Bulletin of TICMI
Vol. 24, No. 02, 2020, 65–73

Energy Transfer by Internal-Gravity Wavy Structures in the Upper Atmosphere with the Shear Flow

Khatuna Elbakidze[1,2*], Oleg Kharshiladze[3,4], Gaetano Zimbardo[5]

[1] *I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[2] *International Black Sea University, Tbilisi, Georgia*
[3] *M. Nodia Institute of Geophysics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[4] *Physics department, I. Javakhishvili Tbilisi State University, Tbilisi, Georgia*
[5] *Department of Physics, University of Calabria, Ponte P. Bucci Cubo 31C, I-87036 Rende, Italy*

(*Received December 1, 2019; Revised October 22, 2020; Accepted November 5, 2020*)

The nonlinear dynamics of internal gravity waves (IGW) in stably stratified dissipative ionosphere with non-uniform zonal wind (shear flow) is studied. Due to the nonlinear mechanism nonlinear solitary, strongly localized IGW vortex structures can be formed. Therefore, a new degree of freedom of the system and accordingly, the path of evolution of disturbances appear in a medium with shear flow. Depending on the type of shear flow velocity profile the nonlinear IGW structures can be the pure monopole vortices, the transverse vortex chain or the longitudinal vortex street in the background of non-uniform zonal wind. Accumulation of these vortices in the ionosphere medium can create the strongly turbulent state.

Keywords: Gravity waves, shear flow, nonlinear vortex structures.

AMS Subject Classification: 35Q35, 37K10, 37K40.

1. Introduction

In recent years an increasing interest is paid to investigation of the properties of internal gravity waves (IGW), arising as a result of vertical density stratification of the gas, and play an important role in the dynamics of both the lower and upper atmosphere and ionosphere of the earth and other planets. Grown interest, first of all, is caused primarily due to the understanding of the fact that these waves can propagate over hundreds or thousands of kilometers from the source without significant attenuation. Propagating with group velocity the IGW provide an efficient transfer of energy, heat and momentum from the troposphere into the upper atmosphere (which exceeds even the energy supplied by the solar wind), where they influence on the thermal and dynamic regimes (Francis, 1975; Kim and Mahrt, 1992; Nakamura et al., 1993; Rishbeth and Fukao, 1995; Fritts et al., 2006; Alexander et al., 2008; Hecht et al., 2009; Alexander, 2010). Latest numerical experiments (Gavrilov and Fukao, 2001; Alexander and Rosenlof, 2003; Alexander

*Corresponding author. Email: khatuna.chargazia@gmail.com

ISSN: 1512-0082 print

et al., 2010) show that an adequate description of climate change and the circulation of the middle atmosphere requires taking into account the accelerations of the background flows and heat inflows generated by the waves (especially by IGW) propagating from the troposphere.

Currently, the results of numerous observations and experiments reveal the wave motion in a wide range of frequencies from the acoustic to the planetary ones in the atmosphere-ionosphere environment on almost all altitudes. In atmospheric acoustics the focus is laid on the study of internal gravity waves (IGW), representing fluctuations of atmospheric and ionospheric layers, the nature of which is mostly determined by gravity force. These oscillations are going with the frequency, at which the wave speed is comparable with the acceleration of gravity force. Therefore, for definiteness, we assume that their periods range from 5 minutes to 3 hours, and the wavelengths - from 100 m to 10 km.

