Ilia Vekua – 100

IUTAM Symposium

23-27 April, 2007, Tbilisi

RELATION OF SHELL, PLATE, BEAM,
AND 3D MODELS

Book of Abstracts

The abstracts are submitted by the authors in camera ready form. Readers are advised to keep in mind that statements, data illustrations, or other items may inadvertently be inaccurate.

IUTAM Symposium, 23-28 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

IUTAM

IUTAM SYMPOSIUM
ON
RELATION OF SHELL, PLATE, BEAM, AND 3D MODELS

Dedicated to Centenary of Ilia Vekua

April 23 – 27, 2007, Tbilisi, Georgia

International Scientific Committee

Philip G. Ciarlet (Hong Kong)
Anatoly Gerasimovich Gorshkov (Russia)
Jorn Hansen (Canada)
George V. Jaiani (Georgia), Chairman
Reinhold Kienzler (Germany)
Herbert A. Mang (Austria)
Paolo Podio-Guidugli (Italy)
Gangan Prathap (India)

IUTAM Representative D. (Dick) H. van Campen (Netherlands)

Local Organizing Committee

Gia Avalishvili (I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University)
Natalia Chinchaladze (I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University), Secretary
David Gordeziani (Iv. Javakhishvili Tbilisi State University)
George Jaiani (I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University), Chairman
Gela Kipiani (Georgian Technical University)
Tengiz Meunargia (I. Vekua Institute of Applied Mathematics of Iv. Javakhishvili Tbilisi State University)
Nugzar Shavlakadze (A.Razmadze Mathematical Institute)
Ilia Tavkhelidze (Iv. Javakhishvili Tbilisi State University)
Tamaz Vashakmadze (Iv. Javakhishvili Tbilisi State University)

Contents

AN ASYMPTOTIC METHOD OF SOLVING THREE-DIMENSIONAL BOUNDARY VALUE PROBLEMS OF STATICS AND DYNAMICS OF THIN BODIES

Lenser A. Aghalovyan
Institute of Mechanics of NAS of Armenia
Marshal Baghramyan ave. 24b
Yerevan, 375019, Armenia

The equations of a space problem of elasticity theory for thin bodies (bars, beams, plates, shells) in dimensionless coordinates are singularly perturbed by a geometrical small parameter. The general solution of similar systems of equations is combined with the solutions of inner problem and boundary layers.

Iteration processes permitting to determine the inner problem solution, as well as the boundary problem solution with beforehand given exactness are built by an asymptotic method. In case of the first boundary value problem (on the facial surfaces of the thin body the stresses tensor components are given) a connection of the asymptotic approach with the classic theory of beams, plates and shells, with more precise theories is established.

In case of a plane first boundary value problem for a rectangular a connection of the asymptotic solution with Saint-Venant principle is established and its correctness is proved.

Asymptotic orders of the stresses tensor components and the displacement vector in the second and mixed boundary value problems for thin bodies are established inapplicability of classical theory hypothesis when solving these problems is proved.

Free and forced vibrations of beams-strips and plates, including anisotropic and layered are considered by an asymptotic method. The connection of free vibrations frequencies values with the velocities of seismic shear and longitudinal waves propagation is established. In a three-dimensional statement forced vibrations of two-layered, three-layered and multilayered plates under the action of seismic and other dynamic loadings are considered, the conditions of resonance rise are established.

Theoretical justification of expediency of using the seismoisolators in seismosteady construction is given.

The areas of mechanics of solid medium, in which the application of the asymptotic method permits us to solve rather complicated three-dimensional problems, are mentioned.

ON THE DIFFERENT POSSIBILITIES TO DERIVE PLATE AND SHELL THEORIES

Holm Altenbach
Lehrstuhl für Technische Mechanik, Zentrum für Ingenieurwissenschaften, Martin-Luther-Universität Halle-Wittenberg, D-06099 Halle (Saale), Germany

Johannes Meenen
Brabantstraße 10 –18, D-52070 Aachen, Germany

Plate and shell theories are two-dimensional representations of thin, three-dimensional bodies. For their derivation, different techniques can be used. In the so-called direct approach, the derivation starts from the postulation of a two-dimensional Cosserat surface, which represents the mechanical behaviour of the plate mid-surface. After that the two-dimensional field equations can be deduced in such strong way like the three-dimensional continuum mechanics, but assumptions are necessary to identify the parameters in the two-dimensional constitutive equations. In the pioneering work of Kirchhoff, the derivation starts from the three-dimensional equations of continuum mechanics and aims at reducing this system of equations to a two-dimensional theory by eliminating the dependence of the independent unknowns from the thickness direction. There are different techniques that can be used for this reduction of complexity. A very common approach is the formulation of assumptions or the use of series expansions, which are often introduced in a weak form such as the principle of virtual displacements. Other authors use mathematical techniques such as the method of asymptotic expansion.

Even with the latest developments in numerics and computer techniques, there is general consent that plate and shell theories are very effective analysis tools which cannot be replaced by full three-dimensional theories. The elimination of the thickness direction however introduces an approximation error, whose magnitude depends on the slenderness and curvature of the shell, the loading pattern and the anisotropy of its material. Several authors have investigated how this approximation error can be reduced. For this purpose, series expansions or assumptions with additional degrees of freedom have been formulated, among which the shear deformation theories of Reissner and Mindlin are well-known examples. Geometrically nonlinear plate and shell theories can be derived by introducing estimates regarding the size of the different components of the displacement gradient into the principle of virtual work. These estimates can also be used to classify the loading into membrane dominated, compression dominated, bending dominated and transverse shear dominated loading. It is shown that von Kármán's plate theory can be derived in a consistent way if the stress resultants are calculated from the second Piola-Kirchhoff stresses. The use of virtual displacements which fulfil the Kirchhoff assumptions and its implications are reviewed for the geometrically nonlinear principle of virtual work.

HIERARCHY AND APPLICATION OF THREE-DIMENSIONAL MODELS OF THICK ANISOTROPIC SHELLS

Alexander A. Amosov, Sergey I. Zhavoronok
Department of of Structural Mechanics
Moscow State University of Civil Engineering
Yaroslavskoe shosse 26, 129337, Moscow, RUSSIA

A based on three-dimensional elasticity problem's reduction generalized approach to construction of some thick anisotropic shell's theories is proposed.

A linear initial-boundary value problem is as linear operator's system in Hilbert spaces of tensor functions considered. Quadratic Euclid norm and metric and tensor function's sets as basis are for these spaces introduced. Linear operator's tensor form with second rank linear transform's tensor which coordinates are by projection operator's type defined is shown with different projectors giving classical Galerkin method, Petrov-Galerkin method etc. The continuum mechanic's problem is to the system of linear tensor equations converted [1].

It is also shown that using the proposed reduction approach for special coordinates with metric $g_{i3} = \delta_{i3}, g^{i3} = \delta^{i3}$ and with Legendre's polynomials as a scalar basis [2], [3] the three-dimensional elasticity problem can be to the set of different shell's theories [2], [4] transformed. Unlike [4], the full quadratic function for the metric of three-dimensional space is introduced [2], [3], the mixed boundary-value problem statement is formulated, and no only middle surface can be for geometry's parameterization used. The N-th order theory of variable-thickness anisotropic laminated shell constructed.

A hierarchy of thick shell's models based on general N -th order theory is discussed. Some applications of the high-order three-dimensional shell theories to static and dynamical problems are shown.

