiroa gamovTvaloT

.

zemoT-iT gvqonda aRniSnuli**** polinomis uf­ro­­sikoeficienti.amis Sesabamisad aRniSnavs -is saTanado koeficients. advili Sesamowmebelia, rom



gamosaxuleba warmoadgens *n*-ze naklebi xarisxis polinoms. mar­Tlac,igiarSeicavsxarisxs, radganiwye­bawevriTda-zemisigamravlebamogvcemsswored**** polinomis ufros wevrs -s. gansaxilav sxvao­ba­Si  wevri gaqreba da amgvarad  iqneba arau­me­tes  xarisxis polinomi.

rogorcukvenaCvenebiiyo,****orTogonaluria-is da amgvarad

.

am ukanasknelis da Semdegi tolobis

,

gaTvaliswinebiT gveqneba, rom



magram, rekurentuli (3) formulis Tanaxmad,

.

Tu am gamosaxulebas SevitanT wina integralis qveS da ga­vi­TvaliswinebT imas, rom  aris -is orTo­go­naluri, miviRebT, rom

.

Tu aq gamoviyenebT (21) tolobas, romelic  da  koe­ficientebs akavSirebs, miviRebT, rom

. (27)

TuamtolobaSiCavsvamT,gamovTvliT -s, amis Sem­deg gamovTvliT -s, -s da, sazogadod,  manor­mi­re­bel mamravls, TanmimdevrobiT gveqneba, rom Tu ,

.

Tu ,

,

da a.S., zogad SemTxvevaSi



amrigad, , sistema warmoadgens orTo­normirebul sistemas.

6. Smidtis orTogonalizaciis meTodi

ganvixiloT  wrfivad damoukidebel funqciaTa mim­devroba, , , sadac  nebismieri Se­mo­saz­Rvruli simravlea -dan. avagoT Semdegi sistema:

 (28)

SevarCioT  mudmivebi ise, rom  sistema orTo­go­na­luri iyos. amisaTvis (28)-is meore toloba skalarulad ga­vamravloT -ze da, orTogonalobis pirobis Tanaxmad, ga­vutoloT 0-s:

.

aqedan

.

analogiurad (28)-is mesame toloba gavamravloT -ze da, orTogonalobis pirobis Tanaxmad, gavutoloT nuls:



.

aqedan  da a.S. davadgineT, rom .

axla (28)-is mesame toloba gavamravloT -ze, ga­mo­vi­ye­noT orTogonalobis piroba da a.S. CavataroT ana­lo­giu­ri operaciebi, Semdeg igive procesi CavataroT -ze gam­rav­lebiT da a.S. -ze gamravlebiT, maSin miviRebT, rom

. (29)

e.i. Tu (28)-Sikoeficientebi gansazRvrulia (29) to­lobiT, maSin  sistema orTogonaluria.

cxadia, orTonormirebisaTvis sakmarisia, TiToeuli maT­ga­ni gavyoT Tavis normaze. maSasadame, miviRebT Semdeg or­To­normirebul sistemas:

. (30)

**Teorema 1**. polinomTa

. (31)

sistemis Smidtis meTodiT orTonormireba gvaZlevs ,  sistemas, sadac  leJandris po­li­nomebia.

*damtkiceba*. sakmarisia, vaCvenoT, rom (31) sistemisTvis (28) sistemas aqvs

, (32)

saxe, sadac . marTlac, maSin mi­vi­Reb­diT, rom

.

cxadia, -sTvis (32) samarTliania, radgan

.

davuSvaT, rom (32) samarTliania  da vaCvenoT, rom igi samarTliania -sTvis. e.i. vuSvebT, rom , ,  da unda davamtkicoT, rom , .

ganvixiloT (ix. (28))

, (33)

sadac



e.i.

 (34)

 warmovadginoT Semdegi saxiT:

, (35)

sadac  koeficientebi Sesabamisadaa SerCeuli.

,  koeficientebis sapovnelad (35) to­lo­bis orive mxare skalarulad gavamravloT -ze, mi­vi­RebT, rom

, (36)

radgan  polinomTa sistema orTogonaluria.

(36)-dan gamomdinareobs, rom

. (37)

(35)-dan gamovTvaloT :

,

anu, (37)-is gaTvaliswinebiT,

.

Tu am ukanasknels SevadarebT (34)-s, davaskvniT, rom

. (38)

radgan, rogorc ukve vaCveneT,  aris -s, , orTogonaluri, (37)-is Tanaxmad, cxadia, rom



sadac  leJandris  polinomis ufrosi koe­fi­cien­­tia, romelic (22)-is Tanaxmad dadebiTia, e.i. . amiT Teorema damtkicebulia.

7. furie-leJandris mwkrivebi

**gansazRvra 1**.

ricxvebs ewodebaT  funqciis furie-leJandris koe­fi­cien­tebi, xolo



mwkrivs ewodeba  funqciis Sesabamiisi furie-leJan­dris mwkrivi, rac ase Caiwereba

.

**Teorema 2**. Tu  segmentze uwyveti  funqcia akma­yofilebs hiolderis pirobas maCvenebliT (e.i. ),maSinfurie-leJandrismwkri­vi-skenkrebadiaTanabrad mTel  segmentze (es krebadobis erT-erTi sakmarisi pirobaa).

**Teorema 3[[17]](#footnote-17)\*)**. vTqvaT,  uwyvetia  segmentze gar­da, SesaZlebelia, wyvetis wertilTa sasruli rao­de­no­bi­sa, sadac funqcias sasruli naxtomi gaaCnia, da fun­qcias yovel wertilSi aqvs rogorc marcxena, ise mar­jvena warmoebuli (cxadia, sadac isini tolia,  war­moebadia). maSin Sesabamisi



furie-leJandris mwkrivi krebadia -sken uwyvetobis yo­vel Siga wertilze, xolo wyvetis yovel Siga wer­til­ze krebadia -sken.

**jeksonis Teorema**. vTqvaT,  aris funqcia sasruli va­ria­ciiT  segmentze, romlis Sesabamis furie-le­Jan­dris mwkrivs aqvs



saxe, sadac

.

Tu  da *V* aRniSnavs -is srul variacias, roca , maSin

,

.

**Teorema 4**. leJandris , , polinomebis sistema srulia -Si.

**lema**. vTqvaT,  da  areebi Semo­saz­Rvru­lia; , , funqciebis sistema orTo­nor­mi­re­buli da srulia -Si da yoveli *j*-sTvis, , , , sistema orTonor­mi­re­bu­li da srulia -Si. maSin

,

funqciaTasistema orTonormirebuli da srulia-Si.

am lemidan da Teorema 4-dan gamomdinareobs

**Teorema 5**., , funqciaTa sis­te­ma orTonormirebuli da srulia -Si.

am damatebasTan dakavSirebiT ix. agreTve [4] da [41].

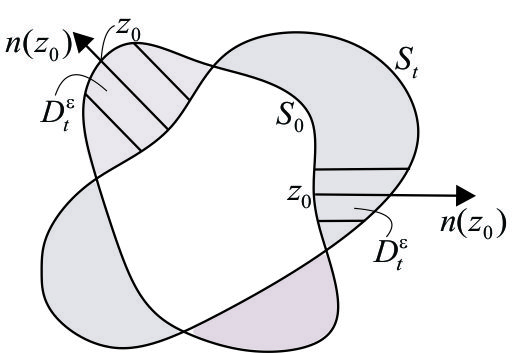
**damateba 3**. **moZrav moculobaze ganxiluli integralis drois mimarT gawarmoebis formula[[18]](#footnote-18)\*)**

vTqvaT,  aris moZravi moculoba (e. i. droze damokide­buli) sakmarisad glu­vi (ix. qvemoT damtkiceba)  sazRvriT;  zedapiris gare normalia,  aris  wertilis siCqare; . maSin samar­Tli­a­nia Semdegi formula:

.

**damtkiceba**. SemoviRoT Semdegi aRniSvna:

.



nax. 1

cxadia (ix. nax. 1),



 (1)

da

, (2)

sadac

,

ganvi­xi­loT -sa da -is simetriuli  sxvaobis (ix. gamuqebuli are, nax. 1) erTi elementaruli  nawili, amoWrili cilin­dru­li zedapiriT, romlis normalia ,  (ix. nax. 2), da aviRoT lokaluri orTogonaluri sakoordinato sis­te­ma ise, rom  RerZi daemTxves -s, xolo ,  RerZebi moxvdes mxeb sib­rtye­Si. vTqvaT,  zedapiris gantolebaa

.

