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Hesitant Fuzzy TOPSIS based Facility Location Selection Problem

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This study develops a decision support methodology for facility location selection problem. The methodology is based on the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) approach in hesitant fuzzy environment. We are focusing on a special case of facility location problem, namely location planning for service centers. Such problem usually involves a set of candidate centers locations (alternatives), which are evaluated considering a set of weighted criteria.

To evaluate criteria our approach implies using of experts' assessments. In the proposed methodology the values of the criteria are expressed in linguistic terms, given by all experts. Then, these linguistic terms are described by trapezoidal fuzzy numbers. Consequently, proposed approach is based on trapezoidal hesitant fuzzy TOPSIS decision-making model.

The case when the information on the criteria weights is completely unknown is considered. The criteria weights identification based on De Luca-Termini information entropy is offered in context of hesitant fuzzy sets.

Following the TOPSIS algorithm, first the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS) are defined. Then the ranking of alternatives is performed in accordance with the proximity of their distances to the both FPIS and FNIS.

Keywords: Facility location selection problem; Multiple criteria group decision making; Hesitant trapezoidal fuzzy set; Information entropy; Fuzzy TOPSIS method.

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1. Introduction

A Multiple Criteria Group Decision Making problem (MCGDM) deals with a selection of one best of the feasible alternatives or several ranked alternatives that are evaluated by a group of experts based on multiple, often conflicting criteria. From this perspective, the Facility Location Problem (FLP) represents a MCGDM problem [6, 11, 12, 16, 17].

Models of FLP have to support the generation of optimal locations of service centers in complex and uncertain situations. The facility location problem usually involves a set of candidate centers locations (alternatives), which are evaluated considering a set of weighted criteria. The alternative that is identified as the best with respect to all criteria, will be chosen for implementation.

Several approaches have been proposed in the literature for solving the facility location problems (see [2] and others). In [1], a hybrid Taguchi-immune approach was presented to optimize an integrated supply chain design problem with multiple shipping. A belief programming model for the location of logistics service

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centers have been proposed in [18]. Multi-attribute facility location models have been investigated in [15]. It should be noted that in most of the above works, the problem of determining the location in a certain environment is studied, that is, the decision-making parameters (criteria, attributes, weights) in the problem are fixed numbers and are known in advance. In reality, often the parameters cannot be obtained with certainty.

There are some publications proposing application of fuzzy methods in the FLP. Because of the inherent uncertainty of expert preferences, as well as due to the fact that decision-making parameters can be fuzzy and uncertain, evaluations of them most often are represent as fuzzy values (fuzzy numbers, triangular fuzzy numbers and so on). Then the methods for facility location problems are developed called in this case Fuzzy Facility Location Problem (FFLP) [4, 9]. In this work we consider a new model of FFLP based on the fuzzy TOPSIS approach [5, 25] for the optimal selection of facility location centers.

Application of fuzzy theory for location planning of facilities has been presented in various studies. In [10], four fuzzy multi-criteria group decision making approaches in evaluating facility locations have been used. An algorithm for facility site selection based on fuzzy theory and hierarchical structure analysis have been developed in [14]. A fuzzy multi-attribute decision making approach for the service centers location selection problem have been developed in [4]. Fuzzy TOPSIS approaches for facility location selection problem have been developed in [5, 13, 21, 25].

Different from other studies, in this paper the novel approach based on hesitant fuzzy TOPSIS decision making model with entropy weights is developed. The case when the information on the criteria weights is completely unknown is considered. The criteria weights are obtained by applying De Luca-Termini non-probabilistic entropy concept [7], which is offered in context of hesitant fuzzy sets. A fuzzy hesitant trapezoidal TOPSIS method is employed to ranking the alternatives. The method is described in Section 2 (Subsection 2.3). The developed method is applied to service centers location selection problem.

2. Preliminaries

2.1. *On the trapezoidal fuzzy numbers*

A trapezoidal fuzzy number $\widetilde{A}$ can be determined by a quadruple $\widetilde{A} = (a, b, c, d)$. Its membership function is defined as

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{if } x < a, \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b, \\ 1 & \text{if } b \leq x \leq c, \\ \frac{d-x}{d-c} & \text{if } c \leq x \leq d, \\ 0 & \text{if } x > d. \end{cases}$$

where $a \leq b \leq c \leq d$ [3].

Let $\widetilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number. Using Graded Mean Integration Representation Method we can get following representation of $\widetilde{A}$ by the equation

$$p(\widetilde{A}) = (a + 2b + 2c + d)/6\,. \tag{2.1}$$

Graded Mean Integration Representation is the defuzzification method that converts a trapezoidal fuzzy number in to a corresponding crisp number.