REFERENCES

[1] **С.И. Жаворонок,** Проекционный подход к решению задач механики // Материалы XI Международного симпозиума «Динамические и технологические проблемы механики конструкций и сплошных сред». – М: Издво МАИ, 2005. Т. 2. С.8992

[2] **А. А. Амосов,** Приближенная трехмерная теория толстостенных пластин и оболочек // Строительная механика и расчет сооружений, 1987, N5. С. 3742

[3] **А. А. Амосов, С. И. Жаворонок,** Редукция трехмерной задачи механики сплошной среды к системе двумерных начальнокраевых задач // Материалы X Международного симпозиума «Динамические и технологические проблемы механики конструкций и сплошных сред». – М: Издво МАИ, 2004. Т. 2. С.1725

[4] **И. Н. Векуа,** Некоторые общие методы построения различных вариантов теории оболочек. – М: Наука, 1982. – 282 с

HIERARCHICAL MODELING OF MULTISTRUCTURES

Mariam Avalishvili, Gia Avalishvili, David Gordeziani
I.Vekua Institute of Applied Mathematics
of Iv. Javakhishvili Tbilisi State University

In the theory of elasticity and mathematical physics lower-dimensional models are often preferred to the three-dimensional ones because of their simpler mathematical structure and better amenability to numerical computations.

One of the widely used approaches for constructing the lower-dimensional models is hierarchical modeling. The main idea of these methods is construction of a sequence of subspaces with special structure approximating the spaces corresponding to the original three-dimensional problem and on these subspaces the lower-dimensional problems are obtained.

One of the methods of constructing the hierarchical models for prismatic shells was suggested by I. Vekua, which was based on the approximation of the components of the displacement vector-function by partial sums of the orthogonal Fourier-Legendre series with respect to the variable of plate thickness. However, I. Vekua considered boundary and initial boundary value problems only in the spaces of classical regular functions and didn't investigate the relation of the constructed two-dimensional models to the original three-dimensional ones. For static boundary value problems, the existence and uniqueness of solutions of the reduced two-dimensional problems obtained by I. Vekua in Sobolev spaces first were investigated by D. Gordeziani and in his papers the rate of approximation of the exact solution of the three-dimensional problem by vector functions restored from the solutions of the reduced problems in the spaces of classical regular functions was estimated. Later on, applying I. Vekua's method and its' generalizations various hierarchical models for shells and rods were constructed and investigated by I. Babuška, V. Vogelius, S. Jensen, C. Schwab, W. Wendland and Georgian mathematicians T. Meunargia, T. Vashakmadze, G. Jaiani and others.

In the present paper we employ and extend I. Vekua's approach for linearly elastic shells, curvilinear rods and multi-structures. We consider static and dynamical three-dimensional problems in curvilinear coordinates for elastic shells and applying variational approach we construct a hierarchy of two-dimensional models. In the case of elastic curvilinear rods we obtain a hierarchy of one-dimensional models. We investigate the existence and uniqueness of solutions of the reduced two-dimensional and one-dimensional problems, prove convergence of the sequence of vector functions of three space variables restored from the solutions of the reduced problems to the exact solution of the original problem and estimate the rate of convergence. Applying the results obtained for shells and rods we construct and investigate hierarchical models for multistructure, which consists of three-dimensional body, shell and rod. We obtain mathematical models for multistructure which are defined on the product of three-dimensional, two-dimensional and one-dimensional domains. Moreover, we study the relation of the constructed hierarchical models to the original three-dimensional ones for dynamical as well as for static problems.

IUTAM Symposium, 23-28 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

3D INVESTIGATION OF BENDING FREE VIBRATIONS IN FERROMAGNETIC RECTANGULAR FREE SUPPORTED PLATES

Alexander G. Bagdoev
Institute of Mechanics NAS Armenia
Yerevan. M. Baghramyan av. 24b, Armenia

Yuri S. Safaryan
Goris state University
Avangard street 2, Goris, Armenia

By developed for elastic plates method [1], consisting in exact solution of three-dimensional (or two-dimensional for plate-layer) equations of motion and satisfying of boundary conditions, the problem of determination of dispersion relation for bending vibrations in ferromagnetic rectangular free supported plates is solved analytically and numerically. The undisturbed magnetic field is constant and perpendicular to middle plane of plate. The exact particular solution of magnetoelastic media equations as well as of electromagnetic induction equation is looked for in form of standing waves, satisfying the free boundary conditions on edges of plate. The resulting relations betweenconstants characterising amplitudes of displacements and magnetic fields are the same as for infinite plate [2,3]. The boundary conditions on plate surface connecting solution in plate with magnetic field in dielectic out of it are satisfied, and the dispersion relation in form of thirth orther determinant equation is obtained. The approximate formula for frequency for relatively small magnetic fields is obtained. Furthermore the mentioned

determinant equation is solved numerically giving exact solution for arbitrary magnetic fields. The obtained results by mentioned methods are in good agreement of one another. The obtained tables for reel parts of frequency are compared with results due to averaged theory [2] based on Kirkhoff hypothesis, and it is shown that as for infinite electroconductivity [2], as for finite conductivity the calculated frequencies by our exact space treatment and by hypothesis are quite different.

REFERENCES

[1] **V. Novatsky,** The elasticity. M.: Mir. 1975. (In Russian).

[2] **Yu. S. Safaryan,** The determination of bending magnetoelastic vibrations frequencies of plates in threedimensional and averaged statement of problem // Doklady of Russian Academy of Sciences. 2002. v.383. N6 pp.767-770 (In Russian).

[3] **A. G. Bagdoev, S. G. Sahakyan,** The stability of non-linear modulation waves in magnetic field for threedimensional and averaged problems // Izvestia of Russian Academy of sciences. *Solid mechanics*. 2001. N5. pp.35-42.

THE ANALYTICAL AND NUMERICAL INVESTIGATION OF FREE BENDING VIBRATIONS OF FERROMAGNETIC CYLINDRICAL SHELL BY EXACT SPACE TREATMENT

Alexander G. Bagdoev, Anna V. Vardanyan, Sedrak.V. Vardanyan

Institute of Mechanics NAS Armenia

Yerevan, M. Baghramyan av. 24b, Armenia

The exact space treatment for derivation of dispersion relation for elastic plate is developed by W.Nowacki [1]. The application of the treatment to bending waves in magnetoelastic plates has been done in [2]. It is shown that in contrast to the purely elastic case the averaged theory, based on Kirchhoff hypothesis in magnetoelastic case is not true. Here the more general case of cylindrical ferromagnetic shell is considered by space treatment. The exact solution of axially symmetric equations of magnetoelasticity satisfying the boundary conditions on shell surface is reduced to solution of dispersion equation for frequency as function of wave number in form of sixth order determinant in left hand side of equation in addition to third order algebraic equation for additional parameters.

Numerical results for various values of axially magnetic fields are carried out. The comparison with results obtained in [3] by formula based on Kirchhoff's hypothesis shows that these hypothesis for magnetoelastic thin bodies is not applicable.

REFERENCES

[1] **W. Nowacki,** Theory of Elasticity. M.: Mir, 1975. 863p. (in Russian).
[2] **A. G. Bagdoev, S. G. Sahakyan,** The stability of nonlinear modulation waves in magnetic field for space and averaged problems// Izvestia of Russian Academy of sciences. MTT. Rigid body mechanics. 2001. N 5, pp. 35-42 (in Russian).
[3] **G. E. Bagdasaryan, M. V. Belubekyan,** Axiasymmetric vibrations of a cylindrical shell in magnetic field// Izv. AN Arm, SSR. Mechanika. 1967. V. 20. N5, pp 21-27 (in Russian).