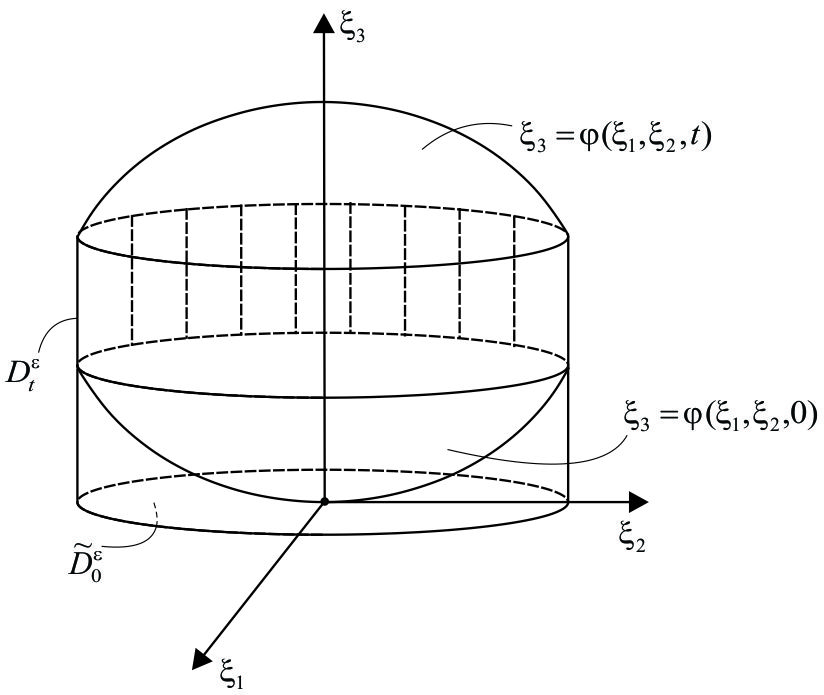
maSin  zedapiris gantoleba iqneba

.

vgulisxmobT, rom*F*sakmaod gluvi fun­­qciaa. igi gansazRvrulia ****-ze (****-is magier unda iyos **** kubSi). maSin, cxa­dia, lo­­ka­lur sistemaSi -isa da -s gantolebebi iqneba

, .

, 



nax. 2

(, cxadia, garkveul pi­robebSi gansazRvravs  ara­cxad funqcias, anu lo­ka­lur sis­te­ma­Si calsaxa  funqci­as  wer­tilis midamoSi). jer vixi­lavT im Sem­Txvevas, ro­ca **-is mi­marTulebiT jer gvxvdeba  da mere . sxva Sem­TxvevaSi Se­iZ­leba piriqiTac iyos (ix. nax. 1).

ganvixiloT



integrali, sadac

.

aq  orTogonaluri matricaa, romlis bolo sveti emTxveva  normals.

gardaqmnas, gadavyavarT lokalur sis­te­maze moculobiTi elementis Se­­ucvlelad, e.i., . cxadia,



(3)

amasTan

, ,

radgan saintegro arec da integralqveSa funqciac  rigisaa, roca . ami­tom -ze gayofisa da  zRvarze gadasvlis Semdeg igi nulad iqceva. amdenad,



 (4)

,

radgan, saSualo mniSvnelobis Teoremis Tanaxmad,



sadac

.

aqve SevniS­noT, rom Tu  normalis gaswvriv jer iqneboda , xolo mere  (ix. nax. 1), maSin (3)-Si gveq­ne­bo­da



da -ze gayofis Semdeg miviRebdiT, rom es gamosaxuleba miiswrafis



zRvrisken, roca .

ase, rom (4) tipis formulaSi simetriuli sxvaobis im wevrebSi, romelSic in­teg­ra­li -s nawilze aiRe­ba, gveqneba niSani “+”, xolo simetriuli sxvao­bis im wev­reb­Si, sadac in­teg­reba -is nawilze xdeba, gveqneba niSani “-”, amitom sabolood yve­la Sesakrebs eqneba niSani “+” (ix. (2)) da gveqneba

 (5)

ajamva xdeba yvela aseTi tipis elementaruli nawilebis mimarT.

gardavqmnaT (5)-Si zedapiruli integrali. radgan  aris dro, amitom  wertilis droiTi warmoebuli iqneba  wertilis siC­qare. -s -Ti gawar­mo­ebiT miviRebT:

,

aqedan

. (6)

 zedapirze mdebare  wertilSi  zedapiris  normalis koordinatebia

.

kerZod,

.

maSin, (6)-is gaTvaliswinebiT, cxadia, rom





,

sadac  wertilis siCqarea. radgan



siCqaris normaluri mdgeneli ar aris damokidebuli orTogonalur gardaqmnaze (ori veqtoris skalaruli nam­rav­li invariantia orTogonaluri gardaqmnis mimarT), gveqneba



(bolo tolobaSi gamoyenebulia ukugardaqmna lokaluri sistemidan). amdenad, radgan -ebis gaer­Ti­aneba iZleva -s, (5)-dan miviRebT:

.

radgan -is nacvlad SegveZlo agveRo nebismieri , amitom gveqneba:



anu

,

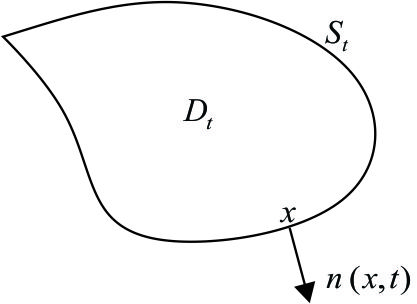
sadac



aris  wertilis veqtoruli siCqare, xolo



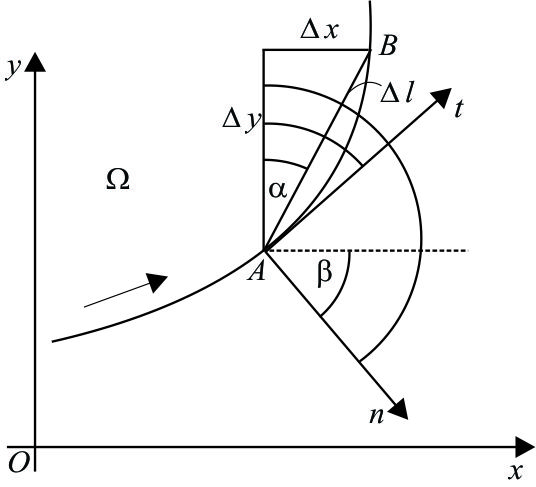
aris  wertilis normaluri siCqare,  gare normalia -s mimarT (ix. nax. 3).



nax. 3

**damateba 4. ori sasargeblo formula sibrtyeze**

moviyvanoT sibrtyeze amocanebis gamokvlevisas sasargeblo ori formula. gan­vi­xi­loT *Oxy* sibrtyeze gluvi wiriT Semo­sa­zRvruli  are. sazRvris gaswvriv moZ­­ra­obisas dadebiT mimarTulebad mivi­RoT is mimarTuleba, romelic ares mar­­cxe­na mxa­res tovebs. *n* gare norma­lia, *t* mxe­bis da­debiTi miamrTulebaa.



nax. 1

**Teorema 1**. samarTliania Semdegi for­­mu­le­bi:

, .

**damtkiceba**. kuTxeebi *n*-sa da *x*-s, *t*-sa da *y*-s Soris aRvniSnoT Sesabamisad -iT da -iT. rogorc es naxa­zi­dan (ix. nax. 1) Cans:

1. radgan , amitom

,

;

2. radgan , amitom ;

3. radgan  da , amitom

.

**damateba 5. prizmuli garsebi**

**1. i. vekuas ierarqiuli modelebis ZiriTadi idea**

drekadobis wrfivi samganzomilebiani Teoriis ZiriTadi damokidebulebebi Sedgeba “moZraobis” gantolebebisgan

, , , ; (1)

hukis ganzogadebuli kanonisgan (izotropuli SemTxveva)

, ; (2)

kinematikuri damokidebulebebisgan

, , (3)

sadac

. (4)

SevniSnoT, rom nakrebi (mTavari) fizikuri momentis nulTan tolobis pirobidan ga­mom­dinareobs Zabvis tenzoris simetriuloba:

, . (5)

yvela sidide, garda koeficientebisa, damokidebulia oTx cvladze (sam sivrciTze , ,  da *t* droze).

sxeuls, romelic zemodan da qvemodan SemosazRvrulia

 da 

zedapirebiT, xolo gverdidan cilindruli zedapiriT  RerZis para­le­luri msaxveliT, prizmuli garsi ewodeba.

, , (6)

polinomebs, kerZod,

, , ,

leJandris polinomebi ewodeba. leJandris polinomTa sistema orTogonaluria  segmentze:

 (7)

(7)-dan cxadia, rom polinomTa

, ,

sistema orTonormirebulia.

(7)-Si movaxdinoT cvladTa

 (8)

gardaqmna, sadac

, ,

 prizmuli garsis sisqea, . cxadia,



da

 (9)

sazogadod,

, , 

orTonormirebuli sistemis mimarT furies mwkrivs aqvs



saxe, sadac

, ,

furies koeficientebia.

Tu davuSvebT, rom  cvladis mimarT , ,  klass  seg­men­tze, maSin

, (10)

sadac



 (11)

da (10)-is marjvena mxare Tanabrad krebadia  segmentze.

ierarqiuli modelebis agebis i. vekuas meTodi mdgomareobs (1)\_(3) damokidebu­le­bebis -ze gamravlebiT da -dan -mde -is mimarT integ­re­biT , , maTematikur momentebze gadas­vlaSi. (2)-dan uSualod gamomdinareobs, rom

,

xolo momentebis mimarT (1) da (3) damokidebulebebidan gamomdinare damokidebulebebis misaRebad ga­mo­iyeneba

,

, ,

damokidebulebebi, romlebic unda Caiweros  seg­men­tisTvis (8) damo­ki­de­bu­le­be­­bis gaTvaliswinebiT. Semdgom, Tu vimoqmedebT ise, rogorc drekadobis TeoriaSi, miviRebT lames sistemis ekvivalentur usasrulo sistemas



momentebis mimarT. amasTan



, . (12)

amis Semdeg, *N*-uri miaxloebis (ierar­qi­u­li modelis) misaRebad *N*-ze meti rigis moments ukuvagdebT (CavTvliT nulis to­lad) da gantolebaTa usasrulo sistemidan ganvixilavT mxolod im gantolebebs, romelTa mTavari (e.i. meore rigis warmoebulebis Semcveli wevrebisgan Semdgari) nawilebi Seicaven  rigamde CaTvliT momentebs.