2.2. *On the hesitant fuzzy sets*

Hesitant fuzzy set (HFS) was introduced by Torra and Narukawa in [20] and Torra in [19] as a generalization of a fuzzy set. In HFS the degree of membership of an element to a reference set is presented by several possible fuzzy values. This allows describing situations when decision makers (DMs) have hesitancy in providing their preferences over alternatives. The HFS is defined as follows:

Definition 2.1 [19, 20]**:** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a reference set, a hesitant fuzzy set E on X is defined in terms of a function $h_E(x)$, which when applied to X returns a subset of $[0, 1]$:

$$E = \{\langle x, h_E(x)\rangle \mid x \in X\},$$

where $h_E(x)$ is a set of some different values in $[0, 1]$, representing the possible membership degrees of the element $x \in X$ to E; $h_E(x)$ is called a hesitant fuzzy element (HFE).

Definition 2.2 [24]**:** Let M and N be two HFSs on $X = \{x_1, x_2, \ldots, x_n\}$, then the distance measure between M and N is defined as $d(M, N)$, which satisfies the following properties:

1) $0 \le d(M, N) \le 1$;
2) $d(M, N) = 0$ iff $M = N$;
3) $d(M, N) = d(N, M)$.

It is clear that the number of values (length) for different HFEs may be different. Let $\ell(h_E(x))$ be the length of $h_E(x)$. After arranging the elements of $h_E(x)$ in a decreasing order, let $h_E^{\sigma(j)}(x)$ be the jth largest value in $h_E(x)$. To calculate the distance between M and N when $\ell(h_M(x_i)) \ne \ell(h_N(x_i))$, it is necessary extend the shorter one by adding any value in it, until both will have the same length. The choice of this value depends on the expert's risk preferences. An optimist expert may add the maximum value from HFE, while a pessimist expert may add the minimal value.

In the present paper the hesitant weighted Hamming distance is used that is defined by the following equation

$$d_{hwh}(M, N) = \sum_{i=1}^{n} w_i \left[\frac{1}{\ell_{x_i}} \sum_{j=1}^{\ell_{x_i}} \left|h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i)\right|\right], \tag{2.2}$$

where $h_M^{\sigma(j)}(x_i)$ and $h_N^{\sigma(j)}(x_i)$ are the jth largest values in $h_M(x_i)$ and $h_N(x_i)$, respectively; $\ell_{x_i} = \max\{\ell(h_M(x_i)), \ell(h_N(x_i))\}$ for each $x_i \in X$; w_i $(i = 1, 2, \ldots, n)$ is the weight of the element $x_i \in X$ such that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 2.3 [23]: For a HFE $h_E(x)$, the score function $s(h_E(x))$ is defined as follows:

$$s(h_E(x)) = \sum_{j=1}^{\ell(h_E(x))} \frac{h_E^{\sigma(j)}(x)}{\ell(h_E(x))}\,, \tag{2.3}$$

where $s(h_E(x)) \in [0, 1]$.

Let h_1 and h_2 be two HFEs. Based on score function it is possible to make ranking of HFEs according to the following rules: $h_1 > h_2$ if $s(h_1) > s(h_2)$; $h_1 < h_2$ if $s(h_1) < s(h_2)$ and $h_1 = h_2$ if $s(h_1) = s(h_2)$.

2.3. *Determination of the criteria weights using De Luca-Termini entropy*

The complexity and uncertainty of the problems of service centers location planning leads to the information on criteria weights usually being incomplete or completely unknown. Here we consider a case when the criteria weights are unknown.

Assume that, we have the hesitant decision matrix $H = (h_{ij})_{m\times n}$, each element of which represents a HFE.

De Luca and Termini [7] defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set X as:

$$E_{LT} = -k \sum_{i=1}^{n} \left[\mu_A(x_i) \ln \mu_A(x_i) + (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))\right], \quad k > 0,$$

where $\mu_A : X \to [0, 1]$; k is a positive constant.

The criteria weights definition method based on the De Luca-Termini entropy can be described as follows:

Step 1: Calculate the score matrix $S = (s_{ij})_{m\times n}$ of the hesitant decision matrix H, where $s_{ij} = s(h_{ij})$ is the score value of h_{ij} (see equation (2.3)).

Step 2: Calculate the normalized score matrix $S' = (s'_{ij})_{m\times n}$, where

$$s'_{ij} = s_{ij} \Big/ \sum_{i=1}^{m} s_{ij}\,. \tag{2.4}$$

Step 3: Determine the criteria weights.

By using De Luca-Termini normalized entropy in context of hesitant fuzzy sets

$$E_j = -\frac{1}{m \ln 2} \sum_{i=1}^{m} \left(s'_{ij} \ln s'_{ij} + (1 - s'_{ij}) \ln(1 - s'_{ij})\right), \quad j = 1, 2, \ldots, n, \tag{2.5}$$

the definition of the criteria weights is expressed by the formula

$$w_j = (1 - E_j) \Big/ \sum_{j=1}^{n} (1 - E_j)\,, \quad j = 1, 2, \ldots, n, \tag{2.6}$$

where the value of w_j represents the relative intensity of x_j criterion importance [12].