STABILITY OF A RECTANGULAR PLATE WITH ACCOUNT OF TRANSVERSE SHEAR DEFORMATIONS

Vagharshak Belubekyan
Yerevan State University
Yerevan, Armenia

Rectangular plate, axially compressed along the edges $x = 0,\ a$ with a uniform load, $p = 2h\sigma_0$ is considered. Two different refined theories of plate bending are applied: the refined theory by S.A. Ambartsumyan [1] and refined theory by E. Reissner [2,3]. The stability equations, resulting from the both theories can be rewritten in a unified form as follows:

$$\Delta^2 w - \frac{1-\nu}{2G}\sigma_0\left(1+\frac{6\kappa}{1-\nu}\right)\frac{\partial^2}{\partial x^2}\Delta w$$
$$+\frac{3\kappa(1-\nu)}{2G^2}\sigma_0^2\frac{\partial^4 w}{\partial x^4}+\frac{2h\sigma_0}{D}\frac{\partial^2 w}{\partial x^2}=0,$$

where w is the deflection, D is the bending stiffness, ν is the Poisson's ratio, G is shear modulus, σ_0 is the compressive load and κ =2/5 in the theory [1] , and κ =1/3 in the theory [2]. Additionally two similar equations for unknown potential functions Φ and Ψ, which describe the transverse shears, are obtained. Even though the equations for the unknown functions w, Φ, Ψ are autonomous, the problem in general is coupled, because all these functions are combined in boundary conditions. At $x = 0$, boundary conditions of hinged edge are assumed. At the edges $y = 0, b$ several different boundary conditions are considered, in particular:

1. At $y = 0$ sliding contact.
2. At $y = 0$ sliding contact , at $y = b$ hinged edge.
3. At $y = 0$ sliding contact, at $y = b$ restricted sliding contact.
4. At $y = 0$ sliding contact, at $y = b$ freely supported edge.
5. At $y = 0, b$ hinged edge.

Characteristic equations for all these cases are obtained. Neglecting the fourth and higher order of relative thickness terms, approximate expressions for the critical loads are derived. These expressions are also numerically verified for several particular cases. It is shown, that, in the most cases, the values of critical load by refined theories may differ from the results of Kirchhoff's theory by a term of square order of relative thickness of the plate. However, in a problem of localized buckling of semi-infinite stripe-plate it was shown that account of transverse shear deformations introduces a refinement term of the first order of relative width [4]. Also for a finite plate, in some special cases of boundary conditions, the refinement term is of the first order with respect to the relative width.

REFERENCES

[1] **S. A. Ambartsumyan,** “Theory of anisotropic plates” Moscow: ”Nauka” 1987 (in Russian).
[2] **E. Reissner,** On the theory of bending of elastic plates. J. Math. and Phys. 1944. V.23, No1. pp. 184-191.
[3] **V. V. Vasilyev,** “Classical theory of plates: history and modern analysis.” Russian Sc. Acad. Publishers, MTT, 1998, No 3, pp. 46-58 (in Russian)
[4] **V. M Belubekyan,** On the problem of stability of plate with account of transverse shear deformations. Proceedings of Russian Sc. Academy, “MTT” No 2, 2004, pp. 126-131.

RECENT DEVELOPMENTS IN THE THEORY OF COSSERAT ELASTIC SHELLS AND APPLICATIONS

Mircea Bîrsan
University of Iasi, Romania
bmircea@uaic.ro

The theory of Cosserat shells is an interesting approach to the mechanics of elastic shell–like bodies, in which the thin three–dimensional body is modeled as a two–dimensional continuum (i.e. a surface) endowed with a deformable director assigned to every point. For a detailed analysis of the theory of Cosserat surfaces and its relation with other (hierarchical) shell theories, we refer to the classical monograph of Naghdi [1] and the modern book of Rubin [2]. According to [1], the Cosserat theory is also called the *direct approach* of shell theory, since its governing equations are deduced directly from the balance laws postulated for these two–dimensional continua (instead of deriving them starting from the three–dimensional theory). One advantage of this approach is that we can use methods analogous to those employed in the three–dimensional theory of elasticity to obtain corresponding results for Cosserat shells. Another feature of the Cosserat theory is that it can easily be extended to account for some important effects in the mechanical behavior of shells, such as thermal effects or porosity effects (see [3, 4]). In our paper, we shall illustrate both of these advantages mentioned above.

In the context of linear theory for anisotropic and inhomogeneous Cosserat elastic shells, we present some recent results concerning the properties of solutions to the boundary–initial– value problems associated to shell's deformation. The existence of solution can be proved on the basis of inequalities of Korn's type for Cosserat surfaces, using the method described by Ciarlet [5] in the classical shell theory. Several general theorems (such as uniqueness, reciprocal and variational theorems) are obtained via the same procedures as in the three–dimensional theory of elasticity.

As an application of the theory, we study the static deformation of thermoelastic cylindrical shells, due to a given temperature distribution in the body. We deal with open or closed cylindrical shells of arbitrary cross–sections. As usually in the treatment of Saint–Venant's problem, we consider a relaxed formulation of the boundary conditions in which the pointwise assignment of mechanical loads on the end edges of the cylindrical shell is replaced by prescribing the corresponding resultant force and resultant moment acting on these boundaries. The method to solve Saint–Venant's problem established in the context of three–dimensional elasticity by Iesan [6] also applies for the theory of Cosserat shells. On the basis of results presented in [7], we determine a closed–form solution to our problem, which can be useful in practical situations.

REFERENCES

[1] **P. M. Naghdi,** The Theory of Shells and Plates, in W. Flügge, Handbuch der Physik, vol. VI a/2 (C. Truesdell, ed.), pp. 425–640, Springer–Verlag, Berlin, 1972.
[2] **M. B. Rubin**, Cosserat Theories: Shells, Rods, and Points, Kluwer Academic Publishers, Dordrecht, 2000.
[3] **A. E. Green and P. M. Naghdi**, On thermal effects in the theory of shells, Proc. R. Soc. Lond. A365, pp. 161–190, 1979.
[4] **M. Bîrsan**, On a thermodynamic theory of porous Cosserat elastic shells, J. Thermal Stresses 29, pp. 879–900, 2006.
[5] **P. G. Ciarlet**, Mathematical Elasticity, Vol. III: Theory of Shells, North–Holland, Amsterdam, 2000.
[6] **D. Ieşan**, Saint–Venant's Problem, Lecture Notes in Mathematics, no. 1279, Springer– Verlag, Berlin, 1987.
[7] **M. Bîrsan**, The solution of Saint–Venant's problem in the theory of Cosserat shells J. Elasticity, 74, pp. 185–214, 2004

VIBRATION OF AN ELASTIC PLATES UNDER ACTION OF AN INCOMPRESSIBLE FLUID

Natalia Chinchaladze
I.Vekua Institute of Applied Mathematics of
Iv. Javakhishvili Tbilisi State University
natalic@viam.sci.tsu.ge

The aim of this paper is to study interaction problems in case of the vibration when in the elastic plate part the $N=0,1$ approximation of Vekua's hierarchical models for cusped elastic plates will be used [1]. The transmission conditions of interaction problems between an elastic plate and a fluid have been established when in the plate part we have the same approximation of I.Vekua's hierarchical model. Cylindrical vibration of a cusped elastic plate caused by an incompressible fluid under these transmission conditions has been studied. Problems of general vibration of the plate with constant thickness under action of an incompressible fluid have been solved in [2, 3].

REFERENCES

[1] **I.N. Vekua,** Shell Theory: General Methods of Construction. Pitman Advanced Publishing Program, Boston-London-Melbourne, 1985.

[2] **N. Chinchaladze, R. Gilbert,** Cylindrical vibration of an elastic cusped plate under action of an incompressible fluid in case of N=0 approximation of I.Vekua's hierarchical models. Complex Variables, vol. 50, No. 7-11 2005, 479-496.

[3] **Chinchaladze, N., Gilbert, R.,** Vibration of an elastic plate under action of an incompressible fluid in case of N=0 approximation of I.Vekua's hierarchical models. Applicable Analysis, vol. 85, No. 9, 2006, 1177-1187.