**2. miaxloeba. ori modeli**

movaxdinoT meTodis ilustrireba  miaxloe­bis­Tvis. (1)-dan cxadia,



, (13)

,

magram, (11)-is ZaliT,

,

(14)

 (15)

.

aq gamoviyeneT parametrze damokidebuli integralis parametriT gawarmoebis wesi:



.

Tu gluvi zedapiris gantolebas aqvs



saxe, maSin normalis  komponentebs, rogorc es kalkulusis kursidanaa cnobili, aqvT Semdegi saxe:

, .

Cvens SemTxvevaSi

,

amitom

,

.

amis gaTvaliswinebiT





. (16)

radgan

,

Tu CavsvamT (14) \_ (16) gamosaxulebebs (13)-Si, miviRebT, rom











magram koSis formulebis Tanaxmad

,



amrigad,  miaxloebaSi

, (17)



; (18)

(12)-dan gveqneba

. (19)

(19)-is gamo, (3) miiRebs Semdeg saxes

. (20)

aqedan

,

[[19]](#footnote-19)\*), , .

marTlac,











 (21)

aqedan, Tu mxedvelobaSi miviRebT (19)-s,







saidanac



analogiurad,

.

cxadia,



 (22)

aqedan, Tu gaviTvaliswinebT (19)-s, gveqneba,rom

.

Tu (20)-s CavsvamT (18)-Si da miRebuls \_ (17)-Si, gveqneba



, .

kerZod, roca ,



, , , (23)

xolo, roca ,



 (24)

modelSi, roca piriT zedapirebze Zabvis veqtoris nacvlad gadaadgilebis veqtoria mocemuli, maSin (15) formulaSi piriT zedapirebze Zabvebis mniSvnelobebad ma­Ti  miaxloebis Sesabamisi



mniSvnelobebi unda CavsvaT, e. i.,



.

.

CavsvaT es da (14) gamosaxulebebi (13)-Si, miviRebT, rom

. (25)

axla gamovTvaloT deformaciis tenzoris nulovani momenti imis gaTva­lis­wi­ne­biT, rom piriT zedapirebze gadaadgilebis veqtoria mocemuli [ix. (21), (22)]





 (26)

 (27)

sadac

2



Tu (26)-sa da (27)-s CavsvamT (18)-Si, miviRebT

 (28)



axla (28) CavsvaT (25)-Si, miviRebT Semdeg saZiebel sis­te­mas:

 (29)

kerZod, roca ,



xolo, roca ,

 (30)

**3. prizmuli garsi xarisxovani wamaxvilebiT**

axla ganvixiloT

, , (31)

sisqis mqone wamaxvilebuli prizmuli garsi. statikis SemTxvevaSi, Tu moculobiT Zalebs da zeda da qveda zedapirebze moqmed zedapirul Zalebs nulis tolad CavTvliT, (24)-dan miviRebT, rom

.

aqedan, Tu CavTvliT, rom  mxolod -zea damokidebuli (e. i., ganvixilavT ci­lin­drul deformacias), miviRebT Semdeg gantolebas

. (32)

mis zogad amonaxsns aqvs

, (33)

saxe. saidanac advilad davaskvniT, rom  sazRvarze, gadaadgilebis SemosazR­vru­lo­bis pirobebSi, gadaadgilebis dasaxeleba SeiZleba mxolod maSin, roca  (am SemTxve­va­Si vambobT, rom gvaqvs blagvi wamaxvileba), e. i., roca  (am Sem­Txve­vaSi vambobT, rom gvaqvs maxvili wamaxvileba),  sazRvari sasazRvro pi­ro­bis­gan Tavisuf­ldeba. wa­maxvilebuli prizmuli garsebis (23), (24) sistemisa da *N*-uri miaxloebis zogadi sis­temisaTvis sasazRvro da sawyis-sasazRvro amocanebi sakmarisad kargadaa Seswavlili (ix. $4.4)[[20]](#footnote-20)\*). wamax­vilebuli prizmuli garsebisaTvis (29), (30) sistema jer-jerobiT ar aris Ses­wav­li­li. Tu ganvixilavT (31) SemTxvevas (30) gantolebis magaliTze advilad davrwmun­de­biT, rom (23), (24) da (29), (30) sistemebi Tvisobrivad gansxvavdeba erTmaneTisagan.

Tu [[21]](#footnote-21)\*\*) da  maSin statikur SemTxvevaSi (30)-dan, cxadia,

, (34)

e.i.



aqedan

 (35)

Tu CavTvliT, rom  mxolod –zea damokidebuli, gaviTvaliswinebT (31)-s da gavyofT , (35)-dan miviRebT

 (36)

es ukanaskneli eileris gantolebis, romlis amoxsnis meTodi cnobilia, kerZo SemTxvevaa da mis zogad amonaxsns aqvs

 (37)

saxe, sadac da  nebismieri mudmivebia.

igive Sedegamde mivalT, Tu (34)-s CavwerT –is mimarT da gaviTvaliswinebT (31)-s,

miviRebT



mis zogad amonaxsns aqvs



saxe. aqedan, cxadia (ix. (19)),



da -sa da -is nebismierobis gamo miviReT igive (37) gamosaxuleba –isTvis.

 amonaxsnis zogadi warmodgenis gamoyenebiT gavaanalizoT  intervalze  CaRunvisaTvis amocanebis dasmis sakiTxi. Tu CavTvliT, rom piriT zedapirebze gadaadgilebebi nulis tolia, bunebrivia  napirze daisvas

 (38)

sasazRvro piroba.

(38) sasazRvro pirobis dakmayofilebisaTvis, roca aucilebelia .  napirze SeiZleba

 (39)

sasazRvro pirobis dakmayofileba. marTlac,

.

saidanac

.

amdenad, (36), (38), (39) sasazRvro amocanis erTaderTi amonaxsnia



es amonaxsni mdgradia (39) sasazRvro pirobis mimarT, magram aramdgradia (38) sasazRvro pirobis mimarT, ramdenadac sasazRvro amocana araerTgvarovani (38) sasazRvro pirobiT araa amoxsnadi. Tu uars vityviT  wamaxvilebul napirze sasazRvro pirobis dakmayofilebaze da sasazRvro pirobas amonaxsnis SemosazRvrulobiT SevcvliT, sasazRvro amocana erTi (39) sasazRvro pirobiT koreqtuli gaxdeba. analogiurad amoixsneba CaRunvisaTvis amocana (38), (39) sasazRvro pirobebSi, roca piriT zedapirebze mocemulia aranulovani gadaadgilebebi, romlebic nuli xdeba wamaxvilebul napirze. sasazRvro amocana ar iqneba amoxsnadi, Tu wamaxvilebul napirze, piriT zedapirebze mocemuli gadaadgilebebi nuli ar xdeba, radgan SemosazRvrulobis pirobiT zogadi amonaxsni nuli xdeba, roca . magram kvlav koreqtuli iqneba sasazRvro amocana erTi (39) sasazRvro pirobiT. amrigad, im modelisaTvis, roca piriT zedapirze gadaadgilebebia mocemuli nebismieri –sTvis koreqtulia amocana, roca wamaxvilebuli napiri ganTavisuflebulia sasazRvro pirobisagan (is icvleba amonaxsnis SemosazRvrulobis pirobiT), xolo arawamaxvilebul napirze gadaaadgilebebia mocemuli. CaRunvisaTvis analogiuri amocana (32) gantolebisaTvis (modeli, roca piriT zedapirebze Zabvebia mocemuli), rogorc es misi zogadi (33) amonaxsnidan gamomdinareobs, Tu  calsaxad amoxsnadia maSinac, roca wamax­vilebul napirze, araerTgvarovani (38) sasazRvro pirobaa mocemuli, amasTan amonaxsni mdgradia orive sasazRvro monacemis mimarT (e.i., uwyvetadaa damo­kidebuli maTze), gansxvavebiT wina [ix. (36)] modeli­sagan. roca , SemosazRvrulobis pirobiT –s mxolod erTi sasazRvro pirobis dakmayofileba SeuZlia.

axla orive modelisaTvis gavaanalizoT sazRvar­ze (prizmuli garsis napirze) Zabvis nulovani momentis, e.i. Zalvis dasaxelebis sakiTxi.

(36) gantolebisaTvis (38) sasazRvro pirobasTan erTad davsvaT

 (40)

sasazRvro piroba, e.i.  napirze davasaxeloT Zalva. maSin, (37)-is da (28)-is Tanaxmad, gveqneba



saidanac, (37)-is gaTvaliswinebiT, miviRebT



e.i.



da

.

amdenad, (36), (38), (40) sasazRvro amocanis erTaderTi amonaxsnia

.