In this paper, a property of internal gravity waves presents particular interest to us - propagating vertically up quite easily in an isothermal atmosphere, IGW tends to increase the amplitude of the hydrodynamic velocity exponentially with height, which follows from the conservation of energy when the density of the medium decreases with height growth (Hines, 1960; Gossard and Hook, 1975). Thus, even for the waves, the initial amplitude of which is small, the nonlinear effects at sufficiently high altitude becomes significant and must be taken into account. Indeed, it is clear that this growth can not be continued indefinitely. At some heights velocity becomes so large that the nonlinear effects can join the game. These effects stop the growth of the oscillation amplitude through the nonlinear interaction between the modes, the perturbations energy redistribution (saturation of the waves) and, for example, self-organization of IGW vortex structures (Aburdjania, 1996, 2006). Nonlinear vortex structures transfer the trapped particles of the medium. Reaching the critical heights, the IGW structures, interacting with each other and medium, may form the atmospheric turbulence (Waterscheid and Schubert, 1990), that creates real threats to aviation safety, but also leads to a mix of chemicals, released from the lower atmosphere, chemical reactions between them and the formation of potentially harmful compounds (Friedrich et al., 2009). Therefore, the IGW structures may also influence the formation of "space weather" by generating irregularities in the ionosphere (Schunk and Sojka, 1996).

2. Nonlinear vortex structures governed by the shear flow

The spontaneously excited internal gravity waves at different layers of the ionosphere intensively draw energy from the shear flow at a certain point (in particular, for a time interval $0 < \tau \leq \tau^*$) in their evolution. Receiving energy, amplitude of IGW increases (by an order of magnitude) and, accordingly, the nonlinear processes come into play. In this case, in the linear IG waves will be translated into nonlinear vortex structures and full nonlinear system has to be investigated given in (Chargazia et al, 2018).

The results of the observations and targeted experiments show (Bengtsson and Lighthill, 1985; Chmyrev et al., 1991; Nezlin, 1994; Sundkvist, et al., 2005) that nonlinear solitary vortex structures can be generated at different layers of the atmosphere-ionosphere-magnetosphere. These structures transfer trapped rotating medium particles. Moreover, the ratio of the rotational speed of the particles U_c to the speed of motion of nonlinear structures U is given by $U_c/U \geq 1$.

We introduce the characteristic time T and spatial scales L of the nonlinear structures. The following relationships are established in [4] between quantities: $U_c \sim V$, $U \sim L/T$. Similarly, for the ratio of the nonlinear term with the inertial one, we have: $(V\nabla)V/(\partial V/\partial t) \sim V/(L/T) \sim U_c/U$. Thus, the nonlinearity plays an essential role for wave processes satisfying the condition $U_c \geq U$. This estimation shows that nonlinear effects play a crucial role in the dynamics of IGW-type wave, the initial linear stage of development of which is considered in previous section. Inequality $U_c \geq U$ coincides with the anti-twisting condition (Williams and Yamagata, 1984). Satisfying just the latter condition the initial nonlinear dynamic equations may have the solitary (vortex) solutions (Williams, Yamagata, 1984; Nezlin, Chernikov, 1999).

From the general theory of nonlinear waves is well known the fact (Whitham, 1977) that if in the system the nonlinear effects are significant, then the principle of superposition cant be applied and the solution in the form of a plane wave is unjust. Nonlinearity distorts the wave profile and the wave form differs from a sinusoid. If in a nonlinear system the dispersion (or non-uniform equilibrium parameters of the medium) is lacked, all small-amplitude waves with different wave numbers k propagate with the same speed and have the opportunity for a long time interaction with each other. So, even a small nonlinearity leads to the accumulation of distortions. Such nonlinear distortion, as a rule, leads to the wave front curvature growth and its upset (breaking) or to the formation of the shock wave. In the presence of dispersion the phase velocities of waves with different k vary with the latter, the waves with different k propagate with different velocities and virtually unable to interact with each other. Therefore, the wave packet tends to spreading. For not very large amplitude the wave dispersion can compete with the nonlinearity. Because of this before breaking the wave may split into separate nonlinear wave packets, and the shock wave will not form. Indeed, in the real atmosphere, the shock wave, as a rule, (spontaneously, without external influence) is not formed spontaneously. Primarily, this means that in the atmosphere-ionosphere medium dispersion effects are strongly pronounced and significantly compete with nonlinear distortion. If the nonlinear steepening of the wave is exactly compensated by the dispersion spreading, there may appear the stationary waves such as solitary vortices propagating in a medium without changing its shape.