This work is made possible in part by Awards No. GTFPF-11 and No. GEP1-3339-TB-06 of the Georgian Research and Development Foundation and the U.S. Civilian Research & Development Foundation for the Independent States of the Former Soviet Union

ON ANISOTROPIC SINGULAR PERTURBATIONS PROBLEMS

Michel Chipot
Angewandte Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich, Switzerland
m.m.chipot@math.unizh.ch

Let $\Omega = (-1,1)^2$. We would like to study the asymptotic behaviour of problems which model could be

$$\begin{cases} -\varepsilon \partial_{x_1}^2 u_\varepsilon - \partial_{x_2}^2 u_\varepsilon = f & in \quad \Omega \\ u_\varepsilon = 0 & on \quad \partial\Omega \end{cases}$$

when $\varepsilon \to 0$ and show in particular that the solution converges toward the solution of the problem in lower dimension

$$\begin{cases} -\partial_{x_2}^2 u_\varepsilon = f & in \quad (-1,1) \\ u_0 = 0 & on \quad \partial\{-1,1\} \end{cases}$$

with a local speed as big as we wish.

IUTAM Symposium, 23-28 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

PARTIAL DIFFERENTIAL EQUATIONS ON HYPERSURFACES AND SHELL THEORY

Roland Duduchava
Razmadze Mathematical Institue,
Tbilisi, Georgia

Partial differential equations on Riemannian manifolds are usually written in intrinsic coordinates, involving metric tensor and Christoffel symbols. But if we deal with a hypersurface, the Cartesian coordinates of the ambient space can be applied. This seemingly trivial idea simplifies the form of many classical differential equations on the surface (Laplace-Beltrami, Lamé, Maxwell etc.), which turn out to have constant coefficients, and enable more transparent proofs of Korn's inequalities, tightly connected with solvability and uniqueness of some boundary value problems.

The above mentioned approach is applied to the Dirichlet and Neumann boundary value problems for the Laplace-Beltrami operator Δ_c to demonstrate simplicity and transparency of the method. An explicit Green formula is derived and proved that the Dirichlet boundary value problems has a unique solution in the Sobolev space $W_2^1(C)$ while the Neumann boundary value problems are solvable under the usual orthogonality constraints on the data. Moreover, herewith we prepare tools for a treatment of more complex boundary value problems for elasticity Lamé operators (isotropic and anisotropic), describing thin shells in the form of an open smooth hypersurface $C \subset S$ with the smooth boundary $\Gamma := \partial C$.

ON THE STABILITY OF NONLINEAR TWO-PHASE SHELLS

Victor A. Eremeyev
South Scientific Center of RASci and Rostov State University,
Milchakova str. 8a, 344090 Rostov on Don, Russia
eremeyev@math.rsu.ru

Within framework of general, dynamically and kinematically exact theory of elastic shells [1, 2] the statement of infinitesimal instability of elastic shells undergoing phase transitions is presented. The non-linear shell model [1, 2, 3] has been based on the exact trough-the-thickness integration of 3D global equilibrium conditions for total force and total torque. The phase transition has been assumed to occur at the singular surface curve which position is not known in advance. The theory of nonlinear shells with phase transitions was developed in [3, 4] by using variational principle of stationary total potential energy. Here the linearized boundary-value problem for two-phase is given. We take into account the permutations of displacements and rotations of the base surface of the shell as well as the permutations of the phase interface. As an example the instability of spherical shell is investigated. The results are compared with the 3D case [5].

This research was partially supported by the Russian Foundation of Basic Research under grant No 04-01-00431 and the Russian Science Support Foundation.

REFERENCES

[1] **J. Chróścielewski, J. Makowski, and W. Pietraszkiewicz**, Statics and Dynamics of Multifold Shells: Nonlinear Theory and

Finite Element Method (in Polish) (Wydawnictwo IPPT PAN, Warszawa, 2004).

[2] **A. Libai, and J. G. Simmonds**, The Nonlinear Theory of Elastic Shells, 2nd ed. (University Press, Cambridge, 1998).

[3] **V. A. Eremeyev and W. Pietraszkiewicz**, The nonlinear theory of elastic shells with phase transitions. J. Elasticity 74 (2004) 67–86.

[4] **W. Pietraszkiewicz, V. A. Eremeyev and V. Konopińska,** Extended nonlinear relations of elastic shells undergoing phase transitions. ZAMM (2006) (submitted).

[5] **V.A. Eremeyev, A.B. Freidin, and L.L. Sharipova,** Nonuniqueness and stability in problems of equilibrium of elastic two-phase bodies. Doklady Physics. (2003) 48(7) 359–363.

VARIATIONAL DIMENSION REDUCTION IN NON LINEAR ELASTICITY: A YOUNG MEASURE APPROACH

Lorenzo Freddi

Dipartimento di Matematica e Informatica

Universitá di Udine, via delle Scienze 206, 33100 Udine

Martensitic thin films have recently attracted much interest because of their applications in the construction of microactuators. The Helmholtz free energy density for these materials is non-convex thus, in general, minimizing sequences develop fine-scale oscillations which manifest themselves as microstructure. It is well known that these fine-scale oscillations can be described mathematically by means of Young measures.

In the past years several theories of thin films and strings have been derived from three-dimensional elasticity. The methodology used has produced limit structures with free energy densities which are quasi-convex and therefore not capable to describe the fine-scale oscillations.

Starting form three dimensional elasticity, we deduce the variational limit of the string and of the membrane on the space of one and two-dimensional gradient Young measures, respectively. The physical requirement that the energy becomes infinite when the volume locally vanishes is taken into account. The rate at which the energy density blows up characterizes the effective domain of the limit energy. The limit problem uniquely determines the energy density of the thin structure.

The talk is based on joint work with R. Paroni.

JOINT VIBRATIONS OF A RECTANGULAR SHELL AND GAS IN IT

Elena Gavrilova
Department of Mathematics,
St. Ivan Rilski University of Mining and Geology,
1700 Sofia, Bulgaria

The thin elastic rectangular plates are often used as structural components of parallelepiped cavities filled with gas and subjected to different dynamic loads. Such systems find application in the glass-skin technology of tall buildings; as outside skin plates of supersonic air crafts; as covers of different tanks in chemical industry; as chambers in hydraulic structures, etc. The main problem of the mechanics of these systems is to determine their response of some specific technological dynamical or to some standard catastrophic loads. As subordinated, but enough important for the engineering practice, appears to be the problem about the determination of the dynamical characteristics of the systems.

A closed rigid rectangular parallelepiped tank, filled with a gas, is under consideration. A part of one of its walls is a thin linearly elastic rectangular plate. The problem about the stationary forced vibrations of the gas and the elastic plate under the action of a source, being situated in the gas tank, is under consideration. Let the source have sizes which are small in comparison with the lengths of the excited waves -then it is possible to be accepted as a point source. It is supposed that the productivity and the frequency of the source are given and they do not experience any back influence of the earlier excited waves. The problem is considered in a linear approximation without giving an account of the dissipating forces.

A combination of the use of the Green function, the method of the crossed strips of *G*. Warburton and the method of Bubnov-Galerkin is made to investigate the dynamic behavior of this gas-structure interaction system in the cases of arbitrary supporting conditions of the plate. An approximate solution is made based on the ignoring the diffracted by the elastic plate waves. Some numerical examples are shown and they are represented graphically.

JUSTIFICATION OF A SHALLOW SHELL MODEL IN UNILATERAL CONTACT WITH AN OBSTACLE

Alain Léger
CNRS, Laboratoire de Mécanique et d'Acoustique 31, chemin Joseph Aiguier, 13402, Marseille cedex 20, France
leger@lma.cnrsmrs.fr

Bernadette Miara
ESIEE, Laboratoire de Modélisation et Simulation Numérique
Cité Descartes, 2, Boulevard Blaise Pascal,
93160 Noisyle Grand Cedex, France
b.miara@esiee.fr

We first recall that structural models such as linearly elastic beams, plates and shells with usual bilateral boundary conditions have received asymptotic or variational justifications. Although frictionless contact problems received a complete mathematical treatment within 2D or 3D linear elasticity (*Signorini problems*), models of structures with unilateral contact conditions (*obstacle problems*) received justifications only in the case of a plate [2]. The present paper deals with the case of a shallow shell and, as a model problem, the analysis is specified to the case where the shell is in unilateral contact with the plane of the reference open set. The objective is to justify the asymptotic limit. More precisely, we start with the 3D Signorini problem (with finite thickness) and obtain at the limit an obstacle 2D problem. The procedure is the same as the one used in the asymptotic analysis of 3D bilateral structures [1], i.e. assumptions on the data, (loads and geometry of the middle surface of the shell) and rescalling of the unknowns (displacement field or stress tensor).