SevniSnoT, rom ganxiluli sasazRvro amocana kvlav aramdgradia (38) sasazRvro pirobis monace­mis mimarT da amocana koreqtuli gaxdeba, Tu (38) sasazRvro piroba  amonaxsnis SemosazRvrulo­biT SevcvliT. (37) amonaxsniT mxolod erTgvarovani woniani



sasazRvro pirobis dakmayofileba SeiZleba wamaxvi­le­bul napirze. amrigad, am modelSi wamaxvilebul napirze arc gadaadgilebis da arc Zalvis nebismierad dasaxeleba ar SeiZleba. rac, faqtiurad, imas niSnavs, rom wamaxvilebuli napiri unda ganTavisufldes sasazRvro pirobisagan. (33)-dan advilad davaskvniT, rom analogiuri sasazRvro amocana orive napirze, sazogadod araerTgvarovani, (38) da (40) sasazRvro pirobebiT koreqtulia (32) modelis SemTxvevaSic, mxolod kvlav aucilebelia  pirobis Sesruleba. SemousazRvrav funqciaTa klasSi mudmivi Sesakrebis sizustiT ganisazRvreba  erTi: an (40) an wamaxvilebul napirze woniani



sasazRvro pirobiT. am SemTxvevaSi mxolod erT na­pir­ze sasazRvro pirobis dakmayofilebis SesaZleb­lo­ba gamowveulia am modelSi cilindruli defor­maciis specifikiT da ara wamaxvilebiT.

**4. Zabvis funqcia. Tavsebadobis gantoleba**

ganvixiloT statikuri mdgomareoba. ugulebelvyoT moculobiTi Zalebi da vigulis­xmoT, rom piriT zedapirebze moqmedi zedapiruli Zalebi nulis tolia. maSin wonaswo­ro­bis (17) gantolebebi gaSlili saxiT Caiwereba Semdegi saxiT

, (41)

. (42)

(41) da (42) pirobebidan Sesabamisad gamomdinareobs, rom arseboben iseTi *A* da *B* fun­qciebi, rom

 (43)

da

, (44)

ramdenadac (41) da (42) imis aucilebel da sakmaris pirobas warmoadgenen, rom Sesa­ba­mi­sad (43) da (44) tolobebis marjvena mxareebi *A* da *B* fun­qciebis srul diferencia­lebs warmoadgendnen. amdenad

, , , , (45)

aqedan



es ukanaskneli ki imis aucilebeli da sakmarisi pirobaa, rom arsebobdes iseTi  fun­qcia, rom

,

saidanac

, . (46)

(45) da (46)-dan pirdapir gamomdinareobs, rom

, , . (47)

hukis ganzogadebuli

 (48)

kanonidan, sadac

, , , (49)

gveqneba

.

maSin (48)-dan, (49)-is gaTvaliswinebiT, gamomdinareobs

, (50)

, (51)

. (52)

(50) da (51)-dan cxadia,

,

,

saidanac integrebiT

 (53)

(53) CavsvaT (52)-Si,

.

aqedan -iTa da -iT gawarmoebis Semdeg miviRebT Tavsebadobis

(54)

gantolebas.

(54)-dan, (47)-is Tanaxmad gamomdinareobs Semdegi gantoleba Zabvis funqciisTvis

.

**5.  miaxloebis Sedareba brtyel deformaciasTan, brtyel daZabul da ganzo­gade­bul brtyel daZabul mdgomareobasTan**

a) **brtyeli deformacia**.

hipoTezebi:

, , .

ZiriTadi damokidebulebebi:

, ,

, ,

,

, , ,

, , ,

, . (55)

b) **brtyeli daZabuli mdgomareoba**.

hipoTezebi:

.

ZiriTadi damokidebulebebi:

, , ,

, ,

, , ,

, ,

, (56)

sadac  parametris rols asrulebs.

g) **ganzogadebulibrtyeli daZabuli mdgomareoba**.

hipoTezebi:

 Zalze mcirea fuZeebis ganzomilebebTan SedarebiT. firfitis fuZeebi Tavisufalia gare Zab­ve­bisgan, xolo gverdiT zedapirebze moqmedeben fuZeebis paraleluri da Sua sibrtyisad­mi simetriulad ganawilebuli gare Zabvebi:

, , .

garda amisa,

, , .[[22]](#footnote-22)\*)

yvela fizikuri da geometriuli sidide icvleba maTi integraluri saSualoTi -is mi­marT:

, ,

, .

ZiriTadi damokidebulebebi:

, ,

, ,

 . (57)

d) . **mudmivi sisqis SemTxveva**.

ZiriTadi hipoTezebi:



da samganzomilebiani drekadobis Teoriis damokidebu­le­be­bis -is mi­marT integrebiT -dan -mde gadavdivarT nu­lovani momentis mimarT Semdeg damokidebulebebze:

, ,

, , , ,



, ,

sadac  damokidebulia piriT zedapirze mocemul Zalebze da moculobiTi Zalis nu­lo­van momentze,

, , (58)

,

.

**daskvna**. Tu , rogorc vxedavT (Seadare (55)-(58)-s), erTi mxriv, maTematikurad yvela modeli erT­ma­neTs emTxveva ( da –is sizustiT), Tumca a) da b) SemTxvevaSi gansaxilveli sidi­deebi samganzomilebiani modelis kerZo SemTxvevebia, xo­lo g) da d) SemTxvevebSi samganzomilebiani modelis sidi­deebi Secvlilia Sesabamisad maTi integ­raluri saSua­loebiT da maTi integrirebuli mniSvnelobebiT -is mi­marT. meore mxriv, masalis fizikuri Tvisebebidan gamom­di­na­re, a) da d) modelebi emTxveva erTmaneTs (orive Sem­Txve­­­vaSi figurirebs ) da b) da g) modelebi emTxveva erTma­neTs (orive SemTxvevaSi -s nacvlad figu­ri­rebs). saerTod ki, maTe­matiku­rad da fizikuradac (radgan, ro­gorc vxedavT, axloa sxva modelebTan) ufro gamar­Tu­lad i. ve­kuas  modeli unda CaiTvalos.

**damateba 6. kolosov-musxeliSvilis formulebi**

**1. Zabvis funqcia**

Tu ugulvebelvyofT moculobiT Zalebs, brtyeli drekadobis Teoriis wonasworobis gantolebebi miiReben (ix. nawili II, Tavi I)

 (1)

saxes. (1)1-dan gamomdinareobs, rom arsebobs iseTi[[23]](#footnote-23)\*), rom

, (2)

xolo (1)2-dan gamomdinareobs, rom arsebobs iseTi[[24]](#footnote-24)\*\*), rom

. (3)

(2)1 da (3)2-dan miviRebT, rom

.

amdenad, arsebobs iseTi[[25]](#footnote-25)\*\*\*), rom

. (4)

(2)2, (3)2, (3)1-dan advilad davaskvniT, rom

. (5)

sen-venanis pirobas moculobiTi Zalebis ugulvebel­yo­fis SemTxvevaSi aqvs Semdegi saxe:

. (6)

(5)1 da (5)3-dan gveqneba

, (7)

xolo (6) da (7)-dan



an gaSlili saxiT

. (8)

**2. biharmoniuli funqciis kompleqsuri warmodgena**

ganvixiloT

 (9)

biharmoniuli gantoleba da Semdegi kompleqsuri opera­to­re­bi

 (10)

Tu –sa da–s mivcemT kompleqsur mniSvnelobebsac (e.i. maTze damokidebul funqciebs gavagrZelebT kompleq­sur sib­rtye­ze), (10) tolobebiT SemoRebuli operatorebi Se­i­Z­­le­ba gavigoT rogorc kompleqsuri cvladebis mimarT ker­Zo­warmoebulebi. (10)–dan, erTi mxriv, cxadia,

, (11)

xolo meore mxriv,

. (12)

(9)da (12)-dan gamomdinareobs

, (13)

xolo (13)-dan -is mimarT integrebiT miviRebT



aqedan -iT kidev erTxel integrebiT gveqneba

,

saidanac z-iT integrirebis Semdeg jer miviRebT, rom

,

xolo Semdeg−

. (14)

imisTvis, rom  namdvili funqcia iyos, unda davuSvaT, rom

 (15)

(14)-dan, (15)-is gaTvaliswinebiT, gamomdinareobs

, (16)

sadac  da  nebismieri analizuri funqciebia.

**3. kolosov-musxeliSvilis formulebi**

cxadia, Tu (16)-s 2-ze gavamravlebT da -is -Ti, xo­lo -is -Ti aRvniSnavT, rac maTi nebismierobis ga­mo dasaSvebia, miviRebT:

 (17)

sadac  da nebismieri analizuri funqciebia.

(11)–is gaTvaliswinebiT, (17)-dan gveqneba

, (18)

. (19)

(18) da (19)-dan miviRebT

 (20)

sadac

.

hukis ganzogadebuli kanonidan gamomdinareobs, rom (ix. [52], gv. 107, formula (12))

 (21)

Tu (21)1-s mivumatebT -ze gamravlebul (21)2-s da mxed­ve­lo­baSi miviRebT (20)-s, gveqneba

 (22)

koSis (2.2.2) formulebidan, (5)-is Tanaxmad, gamomdina­reobs, rom

 (23)

 (24)

magram, Tu  mxebis dadebiTi mimarTulebaa (ix. damateba 4),

(25)

(23) da (24)-dan, (25)-is gaTvaliswinebiT, miviRebT

 (26)

(26)1-s mivumatoT -ze gamravlebuli (26)2:

 (27)

aqedan

. (28)

magram (ix. (20))

 (29)

amitom

 (30)

Tu elements ordinataTa RerZis mimarTuleba aqvs, maSin

 (31)

(30)-dan, misi marjvena mxaris diferencirebiT, (31)-is mxed­ve­lobaSi miRebiT da –ze Sekvecis Semdeg gveqneba

. (32)

Tu elements abscisTa RerZis mimarTuleba aqvs, maSin

(33)

gavamravloT (30) -ze, mis marjvena mxareSi movaxdinoT di­fe­rencireba, gaviTvaliswinoT (33) da SevkvecoT –ze:

. (34)

SevkriboT (32) da (34):

; (35)

(34)–s gamovakloT (32) da  SevcvaloT -–Ti (e.i. miRebul tolobaSi gadavideT SeuRlebulze da gaviTvaliswinoT is, rom Zabvis tenzori namdvili funqciebisgan Sedgeba):

,(36)

sadac



(22)-s, (35)-s da (36)-s ewodeba kolosov-musxeliS­vi­lis for­mule­bi. maTi daxmarebiT brtyeli drekadobis Teo­ri­is sasazRvro amocanebi daiyvaneba analizur funqciaTa Teo­riis sasazRvro amocanebis gamokvlevaze.