It should be noted also, that the results of ground and satellite observations show clearly that in the different layers of the ionosphere the zonal winds (currents) are permanently present, which are non-uniform along the vertical (Gershman 1974; Gossard and Hook, 1978; Kazimirovskii and Kokourov, 1979). As noted in section 3, at interaction with non-uniform zonal flow the wave disturbance obtains an additional dispersion as well as a new source of amplification and the nonlinear effects come into play in their dynamics. Thus, the ionospheric medium with shear flow creates a favorable condition for the formation of nonlinear stationary solitary wave structures.

So, we want to find a solution of the nonlinear model (Chargazia et al, 2018) (a non-dissipative case $\nu = \sigma_p = 0$) in the form of stationary regular waves $\Psi = \Psi(\eta, z)$ and $R = R(\eta, z)$, propagating along the parallel (along the x-axis) with a constant velocity $U = \text{const}$ without changing its form, where $\eta = x - U\tau$. Moreover, we consider the case when the wave structures propagate on the background mean zonal wind, which has the non-uniform velocity.

In the non-dissipative case ($\nu = \sigma_p = 0$), passing to above mentioned auto model variables η and z and considering that in this case $\partial/\partial\tau = -U\partial/\partial\eta$, we get:

$$-U\frac{\partial}{\partial\eta}\left(\Delta\Psi - \frac{\Psi}{4H^2}\right) + \frac{\partial R}{\partial\eta} + J\left(\Psi, \Delta\Psi\right) = 0, \tag{1}$$

$$-U\frac{\partial R}{\partial\eta} - \omega_g^2\frac{\partial\Psi}{\partial\eta} + J\left(\Psi, R\right) = 0. \tag{2}$$

Here we have introduced a new feature of the stream function

$$\Psi(\eta, z) = \Phi_0(z) + \overline{\psi}(x, z), \tag{3}$$

and the velocity potential $\Phi_0(z)$ of the background zonal shear flow through the notation:

$$v_0(z) = -\frac{d\Phi_0(z)}{dz}. \tag{4}$$

Providing the so-called vector integration, according to (Aburjania, 2006), the general solution of equation (70) can be presented as:

$$R(\eta, z) = \omega_g^2 z + F\left(\Psi + Uz\right), \tag{5}$$

where $F(\xi)$ is the arbitrary function of its argument. Next, substituting (5) into (1) and performing the similar transformation we get a nonlinear equation in the form of the Jacobian:

$$J\left(\Delta\Psi + U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z, \Psi + Uz\right) = 0. \tag{6}$$

The general solution of (6) has the form (Aburjania, 2006):

$$\Delta\Psi + U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z = G(\Psi + Uz), \tag{7}$$

where $G(\xi)$- a new arbitrary function of its argument.

As it was mentioned earlier, the results of observations and experiments show that vortex streets of various forms can be generated in ordinary liquid and plasma environment in the presence of the shear flow, as a consequence of the nonlinear saturation of Kelvin-Helmholtz instability. Such structures may occur if the asymptotic form of the function in equation (7) is nonlinear (Petviashvili and Pokhotelov, 1992; Aburjania, 2006).

We assume that the nonlinear structure move by a velocity U that satisfies the following condition:

$$U\int\frac{dz}{4H^2} + \frac{dF(\Psi + Uz)}{d(\Psi + Uz)}z = 0. \tag{8}$$

It is obvious that (8) holds for IGW at only case when the function $F(\xi)$ is a linear function of its argument over the plane x, i.e. $F = -U(\Psi + Uz)/(4H^2)$. In this case, choosing an arbitrary function G as the following nonlinear function $G(\xi) = \psi_0^0\kappa^2\exp(-2\xi/\psi_0^0)$ (Petviashvili and Pokhotelov, 1992; Aburjania, 2006), equation (7) reduces to:

$$\Delta(\Psi + Uz) = \psi_0^0\kappa^2\exp\left[-2(\Psi + Uz)/\psi_0^0\right]. \tag{9}$$

Now lets choose an expression for the stream function of the background shear flow in the form:

$$\Phi_0 = Uz + \psi_0^0\ln(\kappa_0 z). \tag{10}$$

Here ψ_0^0 characterizes the amplitude of the background structure, but $2\pi/\kappa$ and $2\pi/\kappa_0$ presents the characteristic size of the vortex structure and parameter of non-uniform background shear flow, respectively.