The main results are the following:

i) Assume enough regularity on the external volume and surface loads, and on the mapping that defines the middle surface of the shell, then the family of elastic displacements converges strongly as the thickness tends to zero in an appropriate set which is a convex cone.

ii) The limit is a Kirchhoff-Love displacement field given by a variational problem which will be analysed into details. The contact conditions are fully explicited for any finite thickness and at the limit.

REFERENCES

[1] **P.G. Ciarlet, B. Miara**, Justification of the two-dimensional equations of a linearly elastic shallow shell, Comm. Pure Appl. Mathematics, **XLV**, 327–360, 1992.

[2] **J.C. Paumier**, Le probléme de Signorini dans la théorie des plaques minces de Kirchhoff-Love, C.R. Acad. Sci., I, t.335, 567–570, 2002.

VALIDATION OF CLASSICAL BEAM AND PLATE MODELS BY VARIATIONAL CONVERGENCE

Paolo Podio-Guidugli
Dipartimento di Ingegneria Civile
Universita' Di Roma Torvergata
Viale Politecnico, 1 - 00133 Rome, Italy

In the first part of my talk, I plan to give a short account of a method of formal scaling expounded in [1]. This method allows for a unified deduction from three-dimensional linear elasticity of the equations of structural mechanics, such as Reissner-Mindlin's equations for shearable plates and Timoshenko's equations for shearable rods; it is based on the requirement that a scaled energy functional that may include second-gradient terms stay bounded in the limit of vanishing thickness. In the second part, following the developments in [2], I shall provide a justification of the Reissner-Mindlin plate theory, using linear three-dimensional elasticity as framework and Gamma-convergence as technical tool.

REFERENCES

[1] **B. Miara and P. Podio-Guidugli**, Deduction by scaling: a unified approach to classic plate and rod theories. *Asymptotic Analysis* (2007) (to appear)

[2] **R. Paroni, P. Podio-Guidugli and G. Tomassetti**, A justification of the Reissner-Mindlin plate theory through variational convergence. *Analysis and Applications*, Volume 5 (2), pp. 1-18 (2007)

A HIERACHICAL BEAM AND PLATE MODELLING THEORY BASED ON HOMOGENIZATION

Jorn S. Hansen

University of Toronto Institute for Aerospace Studies Toronto, Ontario, Canada

A theory which yields a hierarchy of extremely accurate approximations for layered beams and plates is presented. The beam theory was developed by Hansen and Almeida [1], [2] and extended to plates by Guiamatsia and Hansen [3]. This approach provides a unified theory for laminated composite and sandwich structures. The work utilizes far-field stress and strain solutions corresponding to constant, linear, quadratic, ...n^{th} degree bending states; these solutions are referred to as Fundamental Solutions, can be determined uniquely and are independent of kinematic boundary conditions. Based on the Fundamental Solutions, through-thickness moments of stress and strain yield definitions of homogenized flexural and shear stiffness, homogenized transverse Poisson's ratio as well as a unique definition of a shear-strain-moment correction. Through-thickness stress and strain moments eliminate difficulties commonly associated with discontinuous or non-differentiable solution fields; also, model complexity is independent of the number or type of layers present in a structure. In addition, a simple, well-defined post-processing step based on the Fundamental Solutions used in the model development, allows precise determination of all stress and strain components -including the transverse normal stress and strain. Thus modelling and post-processing are completely consistent.

In the case of beams it is shown that all models adopt a form similar to Classical Timoshenko Beam Theory with the addition of higher order correction terms; however, the displacement representation of the present and the Timoshenko model have different meanings. In the case of layered plates, it is shown that the adopted homogenization approach does not, in general, yield a Reissner/Mindlin type model; an exception occurs for layered plates in which each layer has different material properties but all layers are isotropic and homogeneous.

In order to illustrate this approach, results will be presented for a number of laminated and sandwich beams and plates. These will include both symmetric and non-symmetric laminates. Comparisons are made with precise finite element calculations and closed form results and it is shown that this new approach yields extremely accurate results.

REFERENCES

[1] **J.S. Hansen and S.F.M. Almeida**, A Theory for Laminated Composite Beams, Final Report submitted April 2001, FAPES Grant No. 00/06183-0, S˜ao Paulo, Brasil, 157 pages.

[2] **J.S. Hansen and S.F.M. Almeida**, A Homogenization Based Laminated Beam Theory, 21st International Congress of Theoretical and Applied Mechanics (ICTAM 2004), Warsaw, Poland 16-20 August 2004.

[3] **I. Guiamatsia and J.S. Hansen**, A Homogenization Based Laminated Plate Theory, 2004 ASME International Mechanical Engineering Congress and RD & D EXPO, Anaheim, California, Nov. 13-19, 2004.

PHYSICAL AND MATHEMATICAL MOMENTS AND ANALYSIS OF PECULIARITIES OF SETTING OF BOUNDARY CONDITIONS FOR CUSPED SHELLS AND BEAMS

George Jaiani
I.Vekua Institute of Applied Mathematics
of Iv. Javakhishvili Tbilisi State University
jaiani@viam.sci.tsu.ge

The paper deals with the analysis of the physical and geometrical senses of the N-th (N=0,1,…) order moments and weighted moments of the stress tensor and displacement vector, arising in the theory of cusped prismatic shells [1,2] and beams [3]. There are analyzed the peculiarities of setting of the boundary conditions at cusped edges in terms of moments and weighted moments. The relation of the corresponding boundary conditions to the boundary conditions of the three-dimensional theory of elasticity is also discussed.

REFERENCES

[1] **G. Jaiani**, Elastic Bodies with Non-smooth Boundaries– Cusped Plates and Shells, ZAAM-Zeitschrift fuer Angewandte Mathematik und Mechanik, Vol.76, Supplement 2, pp.117-120, 1996

[2] **G. Jaiani**, **S. Kharibegashvili, D. Natroshvili, W.L. Wendland**, Two-dimensional Hierarchical Models for Prismatic Shells with Thickness Vanishing at the Boundary, Journal of Elasticity, Vol. 77, No. 2, 95-122, 2004

[3] **G. Jaiani**, On a Mathematical Model of Bars with Variable Rectangular Cross-sections, ZAMM-Zeitschrift für Angewandte Mathematik und Mechanik, Vol.81, 3, pp.147-173, 2001

This work is supported by INTAS project 06-100017-8886

MATERIAL CONSERVATION LAWS ESTABLISHED WITHIN A CONSISTENT PLATE THEORY

Reinhold Kienzler and D. K. Bose
University of Bremen, IW3
POB 330440 D-28359 Bremen

Within the framework of the linear theory of elasticity, a consistent second-order plate theory is derived for homogeneous and isotropic materials. To this extent, the displacements are developed in thickness direction into a power series and the strain-displacement relations are satisfied for each power of the thickness coordinate. The strain-energy density is calculated and, in turn, integrated with respect to the thickness direction. Constitutive relations are derived and the equations of equilibrium follow from the principle of virtual work. Finally, all governing equations are approximated uniformly up to the second order. In addition, during the reduction of the systems of differential equations the same approximation is applied. The consistent second-order plate theory takes shear deformations and strains in thickness direction into account. It can be shown that well-established plate theories, like Reissner-Mindlin's [1] or Zhilin's [2] theory are equivalent to the proposed theory within the consistent second-order approximation.