**damateba 7. uwyvet garemoTa**

**momenturi Teoria**

**1. moculobiTi (masobrivi) momenti**

**da momenturi Zabva**

gansxvavebiT uwyvet garemoTa klasikuri Teo­riisagan (ixileT nawili I) *moculobiTi (masob­rivi)* [volume (body) force] da *zedapirul Zalebis* (surface force) da *gadaad­gi­le­bis veqtoris* (displacement vector) cnebebTan (ixileT §1.1) erTad momentur TeoriaSi SemoRebulia *moculobiTi (masobrivi) momentis* [volume (body) couple], *momenturi Zabvis veqtoris* (couple stress vector) da *Sina­ga­ni brunvis (microrotation) cneba* [48, 53].[[26]](#footnote-26)\*) am TeoriaSi igulis­xme­ba, rom moculobis  usasrulod mcire elemen­tze (nawilakze) moqmedi Zalebi *statikurad ekvivalenturia* nawilakis masis centrSi mode­bul *moculobiTi Zalis* da  *mocu­lo­biTi momentis*, romelic wyvilZalas war­moad­gens. am sidideebis -ze gayofiTa da moculobis nulisken miswrafebis Semdeg miviRebT  da  veqtorebs, romlebic Sesabamisad warmoadgenen moculobis erTeulebze gaTvlil *moculobiT Zalas* da *moculobiT moments,* romlebic damo­ki­debulia  wer­til­ze da dinamikur SemTxvevaSi,  drozec. analogiurad, mi­Re­bulia, rom zedapiris usasrulod mcire elementze moq­medi Zalebi *statikurad ekvivalenturia*  zeda­pi­ruli Zalis da  zedapiruli momentis (wyvilZalis). -ze gayofisa da -is farTis nulisken miswrafebis Sem­deg miviRebT  da  veqtorebs, rom­le­bic, Sesabamisad, war­moad­genen zedapiris farTis erTeulz­e gansazRvrul *Zalur Zabvis veqtors an, ubralod, Zabvis veqtors* (traction, stress vector)da *momentur Zabvis veqtors*, romlebic damokidebulia  wertilze da dinamikur Sem­TxvevaSi  drozec. Zaluri da momenturi Zabvis veqtoris komponentebs, Sesa­ba­misad, *Zaluri* da *momenturi Zabvebi* ewodeba*.*

**2. momenturi Zabvis veqtoris damokidebuleba**

**zedapiris orientaciaze**

Tu ganvixilavT §1.2-Si ganxilul tet­raedrs (ix. nax. 1.2.1) da nuls gavutolebT, erTi mxriv, masze moqmedi ze­da­piruli Zalebis (e.i. Zaluri Zabvebis) da mocu­lo­biTi Za­lebis nakrebi veqtorebis jams, xolo meore mxriv, masze moqmedi momen­tebis: moculobiT da zedapiruli Zalebis nakrebi momentebis, moculobiTi momentebisa da momenturi Zabvebis nakrebi veqtorebis jams, Sesabamisad, mivi­RebT (1.2.1) da (ix. [48], gv. 14)

 (1)

formulebs (ix. [48], gv. 14), sadac  aris gansaxilvel wertilSi  normalis mqone zedapiris usasrulod mcire elementze (amas­Tan mniSvne­lo­ba ara aqvs zedapiris formas, mTa­varia yvela am zedapirs, romelic gansazRvrul wer­til­ze gadis, am wertilSi erTi da igive  normali hqon­des) moqmedi momenturi Zabvis veq­to­ris komponenti -ur RerZze, xolo  momenturi Zabvis ten­zo­ria. pir­veli for­mu­lis damtkiceba sityva-sityviT emT­xve­­va (1.2.1)-is gamoy­vanas §1.2-Si, xolo meore formulis damtkiceba, pirveli formulis analogiuria, im gan­sxva­ve­biT, rom amjerad mxo­lod momentebTan gveqneba saqme. mar­Tlac,  tetraedrze (misi zedapiris farTi aRvniS­noT –iT) moqmedi gare Za­le­bis nakrebi (mTavari) momen­tis e.i., moculobiTi Zalebis da Zaluri Zabvebis veq­to­re­bis nakrebi momentebis da moculobiTi momentebisa da mo­men­tu­ri Zabvebis veqtorebis nakrebi veqtoris jamis nul­Tan ga­to­lebiT miviRebT:



,

saidanac





am ukanasknelis -ze gayofisa da -is nulis­ken miswrafebis Semdeg, radgan



(1.2.1)-is gaTvaliswinebiT, miviRebT (1)-s.

**3. wonasworobis gantolebebi**

wonasworobis gantolebebis gamosayvanad §1.3-is analo­giu­rad nuls unda gavutoloT, erTi mxriv, garemodan (sxeu­li­dan) azrobrivad gamoyofil nebis­mier moculobis sxeu­lis  sazRvarze moqmedi gare Zaluri Zabvebis veqtorebis da -ze moqmedi moculobiTi Zale­bis nakrebi veqtorebis jami, xolo meore mxriv,  saz­Rvarze moqmedi Zaluri Zabvebis veqtorebis nakrebi momentis da momenturi Zabvebis veqtorebis nakrebi veqtoris da -ze moqmedi mo­cu­­lo­biTi Zalebis nakrebi momentis da mocu­lo­biTi mo­men­te­bis nakrebi veqtoris jami. pirvel Sem­TxvevaSi §1.3-is msje­lo­bis sityva-sityviT gameo­re­biT miviRebT (1.3.5) formu­lebs, xolo meore Sem­Txve­vaSi, analogiuri msjelobiT mivi­RebT Semdeg for­mu­lebs (ix. [48], gv. 20):

 (2)

sadac



levi-Civatas[[27]](#footnote-27)\*) simbolo (+1)-is ((-1)-is) tolia, Tu indeqsebis  mimdevroba miiReba 1, 2, 3 mimdevrobebidan luwi (kenti) raodenobis tran­spo­ziciiT da nulis tolia, Tu ori mainc indeqsi erTmaneTs emTxveva. Tu CavTvliT, rom momenturi Zabvis  da moculobiTi momentis  komponentebi nulis tolia, maSin gveqneba

 (3)

saidanac gamomdinareobs, rom



da miviRebT Zaluri Zabvis tenzoris simet­riu­lobas, rac (1.3.5) da (1.2.1) formulebTan erTad migviyvans klasikuri Zabvis Teoriis ZiriTad damokide­bu­le­beb­Tan. SevniSnoT, rom Zabvis mo­men­tur TeoriaSi, Zaluri Zabvis tenzori ar aris simetriuli. davam­tki­coT (2). cxadia,







{vinaidan

რadgan



xolo  ortis  RerZze gegmili



(1)-is da gaus-ostogradskis formulis ZaliT ki,



saidanac -s nebismierobis, integralqveSa gamosaxulebis uwyvetobisa da  veqtorTa sistemis wrfivad damoukideblobis gamo gamom­di­nareobs (2).

**4. gadaadgilebis da Sinagani brunvis veqtorebi**

uwyvet garemoTa meqanikis momentur Teo­ria­Si gadaad­gilebis veqtoris cneba Semodis uwyvet garemoTa meqanikis klasikuri Teoriis msgavsad (ix. §1.6), sadac garemo ganxilulia, rogorc uwyve­ti garemo Sedgenili maTematikuri wer­ti­lebis simravlisagan (erToblio­bisagan). Tu mas warmo­vid­genT, rogorc materialuri nawilakebis erTobliobas, suraTi ramdenadme Seicvleba. ganvi­xiloT garemos nebismieri nawilaki. vTqvaT, mosvenebul mdgoma­reo­baSi misi simZimis centri  wertilSi mdebareobs. Semo­vi­RoT am nawilakze xistad mimagrebuli dekartis marTkuTxa koor­di­natTa sistema saTaviT  wertilSi da Tavidan aRebuli uZravi dekartis marTkuTxa koor­di­nat­Ta siste­mis sakoordinato RerZebis paraleluri sakoordinato Rer­ZebiT. nawilaki CavTvaloT abso­luturad myar sxeulad. maSin misi moZraoba daxasiaTdeba Tavisuflebis eqvsi xaris­xiT, eqvsi skalaruli sididiT:  wertilis gadaad­gilebiT, romelic ganisazRvreba uZravi sistemis mimarT gadaad­gi­le­bis veqtoris sami komponentiT da nawilakis brunviT simZi­mis centris garSemo, romelic ganisazRvreba aseve sami skalaruli sididiT: moZravi sistemis Se­mob­­runebis kuT­xeebiT uZravi sistemis sakoor­di­nato RerZebis mi­marT, magaliTad, eileris kuT­xee­biT. erTi wonasworobis mdgo­­ma­reo­bidan meo­re wonasworobis mdgomareobaSi gadas­vlisas moZ­ra­vi sistema daikavebs axal mdebareobas uZ­ra­vi sistemis mi­marT; saxeldobr, Semobrundeba. mob­runebis kuTxeebi avRniS­noT -iT da SemoviRoT Sinagani brunvis



veqtori. axla Tu garemos kvlav warmovadgenT, ro­gorc uw­­yvets, Sedgenils maTematikuri wer­ti­le­bisagan, misi moZ­­­rao­ba (gadasvla erTi wonas­worobis mdgomareobidan meo­­re wonas­worobis mdgomareobaSi) daxasiaTdeba eqvsi ska­la­ru­li si­di­­diT, saxeldobr, gadaadgilebis veqtoris   da Sinagani brunvis veqtoris   kom­­po­nen­tebiT.