Given (3) and using (10), the vorticity equation (9) can be transformed into:

$$\Delta\overline{\psi} = \psi_0^0\kappa_0^2\left[\frac{\kappa^2}{\kappa_0^2}e^{-2\overline{\psi}/=\psi_0^0} - 1\right]. \tag{11}$$

This equation has the solution (Mallier and Maslow, 1993):

$$\overline{\psi}(\eta, z) = \psi_0^0\ln\left[\frac{\mathrm{ch}(\kappa) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}{\mathrm{ch}(\kappa_0 z)}\right], \tag{12}$$

which is a street of the oppositely-rotating vortices. Substituting (12) and (10) into expression (3), we finally obtain the solution:

$$\Psi(\eta, z) = Uz + \psi_0^0\ln\left[\mathrm{ch}(\kappa) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)\right]. \tag{13}$$

Finally, we obtain the following expressions for the components of the medium velocity and shear flow, respectively:

$$V_x(\eta, z) = -U - \psi_0^0\kappa\frac{\mathrm{sh}(\kappa z)}{\mathrm{ch}(\kappa) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}, \tag{14}$$

$$V_z(\eta, z) = -\psi_0^0\kappa\frac{\sqrt{1-\kappa_0^2}\mathrm{sh}(\kappa z)}{\mathrm{ch}(\kappa) + \sqrt{1-\kappa_0^2}\cos(\kappa\eta)}, \tag{15}$$

$$v_0(z) = -U - \psi_0^0\kappa_0\mathrm{th}(\kappa_0 z). \tag{16}$$

At $\kappa_0 = 1$ the solution (13) describes the background flow to the type of shear zonal flow (16). At $\kappa_0^2 < 1$ in the middle of the zonal flow (16) the longitudinal vortex street will form (Fig. 1). Solution (14), (15) with closed streamlines in the form of "cat's eyes" was first obtained by Lord Kelvin.

It must be mentioned that the nonlinear stationary equations (6), (7) also have an analytical solution in the form of a Larichev-Reznik type dipole pair of cyclone-anticyclone (Petviashvili and Pokhotelov, 1992; Aburjania, 2006) and vortex transverse chains (Aburjania et al., 2005).

3. Energy transfer by the vortex structures

In the dynamic equations of IGW structures (Chargazia et al, 2018) the source of convergence of external energy due to shear flow (non-uniform wind), the terms with $v_0(y)$, and divergence sources of energy due to dissipative processes in the environment-terms of induction σ_p and viscous ν damping are included obvious. The above mentioned nonlinear solitary vortex structures can self-sustain only at the existence of an appropriate balance between the convergence and divergence of energy in the wave perturbations in the ionosphere medium.

Energy transport equation for the IGW vortex structures will have the form:

$$\begin{aligned}\frac{\partial E}{\partial t} &= \int v_0'(z)\frac{\partial \overline{\Psi}}{\partial x}\frac{\partial \overline{\Psi}}{\partial z}dxdz - \frac{\sigma_p B_{0y}^2}{\rho_0}\int\left(\frac{\partial \overline{\Psi}}{\partial x}\right)^2 dxdz \\ &-\frac{\sigma_p B_0^2}{\rho_0}\int\left[\left(\frac{\partial \overline{\Psi}}{\partial z}\right)^2 + \frac{\overline{\Psi}^2}{4H^2}\right]dxdz \\ &-v\int\left[\left(\frac{\partial^2 \overline{\Psi}}{\partial^2 x}\right) + \left(\frac{\partial^2 \overline{\Psi}}{\partial^2 z}\right)^2 + 2\left(\frac{\partial^2 \overline{\Psi}}{\partial x \partial z}\right)^2\right]dxdz,\end{aligned} \tag{17}$$