Material conservation laws or path-independent integrals are well established in the theory of elasticity. They are used to determine energy-release rates and material forces connected with the change of configuration of inhomogeneities or defects within the material. The by-now well-known J-Integral [3] characterizes the rate of energy change due to a translation of a defect. It possesses a broad potential of application, especially in fracture mechanics. In addition to J, two further integrals designated as L and M were derived [4], which describe the change of energy of the system due to a rotation and a self-similar expansion of the defect, respectively.

Using, again, the uniform-approximation technique, the associated material conservation laws for the consistent second-order plate theory are established. The corresponding path-independent integrals $\bar{J}$, $\bar{L}$ and $\bar{M}$ may serve to assess the reliability of plates with cracks [5].

REFERENCES

[1] **E. Reissner,** On the theory of bending of elastic plates. J Math Phys 23 (1944) 184-191

[2] **P. A. Zhilin,** On the Poisson and Kirchhoff plate theories from the point of view of the modern plate theory (in Russian). Izvestia Akedemi Nauk Rossii 3 (1992) 48-64

[3] **J. R. Rice,** A path independent integral and the approximate analysis of strain concentration by notches and cracks. ASME J Appl Mech 27 (1968) 379-386

[4] **B. Budiansky, J. Rice,** Conservation laws and energy-release rates. ASME J Appl Mech 40 (1973) 201-203

[5] **D. K. Bose,** Erhaltungssätze der Kontinuumsmechanik für eine konsistente Plattentheorie. Diss Univ Bremen, Germany. Aachen: Shaker 2004

MULTISCALE ASSESSMENT OF LOW-TEMPERATURE PERFORMANCE OF FLEXIBLE PAVEMENTS

Herbert Mang , E. Aigner, R. Lackner, J. Eberhardsteiner
Christian-Doppler-Laboratory for "Performance-based Optimization of Flexible Pavements" Institute for Mechanics of Materials and Structures, Vienna University of Technology, Vienna, Austria

M. Spiegl, M. Wistuba, R. Blab
Christian-Doppler-Laboratory for "Performance-based Optimization of Flexible Pavements" Institute for Road Construction and Maintenance, Vienna University of Technology, Vienna, Austria

The present and future increase of heavy-load traffic within Europe requires the development of appropriate tools for the assessment of existing and new road infrastructure. In this paper, such a tool is presented, combining multiscale material modeling of asphalt with the structural analysis of flexible pavements.

The thermorheological behavior of asphalt provides the low viscosity at T > 135°C necessary for the construction and compaction process of high-quality asphalt layers. The continuous increase of the viscosity with decreasing temperature which, on the one hand, is desirable for the reduction of permanent deformations during summer periods may lead, on the other hand, to so-called top-down cracking in the course of temperature drops during cold winter periods. These cracks, when propagating further into the base layer, significantly reduce the service life of road infrastructure.

Within the presented multiscale model for asphalt (see Figure 1), the viscoelastic properties of asphalt are related to the constituent bitumen, showing the thermorheological behavior, accounting for:

- the large variability of asphalt mixtures, resulting from different mix design, different constituents (e.g. bitumen, filler, aggregate,…), and the allowance of additives, and
- changing material behavior in consequence of thermal, chemical, and mechanical loading.

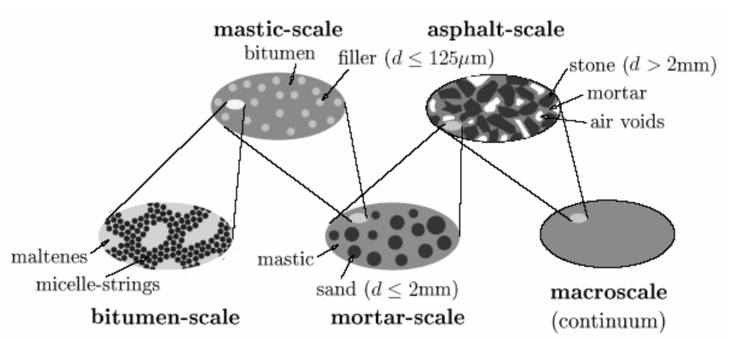

Figure 1: *Multiscale model [1] with four additional observation scales below the macro-scale*

The parameters of the underlying viscoelastic material model for asphalt are obtained from upscaling of parameters identified at the bitumen-scale up to the macro-scale. Hereby, the viscoelastic behavior of bitumen serves as input and the effect of the addition of aggregates, i.e., filler, sand, and stone is investigated. For this purpose, the viscous properties of asphalt are identified at the bitumen scale (see Figure 1), using standard test methods, such as the bending beam rheometer and the dynamic shear rheometer. This set of experiments provides access to the viscoelastic response of bitumen for different temperature regimes. Upscaling of viscoelastic properties is performed in the framework of continuum micromechanics, employing a modified form of the Mori-Tanaka scheme [2]. Based on the correspondence principle, the elastic shear compliance in the employed equations is replaced by the respective Laplace-Carson transform [3] of the viscoelastic compliance. The presented multiscale model is applied to asphalts typically used for surface and base layers of flexible pavements.

The obtained macroscopic model parameters are employed in the numerical analysis of flexible pavements, giving access to stresses resulting from (i) traffic-loading and (ii) a sudden decrease in the temperature in consequence of changing weather conditions (see Figure 2). Comparison of the so-obtained stresses with the tensile strength of asphalt of the respective surface temperature allows the risk assessment of top-down cracking in flexible pavements.

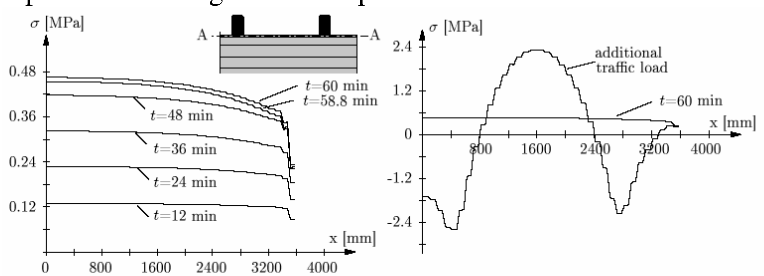

Figure 2: *Stress distribution along section A-A in consequence of temperature changes and additional traffic load*

REFERENCES

[1] **R. Lackner, R. Blab, A. Jäger, M. Spiegl, K. Kappl, M. Wistuba, B. Gagliano, J. Eberhardsteiner,** Multiscale modeling as the basis for reliable predictions of the behavior of multi-composed Materials. In Progress in Engineering Computational Technology, B.H.V. Topping and C. A. Mota Soares (ed.). Saxe-Coburg Publications, Stirling, Chapter 8: 153-187 (2004).

[2] **T. Mori, K. Tanaka,** Average stress in a matrix and average elastic energy of materials misfitting inclusions. Acta Metallurgica, 21:571-574 (1973).

[3] **C. Pichler, R. Lackner, H. Mang,** Multiscale model for early-age creep of cementious materials. In G. Pijaudier-Cabot, B. Gerard, P.A.,editor, Proceedings of the 7th International Conference on Creep, Shrinkage and Durability of Concrete and Concrete Structures (CONCREEP-7). Hermes, pages 361-367 (2005).

THE METHOD OF A SMALL PARAMETER FOR I.VEKUA'S NONLINEAR AND NONSHALLOW SHELLS

Tengiz Meunargia
Tbilisi State University
I.Vekua Institute of Applied Mathematics

I.N. Vekua [1] has constructed several versions of the refined linear theory of thin and shallow shells, containing the regular process by means of the method of reduction of three-dimensional problems of elasticity to two-dimensional ones.

In the present paper by means of the method of I.N. Vekua the system of differential equations for the nonlinear theory of nonshallow shells is obtained. Then the method of a small parameter is used for them and some basic boundary value problems are solved.

REFERENCES

[1] **I.N. Vekua,** Shell Theory: General Methods of Construction. Pitman Advanced Publishing Program, Boston-London-Melbourne, 1985.