uwyvet garemoTa klasikur TeoriaSi nawilakis Tavi­suf­­le­bis xarisxi samia –  wertilis gadaadgilebebi, xo­lo bru­n­vis veq­to­ri ar gani­xi­leba, rogorc da­mou­ki­de­be­li ga­daad­gilebebisgan, ara­med maTTan dakav­Si­re­bu­lia (ix. [48], gv. 16)

 [[28]](#footnote-28)\*)

formulebiT, sadac  xisti brunvis veq­to­ria,  aris  veqtoris -uri kompo­nen­ti. mar­Tlac, ganvi­xi­loT aradeformirebuli garemos wertilis usasrulod mcire midamo da am midamos nebismieri  wer­tili (ix. §1.10). ganvi­xi­loT usasrulod mcire  veqtoris cvlileba defor­ma­ciis dros. am veq­toris sawyisi wertilia  xolo bolo wertilia  veqtoris  cvlileba



gamovTvaloT teiloris formuliT  usasrulod mcire si­di­dis mimarT maRali rigis sidideebis sizus­tiT, mivi­RebT:

 (4)

sadac  veqtoris komponentebia.

cxadia (ix. agreTve (1.10.3)-(1.10.5)),

 (5)

[[29]](#footnote-29)\*)  (6)

da, (4) da (5)-is ZaliT,



Tu , deformacia ar gvaqvs da rCeba mxolod xis­ti brunva.

amrigad, yoveli wertilis midamoSi deformacia SeiZ­le­ba ganvixiloT, rogorc usasrulod mcire veqtoris kom­ponentebis wrfivi da erTgvarovani funqcia.

**5. moZraobis gantolebebi, deformaciis**

**da grexa-Runvis komponentebi momentur TeoriaSi, deformaciis energia**

vTqvaT, sawyis momentSi garemo imyofeba arade­for­­­­mi­rebul mdgomareobaSi. ganvixiloT deformi­re­bu­li mdgo­­­ma­reo­ba  momentSi da ga­mov­TvaloT misi gamomwvevi ga­­re zemoq­me­de­bebis mu­Sao­ba. aseT zemoqmedebebs warmoad­ge­nen gare Za­luri da momenturi Zabvis veqtorebi da mo­cu­­­lo­biTi Zalebi da moculobiTi momentebi (ix. [48], gv. 29).

ramdenadac momenturi Zabvebis da moculobiTi momen­te­bis SemoReba dakavSirebulia Sinagan brun­vasTan, amitom bunebrivia CavTvaloT, rom momen­turi Zabvebi da mocu­lo­biTi momentebi mu­Saoben mxolod Sinagan brunvaze, xolo Zaluri Zabvebi da moculobiTi Zalebi mxolod gadaad­gi­lebaze.

-Ti aRvniSnoT aRniSnuli Zalebis mier Sesrule­bu­li muSaoba drois  SualedSi, xolo -Ti −  SualedSi.

Zaluri Zabvebisa da moculobiTi Zalebis mier Sesru­lebuli muSaoba gamovTvaleT parag­rafSi 1.18 (ix. (1.18.6) da (1.18.13)). axla gamov­TvaloT momenturi Zabvebisa da moculobiTi momentebis mier Sesrulebuli muSaoba. ganvi­xi­loT garemos  wertili mosvenebul mdgo­ma­reo­baSi ( momentSi).  momentSi misi mdgomareoba xasiaTdeba  gadaad­gilebis da  Sina­ga­ni brunvis veqtoriT.  drois Sua­ledSi -s nazrdi aRvniSnoT -Ti:



-s mimarT maRali rigis usasrulod mcire sidideebis sizustiT.

gare momenturi Zabvebis, romlebic zedapiris usasrulod mcire elementze moqmedeben, muSaoba  SualedSi

-s

tolia, aq  aris  zedapiris  wertilSi erTeu­lovani normali (gare  aris mimarT, romelic garemos ukavia), amasTan  wertili  usasrulod mcire elementis simZimis centria.

formula (1)-is Tanaxmad,  drois Sua­led­Si gare momenturi Zabvebis mier Sesru­lebuli muSaoba

 (8)

-ze moqmedi moculobiTi momentebis mier  SualedSi Sesrulebuli muSaoba

-s (9)

tolia.

amdenad, Tu gaviTvaliswinebT (1.18.6) tolobebSi bolos wina gamosaxulebas, (8)-s da (9)-s gveqneba, rom



 (10)

moZraobis gantolebebis misaRebad, dalam­be­ris principis Tanaxmad, moculobiT Zalas (1.3.5)-Si unda daematos inerciis Zala



xolo moculobiT moments (2)-Si - ”spinuri” momen­ti[[30]](#footnote-30)\*)



romelic Seesabameba Sinagan brunvas. aq aris spe­cia­lu­ri dinamikuri dadebiTi maxasiaTebeli (ix. [48], gv. 20) romelic masis (inerciis zomis) brunviTi analogia e.w. "brunviTi masa" (wriuli moZraobis dros , sadac  wrewiris radiusia). amdenad, (1.3.5)-dan gamomdinareobs (1.18.1), xolo (2)-dan

 (11)

erTad es ori sistema aRwers mikropolaruli uwyveti garemos moZraobas.

Tu (10)-is pirvel or integralSi, Sesabamisad, gaviTvaliswinebT (1.18.1)-s da (11)-s, (10) miiRebs

 (12)

saxes, sadac

 (13)

kinetikuri energiaa. Tu  (13)-dan miviRebT klasikuri Teoriis  kinetikuri energiis (1.18.8) gamosaxulebas.

-s mimarT maRali rigis usasrulod mcire sidideebis sizustiT



sadac  zemoaRniSnuli sizustiT fiqsirebuli -sTvis  funqciis -s mimarT nazrdis tolia. e.i., amave sizus­tiT, qvemoT moyvanil tolobebSi marcxena mxareebis -ze gam­ravlebiT da gayofiT, gveqneba:





am ukanaskneli tolobebis gaTvaliswinebiT, (12)-Si integ­ral­qveSa gamosaxuleba miiRebs

 (14)

saxes, sadac (5)-is da (6)-is Tanaxmad,

[[31]](#footnote-31)\*)  (15)

(am sidideebs ewodeba momenturi Teoriis defor­ma­ciis komponentebi), xolo



(am sidideebs grexa-Runvis komponentebi ewo­de­ba).

(12) da (14)-dan gamomdinareobs, rom

 (16)

sadac

 (17)

ramdenadac garemos  kinetikuri energia ar aris damokidebuli garemos deformirebul mdgo­mareo­ba­ze drois  momentSi, amitom  war­moad­gens kine­tiku­ri energiis nazrds drois  SualedSi. ma­Sin rodesac  damo­ki­de­bulia mxolod defor­mi­re­bul mdgomareobaze. mar­Tlac, Tu garemo bunebrivi mosvenebuli mdgomareobidan gamoyvanilia nebis­mie­ri defor­mi­rebiT (momenturi Teoriis Tvalsaz­ri­siT) da Semdeg ( momentSi) gadavida axal mosvenebul mdgomareobaSi, maSin  da aqedan gamom­di­na­re  amdenad,  warmoadgens defor­ma­ciis Sesabamis muSaobas drois  SualedSi, an rac igi­vea,



sadac  im muSaobis tolia, rac unda Seas­ru­los gare zemoqmedebebma (Zalurma da momenturma Zabvebma, moculo­biT­ma Zalebma da moculobiTma momentebma), rom gamoiwvion is defor­mire­buli mdgomareoba, romelic Seesabameba drois  mo­ments.

amrigad, (16) warmoadgens energiis Senaxvis ka­nons:  simZlavre kinetikuri da poten­ciu­ri energiebis cvlilebis siCqareTa jamis to­lia.

