

## HIGH ORDER DOMAIN DECOMPOSITION METHODS

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The aim of our contribution is an efficient method which is also parallelizable providing the following features:

1. The computation of boundary displacements and boundary tractions with high accuracy either on the whole boundary curve ( $n = 2$ ) or boundary surface ( $n = 3$ ) or on an a-priori chosen subregion of the boundary and
2. the computation of displacements, strains and stresses near and up to the boundary with high accuracy.

Our method is based on the decomposition idea applied to rather different parts of the boundary integral equation method:

- a) the decomposition of the boundary curve or boundary surface into an overlapping geometric partition,
- b) the decomposition of the trial space for the desired quantities into a regular coarse grid space and local spaces on fine grids; and
- c) the additive decomposition of the boundary integral operators involved into a standard principal part and a smoothing remainder.

As an example of our approach we consider the Lamé equations of linearized isotropic homogeneous elasticity. Near to the boundary we use Hadamard's natural local coordinates in normal and tangential directions of the boundary, respectively.

To exemplify our method, we consider the Dirichlet problem where we have to solve a boundary integral equation for the conormal derivative, e.g. the integral equation of the first kind,

$$V\vec{\psi} = \vec{F} := \left(\frac{1}{2}I + K\right)\vec{\varphi} \text{ on } \Gamma \quad (0.1)$$

In a first step, this equation is solved on the whole boundary where we use coarse grid trial functions on a regular grid associated with the mesh width  $H$ . In order to obtain high order convergence, we use the method proposed in [1], [2], where we approximate simultaneously tangential derivatives of  $\vec{\psi}$  on a subdomain  $\Gamma_0 \subset \Gamma$ , similar as in the partition of the unity method. In addition, we split the operator into a simplified principal part  $V_0$  and a remainder that has a kernel which is less singular. Then we solve the local boundary integral equation for  $\tilde{u} \in \tilde{H}^{-\frac{1}{2}}(\Gamma_0)$  on the fine grid where the simplified operator, i.e.

$$\langle V_0\tilde{u}, \tilde{v} \rangle = \langle V - 0\omega u_H, \tilde{v} \rangle - \langle \omega(Vu_H - F), \tilde{v} \rangle \text{ for all } \tilde{v} \in \tilde{H}^{-\frac{1}{2}}(\Gamma_0) \quad (0.2)$$

defines the corresponding local influence matrix whereas the modified right-hand side contains all the global information. Now, we use simultaneous approximation of tangential derivatives and recovery for improving the efficiency of the local method. For more general boundary conditions, we employ hybrid boundary element approximations of the Steklov-Poincaré operator [3].

#### R E F E R E N C E S

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