2D VERSUS 1D MODELLING OF VIBRATING CARBON NANOTUBES

Paola Nardinocchi
University of Rome "La Sapienza", Rome, Italy

Given the increasing demand of novel experiments at the nanoscale and because of the limits of molecular dynamic simulations in analyzing large scale systems, continuum modelling of CNTs has represented an useful tool to gain insigth in some typical mechanical behaviour of single-walled and multi-walled carbon nanotubes.

Both one-dimensional and two-dimensional continuum models borrowed from structural mechanics have been developed with the aim to simulate mechanical behaviour of CNTs. Typically, the capabilities of cylindrical shell models and Euler or Timoshenko beam models in simulating deformation modes and buckling behaviour of carbon nanotubes under different conditions have been compared.

Moreover, granted for the scientific interest in the application of CNTs as resonators and as strengthening in fiber-reinforced composites, the analysis of waves propagation, resonant frequencies and associated vibration modes of single-walled and multi-walled carbon nanotubes is the subject of many researches.

Here, I compare the performances of suitable shell and beam models in analyzing the vibration characteristics of different carbon nanotubes such as single-walled CNTs, nanotube spirales and nanotube strands. As far as concerning the firsts, a one-dimensional continuum model apt to simulate breathing modes of single-walled carbon nanotubes is developed and put in perspective with cylindrical shell models. Nanotube spirales are modelled as ribbon-like shells and the wave characteristics of the resulting model are compared with those associated to one-dimensional helicoidal beam models. As far as concerning nanotube strands, they are viewed as a collection of three single-walled carbon nanotube twisted to form a unit nanotube for which a one-dimensional model is derived.

THE EXTENSION AND APPLICATION OF THE HIERARCHICAL BEAM THEORY TO PIEZOELECTRICALLY ACTUATED BEAMS

Donatus C.D. Oguamanam
Dept. of Mechanical Engineering,
Ryerson University, Toronto, Ontario Canada, M5B 2K3

C. McLean
Nuclear Safety Solutions Ltd.
4/F-700 University Avenue, Toronto,
Ontario Canada, M5G 1X6

J.S. Hansen
Institute for Aerospace Studies, University of Toronto
4925 Dufferin Street, Downsview, Ontario Canada, M4S 1C4

In order to realize the full compatibility of advanced composite and sandwich structures an internally consistent and accurate modelling process is needed. Hansen and de Almeida developed a unified hierarchical theory for layered beams which has the ability to accurately predict through-the-thickness stress and strain distributions, as well as displacement moments of all combinations of symmetric and asymmetric laminates and sandwich structures. This theory asserts that these predictions can be developed as a superposition of sets of well chosen fundamental states, where a hierarchical sequence of bending states occurs. Fundamental states are described as numerical experiments performed on an infinitesimal segment of the physical beam. Here, piezoelectric actuation effects are considered and a new fundamental state

to model the actuation within the composite or sandwich structure through the application of a uniform electric field is developed. The capabilities of this fundamental state are demonstrated within the confines of the hierarchical beam theory by solving three problems: a purely actuated system, a system subjected to both electrical and mechanical loads and a system of piezoelectric patches that are embedded within a composite structure. The calculated through-the-thickness stress and strain distributions and the displacement moments yield comparable results to 2-D ANSYS finite element predictions.

THIN WALLED ELASTIC BEAMS: A RIGOROUS JUSTIFICATION OF VLASOV THEORY

Roberto Paroni
University of Sassari

We discuss the asymptotic analysis of the three-dimensional problem for a linearly elastic cantilever having an open cross-section which is the union of rectangles with sides of order h and *h*h*, as h goes to zero. Under suitable assumptions on the given loads and for homogeneous and isotropic material, we show that the three-dimensional problem Gamma-converges to the classical one-dimensional Vlassov model for thin-walled beams.

The talk is based on joint work with L. Freddi and A. Morassi.

THE CONTACT PROBLEMS OF THE MATHEMATICAL THEORY OF ELASTICITY FOR PLATES WITH AN ELASTIC INCLUSION

Nugzar Shavlakadze
Razmadze Mathematical Institute
1, Alexidze St., 0193 Tbilisi, Georgia

Contact problems of the plane theory of elasticity and bending theory of plates for finite or infinite plates with an elastic inclusion of variable rigidity are considered. The problems are reduced to the integro-differential equation or to the system of integro-differential equations with variable coefficient at the singular operator. If such coefficient varies with power law we can investigate the obtained equations and get exact or approximate solutions, and establish behaviour of unknown contact stresses at the end points of elastic inclusion.

ON THE SIMULATION OF TEXTILE REINFORCED CONCRETE LAYERS BY A SURFACE–RELATED SHELL FORMULATION

Rainer Schlebusch, Bernd W. Zastrau
Technische Universität Dresden
01062 Dresden, Germany
Rainer.Schlebusch@tu-dresden.de
Bernd.W.Zastrau@tu-dresden.de

This work is embedded in the Sonderforschungsbereich 528 (Collaborative Research Centre): ”Textile Reinforcement for Structural Strengthening and Retrofitting” at Technische

Universität at Dresden. The stress–oriented arrangement of glass or carbon fibres, respectively, having an excellent load–bearing capacity, leads to technical textiles which might be incorporated into a concrete matrix. Therewith, a new innovative composite material, textile–reinforced concrete, is developed from being used for both the production of new concrete members and for the restoration and strengthening of existing structures. Since the materials used are noncorrosive compared to steel and as they show great strength at the same time, textile–reinforced concrete can be used for strengthening tasks of small dimensions, i. e. in thin strengthening layers applied to the surface of existing structures.

The solution of the resulting structural analysis problems demands for an efficient and reliable numerical solution strategy. Since contact problems are involved for mapping the interface between the existing structure and the strengthening layer, the shell model for the strengthening layer is formulated with respect to one of the outer surfaces, i.e. the shell formulation is surface–related, cp. [2].

Since shells are three–dimensional structures, i.e. bodies, the field equations of continuum mechanics must be the starting point. This set of partial differential equations with pertinent boundary conditions has to be solved. An efficient numerical solution of this problem becomes easier, if the problem is reformulated against a background of variational calculus. The discretization of the resulting variational formulation is, among others, the source of several locking phenomena.

The presented shell formulation uses linear shell kinematics with six displacement parameters. This low– order shell kinematics produces parasitical strains and stresses leading to wrong or even useless results, i. e. to locking as well. Therewith, an extension and/or adjustment of well–known techniques to prevent or reduce locking like the assumed natural strain (ANS) method, cp. [3], and the enhanced assumed strain (EAS) method, cp. [4], is accomplished.

Using these adapted methods, a reliable and efficient solid–shell element with tremendously reduced locking properties is obtained. This concept additionally comprises the utilization of unmodified three–dimensional constitutive relations by a minimal number of kinematical parameters, cp. [1]. With the aid of different nonlinear examples, the reliability and the efficiency of the new solid–shell element is finally verified.

REFERENCES

[1] **N. Buchter, E. Ramm**, 3d–Extension of Nonlinear Shell Equations Based on the Enhanced Assumed Strain Concept, in C. Hirsch, ed., *Computational Methods in Applied Sciences*, Elsevier, 55–62, 1992.

[2] **R. Schlebusch, J. Matheas, B. Zastrau**, On Surface–Related Shell Theories for the Numerical Simulation of Contact Problems, *Journal of Theoretical and Applied Mechanics*, 41(3), 623–642, 2003.

[3] **J.C. Simo, T.J.R. Hughes**, On Variational Foundations of Assumed Strain Methods, *Journal of Applied Mechanics*, 53, 52–54, 1986.

[4] **J.C. Simo, M.S. Rifai**, A class of mixed assumed strain methods and the method of incompatible modes, *International Journal for Numerical Methods in Engineering*, 29, 1595 -1638, 1990.