*deformaciis*  *xvedriTi energia* ewodeba defor­mi­rebis muSaobas, an rac igivea, *defor­ma­ciis poten­ciur energias,* gaTvlils moculobis erTeulze  wer­tilSi, drois  momentSi.

e.i. mTeli moculobis potenciuri energiaa



(17)-dan, cxadia, rom -s nazrdi drois  SualedSi

 (18)

gavSaloT  xarisxovan mwkrivad -is da -is mi­marT mosvenebuli mdgomareobis  mi­da­moSi da vixilavT ra mcire deformaciebs, ugul­vebel­vyoT -is da -is xarisxebi dawyebuli mesame rigidan:



 (19)

roca  da  maSin  amitom 

(18)-dan, cxadia,

 (20)

magram, (20)-is marcxena mxareSi miTiTebuli kerZo warmoebulebi SeiZleba gamovTvaloT (19)-is gawar­moe­biT; miRebuli gavutoloT (20)-is marjvena mxares:

(21)

 (22)

roca  da , maSin  da , ami­tom (21) da (22)-dan miviRebT, rom

 (23)

amdenad, (19)-dan gamomdinareobs, rom

 (24)

**6. konstituciuri damokidebulebebi.**

**mmarTveli gantolebebi gadaadgilebebSi da Sinagan brunvebSi. mikropolaruli siTxeebis hidromeqanikis ZiriTadi damokidebulebebi**

aqamde moyvanili yvela formula samarT­lia­nia nebismieri uwyveti garemosTvis. ganvixiloT *drekadi deformadi myari sxeuli simetriis centriT*. mas axasiaTebs drekadi mudmivebis invariantoba sakoordinato RerZebis mimar­Tu­le­bis maTi sawi­naaR­mdegoTi Secvlis dros. am dros -ebi ar icvleba, xolo -ebi niSans icvlian[[32]](#footnote-32)\*) da (24)-dan miviRebT, rom

 (25)

da, amdenad,

 (26)

radgan  da  ar icvleba  da  indeqsebis gadas­mis dros, zogadobis SeuzRudavad[[33]](#footnote-33)\*), SegviZlia CavTva­loT, rom

 da  (27)

(23), (25) da (27)–is Tanaxmad, (21) da (22)-dan, drekadobis momenturi TeoriisaTvis miviRebT hukis Semdeg ganzogadebul kanons

 (28)

sadac  drekadi mudmivebia, rom­le­bic eqsperimentulad dgindeba. dre­ka­do­bis klasikur Teo­ria­Si isini dadgenilia, xolo momentur TeoriaSi jer­je­ro­biT ar aris dad­ge­nili.

cxadia, es formulebi mxolod myari drekadi sxeu­le­bis­Tvi­saa samarTliani. izotropuli (si­met­riis centriT) sxeu­lis­Tvis hukis gan­zo­gadebul kanons aqvs (ix. [48], gv. 33):

 (29)

 (30)

saxe, sadac  drekadi mudmivebia. Tu Cav­TvliT, rom Sinagani brunvis veqtori nulis tolia, dre­ka­dobis mo­menturi Teoriis hukis ganzogadebuli kanonidan mivi­RebT klasikuri drekadobis Teoriis hukis ganzo­ga­de­bul kanons.

gansaxilvel SemTxvevaSi deformaciis xvedriT ener­gias aqvs (ix. [48], gv. 34)



saxe. am kvadratuli formis dadebiTad gansazRvrulo­bi­saT­­vis aucilebeli da sakmarisia Sesruldes Semdegi piro­bebi:



mis misaRebad (29) da (30) unda CavsvaT (SeadareT (18)-s)



formulaSi.

SevniSnoT, rom iqidan, rom (ix. [48], gv. 110)



gamomdinareobs



sadac  da  nebismieri mudmivi veqtorebia, rac Seesaba­me­­ba xist moZraobas (xist gadaadgilebas), e.i. xist mob­ru­­­nebas da xist gadatanas RerZebis paralelurad. rogorc mo­sa­­lod­neli iyo deformaciis (xvedriTi) energiis nul­Tan to­­lobis SemTxvevaSi gvrCeba mxolod xisti gadaad­gi­le­ba, amasTan Sinagani brunva mudmivia da emTxveva xist mob­ru­ne­bas.

Tu (15)-s CavsvamT (29)-Si da miRebuls (30)-Tan erTad CavsvamT (1.18.1)-sa da (11)-Si, miviRebT drekadobis momenturi Teoriis mmarTvel gan­to­lobebs gadaadgilebebSi da Sinagan brunvebSi (ix. [48], gv. 44):

 (31)



 (32)

marTlac, (15)-is Casma (29)-Si gvaZlevs





gadavsvaT  da :

 (33)



es ukanaskneli CavsvaT (1.18.1)-Si:





aqedan





-ze gamravlebis da -s mimarT ajamvis Semdeg miviRebT





saidanac gamomdinareobs (31).

(30)-is da (33)-is (11)-Si Casma da (15)-is gaTvaliswi­ne­ba gvaZlevs







aqedan



e.i.,

 (34)



radgan, Tu gaviTvaliswinebT (6\*)-s,

 (35)

(34) igivea rac





-ze gamravlebis da -s mimarT ajamvis Semdeg miviRebT (32)-s.

samecniero literaturaSi drekadobis momen­turi Teo­ria gvxvdeba sxvadasxva saxelwodebiT: asimet­riuli dreka­do­bis Teoria, koserebis (Cosserats’) Teoria, dreka­do­bis Teo­ria nawi­la­kebis brunviTi zemoqmedebiT, *mikro­pola­ru­li dre­kadobis Teoria*, aralokaluri drekadobis Teoria, meore klasis garemoTa drekadobis Teo­ria.

drekadobis momenturi Teoria ar warmoad­gens kla­sikuri dre­kadobis Teoriis maTema­ti­kur gan­zo­gadebas Teoriuli inte­residan gamom­dinare. mis Seqmnas biZgi misca, magaliTad, iman, rom mokle akustikuri talRebis gavrceleba maRa­li ri­gis polimerebSi[[34]](#footnote-34)\*) da sxva msxvil ­mar­cvlo­van sxeulebSi arada­mak­mayofileblad aRi­we­reba kla­si­­kuri dre­ka­do­bis Teoriis far­gleb­Si.

aseve uZluria navie-stoqsis klasikuri Teo­ria axsnas is efeqti, rom arsebobs minarevis saxiT polimeruli nawi­la­kebis Semcveli iseTi siTxe, romliTac myari sxeu­lis gar­sdenisas war­moSobili zedapiruli xaxuni 30-35%-iT nak­­le­bia, vidre iseTi siTxiT garsdenisas, romelic ar Sei­cavs aseT poli­me­rul minarevs. es da sxva magaliTebi gaxda siTxeebis momen­tu­ri Teoriis Seqmnis safuZveli. am Teo­rias ewodeba *mikro­po­la­ru­li siTxeebis hid­ro­me­qa­nika.*

blanti arakumSvadi mikropolaruli siTxis aras­ta­cio­­naluri dinebis ZiriTad gantolebaTa sistema miiReba (1.18.1), (11) sistemidan da mas aqvs

 (36)



 (37)

saxe, sadac  da  Sesa­ba­mi­sad, warmoadgenen mikropolaruli siT­xis yoveli werti­lis moZraobis damaxasiaTebel siCqarisa da misgan damou­ki­de­bel Sinagani mikrobrun­vis  siCqaris veqtorebs, aris inerciis momenti.

(36), (37) gantolebebs unda daematos uwyve­to­bis gantolebac (ix. 1.23.11), romelsac arakum­Sva­di siTxis  SemTxvevaSi aqvs

 (38)

saxe.

blanti, erTgvarovani, izotropuli, arakum­Sva­di[[35]](#footnote-35)\*) mikropolaruli siTxis SemTxvevaSi kon­sti­tu­ciur damoki­de­bulebebs – niutonis gan­zo­ga­debul ka­nons aqvs

 (39)

 (40)

saxe, sadac  aris wneva,  da  mudmivebi axasia­Teben siTxis fizikur Tvisebebs, xolo

 (41)

SevniSnoT, rom Tu



maSin dadebiTad gansazRvrulia deformaciis



 (42)

xvedriTi energia[[36]](#footnote-36)\*\*).

Tu (41)-s CavsvamT (39)-sa da (40)-Si, xolo Semdeg miRebul gamosaxulebebs, Sesa­ba­misad, (36)-sa da (37)-Si – miviRebT blanti arakumSvadi, er­T­gva­­ro­vani, izotropuli mikro­po­laruli siTxis arasta­cio­­na­lu­ri dinebis mmar­Tvel



 (43)



 (44)

 (45)

sistemas.

marTlac, Tu (41)-dan pirvels CavsvamT (39)-Si, xolo miRebuls – (36)-Si, gveqneba





aqedan, Tu gaviTvaliswinebT (38)-s, miviRebT





saidanac, -ze gamravlebisa da -s mimarT ajamvis Semdeg, gamomdinareobs (43).

Tu (41)-dan pirvels da meores CavsvamT, Sesabamisad, (39)-sa da (40)-Si, xolo miRebul gamosaxulebebs – (37)-Si, gveqneba









aqedan









e.i., Tu mxedvelobaSi miviRebT (35)-s,





saidanac, -ze gamravlebisa da -s mimarT ajamvis Semdeg, gamomdinareobs (44).

**7. drekadobis momenturi Teoriis ZiriTadi amocanebi**

 sivrce gluvi Sekruli  zedapiriT davyoT or Semo­sazuRvrul  da SemousazRvrav  ared.