where

$$E = \frac{1}{2}\int\left[\left(\nabla\overline{\Psi}\right)^2 + \frac{\overline{\Psi}^2}{4H^2} + \frac{R^2}{\omega_g^2}\right]dxdz, \tag{18}$$

presents energy of the nonlinear internal-gravity vortex structure.

It should be mentioned, that the equation (17) is valid for linear as well as for nonlinear stage of evolution of IGW perturbations. In this equation the first term of the right hand side describes transient sway-generation of the IGW structures due to the shear instability; the second and the third terms an induction damping of the wave disturbances due to Pedersen conductivity, and the last term describes the viscous damping of the perturbations. According to (17), for generation of the structures it is necessary the velocity of the shear flow to have at least the first derivative with respect to vertical coordinate different from zero ($v_0'(z) \neq 0$).

The considered IGW perturbations in the linear modes propagate along the Earths parallel (along the x axis). The induction and viscous damping takes energy from these IGW structures and heat the ionospheric environment. In this case, shear flow temporarily supply the medium with energy, causing generation - swing of IGW structures and the development of shear instability.

Thus, the revealed internal gravity vortices in the ionosphere are sufficiently long-lived, so they can play a significant role in the transport of solid matter, heat, energy and form strong turbulence state in the medium (Aburjania et al., 2009).

4. Energy transfer by the vortex structures

In this article the nonlinear evolution of IGW structures in the dissipative stably stratified ($\omega_g^2 > 0$) ionosphere in the presence of shear flow (non-uniform zonal wind) is studied. The self-organization of nonlinear wave perturbations into the solitary vortex structures and the transformation of the perturbation energy into heat is revealed.

The equation of energy transfer by nonlinear wave structure in the dissipative ionosphere is established. Based on the analysis of this equation it is revealed that the IGW structure effectively interacts with the local background non-uniform zonal wind and self-sustained by the shear flow energy in the ionosphere.

On the basis of analytical solutions of nonlinear dynamical equations it's shown that the internal-gravity waves organize themselves (due to the shear flow energy) in the form of stationary solitary vortex structures. The solution of the nonlinear equations has an exponential asymptotic behavior $\sim \exp(-\kappa|r|)$ at $|r| \to \infty$, i.e. structures are strongly localized along the plane transverse to the Earth's surface. Depending on the type of velocity profile of the zonal shear flow (wind) $v_0(z)$, the generated nonlinear structures maybe the monopole solutions, cyclone, anticyclone, dipole cyclone-anticyclone pair, longitudinal vortex streets or transverse vortex chain in the background of non-uniform zonal wind (see also Aburjania, et al., 2005). The presence of spatially non-uniform winds in the ionosphere gives IGW the properties of self-organization and self-sustaining in the form of the aforementioned nonlinear solitary vortex structures of different shapes.

It should be noted that the discussed nonlinear two-dimensional vortex structures are very different from the atmospheric Rossby-type vortices (Larichev and Resnick, 1976; Aburjania, 2006). The main difference is that the motion velocity of our vortices is completely symmetric, i.e. the structures can move with velocities greater than the maximum phase velocity of linear waves in any horizontal direction. While Rossby vortices can move to the west only at the velocities exceeding the maximum velocity of Rossby waves. In the East such vortices can move with any speed as far as the linear Rossby waves do not propagate in this direction. In addition, we assumed that the atmospheric-ionospheric medium is isothermal.