ON THE VALUE OF SHEAR CORRECTION FACTOR IN BEAM, ARCH, PLATE AND SHELL MODELS

Sheng Zhang
Department of Mathematics, Wayne State University, Detroit, Michigan 48202, USA, sheng@math.wayne.edu

The Timoshenko beam, Reissner–Mindlin plate and Naghdi arch and shell models are thickness-dependent lower dimensional models that allow for transverse shear deformabilities. There is an issue about the shear correction factors involved in these models. The value 5/6 is often viewed as the best. We show that 1 is actually better: We prove that when a dominant shear effect arises a factor that differs from 1 forces the model solutions diverge from that of the three-dimensional elasticity when the thickness tends to zero, while in the bending dominated case, as is well known, the value of shear correction factor has virtually no effect on the model validity. The talk is based on some published works and some ongoing researches.

REFERENCES

[1] **D.N. Arnold, A. Mardureira**, Sheng Zhang, On the range of applicability of the Reissner–Mindlin and Kirchhoff–Love plate bending models, *J. Elasticity*, 67:171-185, 2002.
[2] **Sheng Zhang**, A linear shell theory based on variational principles, *PhD Thesis*, Penn State University, 2001.
[3] **Sheng Zhang**, Equivalence estimates for a class of singular perturbation problems, *C. R. Acad. Sci. Paris, Ser. I*, 342:285-288, 2006.

[4] **Sheng Zhang**, On the accuracy of Reissner–Mindlin plate model for stress boundary conditions, *Math. Modelling Numer. Anal.*, 2006, to appear.

This work was partially supported by NSF grant DMS-0513559.

EQUIVALENCE BETWEEN THE VANISHING OF THE 3*D* RIEMANN-CHRISTOFFEL TENSOR AND THE 2*D* GAUSS-CODAZZI-MAINARDI COMPATIBILITY CONDITIONS IN SHELL THEORY

Claude M. Vallee
LMS, UMR CNRS 6610, SP2MI,
University of Poitiers, France

Well-known necessary and sufficient condition for the Partial Differential Equation

$$\delta_{kl}\frac{\partial\Theta^k}{\partial x^i}\frac{\partial\Theta^l}{\partial x^j} = g_{ij}(x^1, x^2, x^3)$$

to be integrable, is the vanishing of the Riemann-Christoffel curvature tensor associated to the g_{ij} regarded as metric tensor field. In the special case of 3*D* euclidean space, the four times covariant version of the Riemann-Christoffel tensor takes the factorised expression $R_{ijkl} = \varepsilon_{mlk}\varepsilon_{nij}[B(CofU)]^m_q\,\delta^{qn}$ where U is an invertible solution of $\delta_{kl}U^k_i U^l_j = g_{ij}$ and $B = Rot(\Lambda^T) + Cof(\Lambda^T)$ with

$$(\det U)\Lambda = U[Rot(U^T)]U - \frac{tr[URot(U^T)]}{2}U$$

When $\Theta(x^1, x^2, x^3) = \theta(x^1, x^2) + x^3 n(x^1, x^2)$ parameterises shell, with as parameterisation of the middle surface and as the unit normal, question arises: does the Riemann-Christoffel conditions associated to the metric reduce to the Gauss-Codazzi-Mainardi compatibility conditions associated to the 1^{st} and 2^{nd} fundamental forms and of the middle surface This communication is devoted to give short proof of the equivalence between both $2D$ and $3D$ compatibility conditions. If one expresses the G-C-M conditions, as usual, in terms of the Christoffel symbols of the middle surface, the proposed equivalence will reveal to be very burdensome to prove. Readable version of the G-C-conditions was beforehand produced by the author and simultaneously by Athanassios Fokas and Israel Gelfand. The key idea is to prefer to the symbols proposed by Elwin Bruno Christoffel the components of the two $3D$ "instantaneous rotation vectors" and proposed by Gaston Darboux. The first and second components of these two Darboux vectors are determined in terms of the two fundamental forms by solving the linear Equation

$$a^{\frac{1}{2}} J \begin{bmatrix} Y^1 & Z^1 \\ Y^2 & Z^2 \end{bmatrix} = b, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

as the square root of the 1^{st} fundamental form. The third components are determined in turn by solving again linear equation:

$$\left[Y^3 Z^3\right] J a^{\frac{1}{2}} J = \left[\frac{\partial a_{11}^{\frac{1}{2}}}{\partial x^2} - \frac{\partial a_{21}^{\frac{1}{2}}}{\partial x^1} \quad \frac{\partial a_{12}^{\frac{1}{2}}}{\partial x^2} - \frac{\partial a_{22}^{\frac{1}{2}}}{\partial x^1} \right]$$

Following the idea of Darboux, the G-C-conditions take the simple expression

$$\frac{\partial Y^k}{\partial x^2} - \frac{\partial Z^k}{\partial x^1} = Y^i Z^j - Y^j Z^i$$

with (*i,j,k*) as any even permutation of (1,2,3). In the case of shell, solution for U is

$$U(x^1, x^2, x^3) = \begin{bmatrix} a^{\frac{1}{2}}(x^1, x^2) - x^3 a^{-\frac{1}{2}}(x^1, x^2) b(x^1, x^2) & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the determinant of U being

$$\det U = [\det(a^{\frac{1}{2}})][1 - 2Hx^3 + K(x^3)^2],$$

with H and K as the mean and gaussian curvatures of the middle surface. Therefore, the definition of Λ leads for shell to

$$\Lambda = \begin{bmatrix} Y^1 & Z^1 & 0 \\ Y^2 & Z^2 & 0 \\ Y^3 & Z^3 & 0 \end{bmatrix}$$

and it is then rather easy to become aware that the Riemann-Christoffel conditions expressed as $Rot(\Lambda^T) + Cof(\Lambda^T) = 0$

are equivalent to the Gauss-Codazzi-Mainardi conditions in their already mentioned readable version.

ON BASIC SYSTEMS OF EQUATIONS OF CONTINUUM MECHANICS AND SOME MATHEMATICAL PROBLEMS FOR ANISOTROPIC THIN-WALLED STRUCTURES

Tamaz S. Vashakmadze
Iv.Javakhishvili Tbilisi State University
I.Vekua Institute of Applied Mathematics
2, University Str., 0143, Tbilisi, Georgia,
tamazvashakmadze@yachoo.com

1. We construct three-dimensional (respect to spatial coordinates) nonlinear dynamical system of partial differential equations (PDE) which contains as particular cases Navier-Stokes' equations and nonlinear PDEs of the elasticity theory. By this presentation we prove that nonlinear appearances, observed in problems of solid mechanics, may be detected in the Navier-Stokes' type equations and vice versa.

2. The purposes of the second part are creation and justification of new mathematical models for anisotropic thermopiezo- and poro-elastic media, its application to a variety of dynamical and steady state nonlinear problems for thin-walled structures of binary mixtures with variable thickness.

The well-known and wide-spread model is Biot's theory for poro-elastic media. However, this theory contains some contradictions. We develop a nonlinear mathematical model for anisotropic poroelastic media. As special cases of this theory we can derive the Biot's theory as well as the modern nonlinear theory of elasticity. Further, we use these theoretical investigations for construction of von Karman-Reissner type evolutionary models for anisotropic beams, plates and shallow shells with variable thickness, correspondingly.

We study problems of construction of mathematical models for continuum media without simplifying assumptions. Our methodology is different from the asymptotic methods developed by Friedrichs-Goldenveiser-Ciarlet for constructing refined plate theories of van Karman-Mindlin-Reissner type. We also explain "Physical Soundness" in the Truesdell-Ciarlet sense of some dynamic nonlinear models of thermopiezo-elasticity. This problem even in the case when elastic plates are isotropic with constant thickness had been an open problem (see e.g. details in monograph of P.Ciarlet: Mathematical Elasticity, II: Theory of Plates, Elsevier, 1997, ch.5, pp. 367-406).

Further, we constructed new numerical algorithms and created corresponding software using the schemes of Ch.III of the monograph T.Vashakmadze. The Theory of Anisotropic Elastic Plates, Kluwer Acad. Publ., 1999.