**dinamikis amocanebi.**  areSi ganvsazRvroT mocemuli moculobiTi Zalisa da moculobiTi momentis Sesabamisi drekad-dinamikuri mdgomareoba drois  SualedSi Semdegi sawyisi pirobebisa





da erT-erTis saSualebiT Semdegi sasazRvro pirobebidan



(amocana ) an



(amocana ) roca  sadac “+” da “-” Seesabameba  da  areebidan zRvrul mniSvnelobebs, xolo  da  mocemuli veqtor-funqciebia.

drekad-dinamikuri mdgomareobis gansazRvra maTematiku­rad gulisxmobs (31), (32) sistemis amonaxsnis povnas veq­tor-funqciaTa garkveuli klasidan, romlebic akmayofi­le­ben zemoT miTiTebul sawyis da sasazRvro pirobebs  aris SemTxvevaSi usasrulobaSi garkveul damatebiT piro­beb­sac).

sasazRvro pirobebi SeiZleba Seicvalos Sereuli sa­saz­­Rvro pirobebiTac (ix [48], gv. 57).

statikis SemTxvevaSi ganixileba drekadi wonas­wo­ro­bis gansazRvris amocanebi mxolod zemoT miTiTebuli sasaz­Rvro pirobebiT sawyisi pirobebis gareSe, radgan amo­nax­sni  droze ar aris damokidebuli, amasTan (31), (32) sis­temis marjvena mxareSi miTiTebuli sidideebi nulebiT icvleba.

amocanebis amonaxsnebis erTaderTobis sakiTxis gamok­vle­va klasikuri drekadobis Teoriis SemTxvevis analo­giu­ria da Sedegic emTxveva (ix. [48], gv. 110). dinamikis amo­ca­nebs erTaderTi amonaxsni aqvs, xolo statikis Sem­Txve­va­Si, amocana -s aqvs erTaderTi amonaxsni, amo­ca­na -is amonaxsni ki ganisazRvreba xisti gadaad­gi­le­bis si­zus­tiT, Tu  sakma­ri­sad gluvia da . amo­cana -s da amo­ca­na-s aqvs erTaderTi amonaxsni, Tu sakmarisad glu­via;  da usasrulobis mida­mo­Si srul­de­ba

; ,

pirobebi. amonaxsnebis arsebobis Taobaze ix. [48], gv. 351.

**8. mikropolaruli siTxeebis Teoriis ZiriTadi amocanebi**

wina paragrafSi gansazRvruli  da  areebSi ve­ZebT (43)-(45) sistemis  amonaxsnebs garkveuli kla­sidan, romlebic akmayofileben



sawyis da  sazRvarze an



an



[[37]](#footnote-37)\*)



sasazRvro pirobebs, sadac  mocemuli veqtor-fun­qciebia (aris SemTxvevaSi moiTxoveba damatebiTi pi­ro­bebi usasrulobaSic)[[38]](#footnote-38)\*).

1. \*) mas uwodeben agreTve koSis utolobas. [↑](#footnote-ref-1)
2. \*)evklide (daax. 340-daax. 287 Cv. w. aR.-mde). [↑](#footnote-ref-2)
3. \*)d. hilberti (1862-1943). [↑](#footnote-ref-3)
4. \*\*)s. banaxi (1892-1945). [↑](#footnote-ref-4)
5. \*)i.h. grami (1850-1916). [↑](#footnote-ref-5)
6. \*)i.p. Sauderi (1896-1943). [↑](#footnote-ref-6)
7. \*) f.v. beseli (1784-1846). [↑](#footnote-ref-7)
8. \*\*)m.a. parsevali (1755-1836). [↑](#footnote-ref-8)
9. \*)f. risi (1880-1956). [↑](#footnote-ref-9)
10. \*\*)h. hani (1879-1954). [↑](#footnote-ref-10)
11. \*)k.o. fridrixsi (1901-1982). [↑](#footnote-ref-11)
12. \*)r.o.s. lipSici (1832-1903). [↑](#footnote-ref-12)
13. \*)a.l. lebegi (1875-1941). [↑](#footnote-ref-13)
14. \*)l. Svarci (1915-2002). [↑](#footnote-ref-14)
15. \*)b.o. rodrigi (1794-1851). [↑](#footnote-ref-15)
16. \*) radgan leJandris (17) gantolebis zogad amonaxsns aqvs (ix [45], gv. 472)

    saxe, sadac  aris leJandris mikavSirebuli funqcia, romelic ar aris polinomi. [↑](#footnote-ref-16)
17. \*) saSualod krebadobisTvis (e.i. -Si) warmoebulze piroba saWiro ar aris (ix. Teorema 4 da [37], gv. 502). ufro metic, Tu , Sesabamisi furie-leJandris mwkrivi krebadia *f*–sken, -is normiT. [↑](#footnote-ref-17)
18. \*) damtkiceba mogvawoda d. natroSvilma. [↑](#footnote-ref-18)
19. \*)am ukanasknelis gaTvaliswinebiT, (18)-dan, cxadia,

    da

    ,

    sadac  puasonis koeficientia. [↑](#footnote-ref-19)
20. \*)Jaiani G. Cusped Shell-like Structures, Springer, Heidelberg

    -Dordrecht-London-New York, 2011 [↑](#footnote-ref-20)
21. \*\*)e.ი.

    kerZod, esukanaskneli sruldeba, Tu

    an da , roca. es ukanaskneli, ki Sesruldeba, Tu , roca . Tu -smivaswrafebT nulisken (e.i. wamaxvilebuli napirisken), miviRebT , rac mxolod maSin Sesruldeba, roca , radgan . [↑](#footnote-ref-21)
22. \*) am modelSi iTvleba, rom misi simciris gamo  (Tumca vTvliT, rom ), , , , kenti funqciebia (amdenad, , ). [↑](#footnote-ref-22)
23. \*) 

     funqciis ageba xdeba meore gvaris wiriTi integraliT, romlis gzisgan damoukideblobis aucilebeli da sakmarisi piroba, rogorc es analizidanaa cnobili, aris (1)1. [↑](#footnote-ref-23)
24. \*\*)  [↑](#footnote-ref-24)
25. \*\*\*)  [↑](#footnote-ref-25)
26. \*)ix. agreTve

    1. Cosserat E., Cosserat. F., Sur leséquations de lathéorie de I’élasticité, C.R. Acad. Sci.Paris 126 (1898), 1129-1132

    2. Cosserat E., Cosserat. F., Théorie des corps déformable, Hermann, Paris, 1909

    3. Voigt W., Theoretische Studien über die Elastizitätsverhältnisse der Kristalle, Abh. Der Königl. Ges. Wiss., Göttingen, 34, 1887

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    5. Dyszlewicz J., Micropolar Theory of Elasticity, Lecture Notes in Applied and Computational Mechanics,  vol. 15, Springer-Verlag Berlin Heidelberg GmbH, 2004

    6. Ieşan D., Classical and Generalized Models of Elastic Rods, CRC Press, A Chapman and Hall Book, 2009 (Chapters 5 and 6) [↑](#footnote-ref-26)
27. \*) tulio levi-Civita (Tullio Levi-Civita, 29.3.1873-29.12.1941) [↑](#footnote-ref-27)
28. \*) radgan . [↑](#footnote-ref-28)
29. \*) cxadia,

    aqedan

    analogiurad,

    aseve cxadia, radgan, (6)-is Tanaxmad,

    vinaidan

     e.i. 

    analogiurad,

    da, sabolood,

     (6\*) [↑](#footnote-ref-29)
30. \*) aq simartivisaTvis elementarul moculobad aRebulia birTvi Tanabrad ganawilebuli masebiT (simetriuli bzriala). zogad SemTxvevaSi  icvleba  veqtoriT, sadac  deformaciamdel mdgomareobaSi simkvrivea, xolo  inerciis matrici. -is ewodeba inerciis koeficientebi. inerciis “spinuri” momenti  da  matrici masis brunviTi analogebia. [↑](#footnote-ref-30)
31. \*) cxadia , magram, sazogadod,  [↑](#footnote-ref-31)
32. \*) radgan RerZebis mimarTulebis sawinaaRmdegoTi Secvlisas ga­daa­d­gilebebi da warmoebulebi niSans icvlian, xolo brunvis kuT­xe ni­­Sans ar icvlis, vinaidan sistemis marjvena orientacia mar­cxe­naTi icvleba da kuTxis aTvlis dadebiTi mimarTuleba saa­Tis is­ris moZraobis sawinaaRmdego mimarTulebidan icvleba saa­Tis is­ris moZraobis mimarTulebiT. SevniSnoT, rom sibrtyeze Rer­Ze­bis mimarTulebis Secvla sistemis orientaciis Secvlas ar iw­vevs. [↑](#footnote-ref-32)
33. \*) radgan, magaliTad, Tu  maSin

    sadac

     [↑](#footnote-ref-33)
34. \*) polimerebi warmoadgens maRali molekuluri masis (ramdenime aTasidan ramdenime milionamde) qimiur naerTebs, romelTa mole­ku­lebi (makromolekulebi) Sedgeba ganmeorebadi mravalric­xo­vani dajgufebebisagan (monomeruli rgolebisagan). ganas­xva­ve­ben buneb­riv (magaliTad, cilebi) da sinTezur (maga­li­Tad, po­lie­Tileni, poliamidi, epoqsiduri fisebi). [↑](#footnote-ref-34)
35. \*) radgan am SemTxvevaSi (ix. (41)), (38)-is da -s gamo,

    amitom (39)-Si  (Seadare (1.13.7)-s) Sesakrebi ar gveqneba.

    [↑](#footnote-ref-35)
36. \*\*) (42)-s miviRebT, Tu  formulaSi CavsvamT (39) da (40) gamosaxulebebs [↑](#footnote-ref-36)
37. \*) radgan, (1.2.1)-is, (39)-is da (41)-is gaTvaliswinebiT,

    xolo , aq  aris –s normaluri mdgeneli. [↑](#footnote-ref-37)
38. \*) Teoria da gamoyenebebi ix. wignSi Lukaszewicz Gz., Micropolar Fluids - Theory and Application, Birkhaüser, 1999. [↑](#footnote-ref-38)