IGW structures are eigen degrees of freedom of the ionospheric resonator. Therefore, influence of external sources on the ionosphere above or below (magnetic storms, earthquakes, artificial explosions, etc.) will excite these modes (or intensified) in the first, (Aburjania and Machabeli, 1998). For a certain type of pulsed energy source the nonlinear solitary vortical structures will be generated (Aburdjania, 1996; Aburdjania, 2006), which is confirmed by experimental observations (Ramamurthy et al., 1990; Cmyrev et al., 1991; Nezlin, 1994; Shaefer et al., 1999; Sundkvist et al., 2005). Thus, these wave structures can also be the ionospheric response to natural and artificial activity.

Acknowledgements.

This work was supported by Shota Rustaveli National Science Foundation project No FR17_279.

References

[1] G.D. Aburjania, Kh. Chargazia, L.M. Zeleny, G. Zimbardo, *Model of strong stationary vortexturbulence in the space plasma*, Nonlinear Processes in Geophysics, **16**, 1 (2009), 11–22

[2] G.D. Aburjania, *Self-organization of the vortex structures and the vortex turbulence in the dispersed media* (in Russian), Komkniga, URSS, Moscow, 2006

[3] G.D. Aburjania, A.G. Khantadze, O.A. Kharshiladze, *Mechanism of the planetary Rossby wave energy amplification and transformation in the ionosphere with an inhomogeneous zonal smooth shear wind*, Journal of Geophysical Research, **111**, A09304 (2006), Doi:10.1029/ 2005JA011567

[4] G.D. Aburjania, Kh.Z. Chargazia, G.V. Jandieri, O.A. Kharshiladze, *Generation and propagation of the ULF planetary-scale electromagnetic wavy structures in the ionosphere*, Planetary and Space Science, **53**, 9 (2005), 881–901

[5] G.D. Aburjania, *Self-organization of acoustic gravity vortices in the ionosphere before the earthquake*, Plasma Phys. Rep., **22**, 10 (1996), 954–959

[6] M.J. Alexander, *Gravity waves in the stratosphere*, in: The Stratosphere: Dynamics, Chemistry and Transport. L.M. Polvani, A. Sobel, D.W. Waugh (Eds.). Geophys. Monogr. Ser., (2010), Chapter 5. Doi: 10. 1029/ 2009GM000887

[7] M.J. Alexander, M. Geller, C. McLandress, et al., *Recent developments in gravity wave effects, in climate models and global distribution of gravity wave momentum flux from observations and models*, Q. J. R. Meteorol. Soc. **136**, (2010), 1103–1124

[8] M.J. Alexander, J. Gille, C. Cavanaugh, et al., *Global estimates of gravity wave momentum flux from High Resolution Dynamics Limb Sounder observations*, Journal of Geophysical Research, **113**, D15S18 (2008), Doi:10.1029/2007JD008807

[9] M.J. Alexander, K.H. Rosenlof, 2003. *Gravity wave forcing in the stratosphere: Observational constraints from the Upper Atmosphere Research Satellite and implications for parametrization in global models*, Journal of Geophysical Research, **108**, D19,4597, (2003), Doi:10.1029/2003JD003373

[10] L. Bengston, J. lighthill, *Intense Atmospheric Vortices*, Springer-Verlag, Beerlin-Heidelberg, 1982

[11] K. Chargazia, O. Kharshiladze,D. Kvaratskhelia, *Linear Evolution of IGW Structures in the Ionospheric Plasma at Interaction with Shear Flows*, Bulletin of TICMI, **22**, 1 (2018), 11-23

[12] V.M. Cmyrev, V.A. Marchenko, O.A. Pokhotelov, et al., *Vortex structures in the ionosphere and magnetosphere of the Earth*, Planet. Space Sci. **39**, (1991), 1025–1030

[13] S.H. Francis, *Global propagation of atmospheric gravity waves: A review*, Journal of Atmospheric and Terrestrial Physics. **37**, 6-7 (1975), 1031–1054

[14] M. Friedrich, K.M. Torkar, W. Singer, et al., *Signatures of mesospheric particles in ionospheric data.* Ann. Geophys, **27**, (2009), 823–829

[15] D.C. Fritts, D. Janches, D.M. Riggin, et al., *Gravity waves and momentum fluxes in the mesosphere and lower thermosphere using 430 MHz dual-beam measurements at Aresibo: 2. Frequency spectra, momentum fluxes, and variability.* Journal of Geophysical Research, **111**, D18108 (2006), Doi:10.1029/2005JD006883

[16] N.M. Gavrilov, S. Fukao, *Hydrodynamic tropospheric wave sources and their role in gravity wave climatology of upper atmosphere from the MU radar observations*, J. Atmos. Solar-Terr. Physics. **63**, 9 (2001), 931–943

[17] E. Gossard, W. Hooke, *Waves in the Atmosphere*, Elsevier, Amsterdam, 1975

[19] J.H. Hecht, M.J. Alexander, R.L. Walterscheid, et al., 2009. *Imaging of atmospheric gravity waves in the stratosphere and upper mesosphere using satellite and ground-based observations over Australia during the TWPICE campaign*, Journal of Geophysical Research, **114**, D18123 (2009), Doi: 10.1029/2008 JD011259

[19] C.O. Hines, *Internal atmospheric gravity waves at ionospheric heights*, Canad. J. Phys., **38**, 11 (1960), 1441–1481

[20] J. Kim, L. Mahrt, *Momentum transport by gravity waves*, J. Atmos. Sci. **49**, (1992), 735–748

[21] T. Nakamura, T. Tsuda, M. Yamamoto, et. Al., *Characteristic of gravity waves iv the mesosphere observed with the middle and upper atmosphere radar. 1. Momentum flux*, Journal of Geophysical Research, **98**, D5 (1993), 8899–8910

[22] M.V. Nezlin, G.P. Chernikov, *Analogies of the drift vortices in plasma and geophysical hydrodynamics*, Plasma Phys. Rep., **21**, 11 (1999), 975–999

[23] M.V. Nezlin, *Rossby solitary vortices, on giant planets and in the laboratory*, CHAOS. 4, (1994), 187–202

[24] J. Pedlosky, *Geophysical Fluid Dynamics*, Springer-Verlag, New York, 1979

[25] V.I. Petviashvili, O.A. Pokhotelov, *Solitary Waves in Plasmas and Atmosphere*, Gordon and Breach, London, 1992

[26] M.K. Ramamurthy, B.P. Collins, R.M. Rauber, et al., *Evidence of very-large-amplitude solitary waves in the atmosphere*, Nature, **348**, (1990), 314–317, Doi: 10.1038/348314A0

[27] H. Rishbeth, S. Fukao, *A review of MU radar observation of the thermosphere and ionosphere*, J. Geomag. Geoelectr, **47**, (1995), 621–637

[28] R.W. Schunk, J.J. Sojka, *Ionosphere thermosphere space weather issues*, J. Atmos. Terr. Physics, **58**, 14 (1996), 1527–1574

[29] L.D. Shaefer, O.R. Rock, T.P. Levis, *Detection of explosive events by monitoring acoustically-induced geomagnetic perturbations*, Lawrence Livermore Laboratory. CA USA, 94550, 1999

[30] D. Sundkvist, A. Vaivads, M. Andre, et al., *Multi-spacecraft determination of wave characteristics near the proton gyrofrequency in high-altitude cusp*, Ann. Geophys., **23**, (2005), 983–995

[31] R.L. Waterscheid, G. Schubert, *Nonlinear evolution of an upward propagating gravity wave: Overturning, Convection, Transience and Turbulence*, J. Atmos. Sci. **47**, 1 (1990), 101–125

[32] G.B. Whitham, *Linear and Nonlinear Waves*, John Wiley, New York, 1977

[33] G.P. Williams, T. Yamagata, *Geostrophic regimes, intermediate solitary vortices and Jovian Eddies*, J. Atmos. Sci. 41, (1984), 453–468