2.4. *On the hesitant trapezoidal fuzzy sets*

Definition 2.4 [23]**:** Let Xbe a reference set, a hesitant trapezoidal valued fuzzy set T on X is defined in terms of a function $f_T(x)$ as follows:

$$T = \{\langle x, f_T(x)\rangle \mid x \in X\},$$

where $f_T(x)$ is a set of several trapezoidal fuzzy numbers, representing the possible membership degrees of the $x \in X$ element to the HTFS T; $f_T(x)$ is called a hesitant trapezoidal fuzzy element (HTFE).

3. Formulation of Facility Location Selection MCGDM Problem in Hesitant Fuzzy Environment

Consider MCGDM problem for location planning of service centers. The proposed framework for location planning of candidate centers comprises following steps presented in detail.

3.1. *Selection of location criteria*

Involves the selection of location criteria to assess potential locations for candidate centers. These criteria are obtained from a literature review and discussion with other experts and members of the city transportation group. For example, the set of criteria may be presented (see Table 1) to determine the best location for implementing service centers.

As can be seen from Table 1, criteria of two types are considered:

a) the benefit type criteria - this means that the higher the criterion's value, the more preferable is the alternative for the best location;
b) the cost type criteria - that is, the lower the criterion's value, the better the alternative for the best location.

3.2. *Selection of potential locations for service centers*

Involves selection of potential locations for implementing service centers. The experts use their knowledge, prior experience with the transportation or other conditions of the geographical area and the presence of freight regulations to identify candidate locations for implementing service centers. For example, if certain areas are restricted for delivery by municipal administration, these areas are barred from being considered as potential locations for implementing urban service centers. Ideally, potential locations are those that satisfy the interests of all stakeholders in the city, such as city residents, logistics operators, municipal administrations, etc.

Assume that there are m locations of candidate centers – decision making alternatives $A = \{A_1, A_2, \ldots, A_m\}$, and the group of k experts $E = \{e_1, e_2, \ldots, e_k\}$ evaluates them with respect to an n criteria $X = \{x_1, x_2, \ldots, x_n\}$. Experts give the

Table 1. The Criteria for Location Centers Selection

Criteria	Definition	Criteria type
Accessibility	Access by public and private transport modes to the location	Benefit (the more the better)
Security	Security of the location from accidents, theft and vandalism	Benefit (the more the better)
Connectivity to multimodal transport	Connectivity of the location with other modes of transport, e.g. highways, railways, seaport, airport etc.	Benefit (the more the better)
Costs	Costs in acquiring land, vehicle resources, drivers and etc. for the location	Cost (the less the better)
Environmental impact	Impact of location on the environment, for example, air pollution, noise	Cost (the less the better)
Proximity to customers	Distance proximity of location to customer locations	Benefit (the more the better)
Proximity to suppliers	Distance proximity of location to supplier locations	Benefit (the more the better)
Conformance to sustainable freight regulations	Ability to conform to sustainable freight regulations imposed by municipal administrations for e.g. restricted delivery hours, special delivery zones	Benefit (the more the better)
Possibility of expansion	Ability to increase size to accommodate growing demands	Benefit (the more the better)

evaluations over criteria in form of lingual assessments – linguistic terms. Then, these assessments are expressed in trapezoidal fuzzy numbers (TrFNs) using 5-point linguistic scale [22] (see Table 2).

Table 2. Linguistic terms for criteria ratings

Linguistic term	Corresponding TrFNs
Very low (VL)	(0, 0.1, 0.2, 0.3)
Low (L)	(0.1, 0.2, 0.3, 0.4)
Medium (M)	(0.3, 0.4, 0.5, 0.6)
High (H)	(0.5, 0.6, 0.7, 0.8)
Very high (VH)	(0.7, 0.8, 0.9, 1.0)

After those transformations of lingual expressions, experts' joint assessments concerning each alternative represent HTFS:

A HTFS A_i of the ith alternative - location of candidate centers - on X is given by

$$A_i = \{\langle x_j, f_{A_i}(x_j)\rangle \mid x_j \in X\},$$

where $f_{A_i}(x_j)$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$, indicates the possible membership degrees of the ith alternative A_i under the jth criterion x_j, and it can be expressed as a HTFE $\widetilde{t}_{ij}$. All HTFEs create the aggregate fuzzy hesitant trapezoidal decision matrix $\widetilde{T} = (\widetilde{t}_{ij})_{m\times n}$.

Considering that the criteria have different importance degrees, the weight vector of all criteria, given by the experts, is defined by $w = (w_1, w_2, \dots, w_n)$, where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$ and w_j is the importance degree of jth criterion.

3.3. *Locations evaluation using fuzzy TOPSIS*

This Subsection presents a MCGDM approach for location planning of service centers based on the hesitant fuzzy TOPSIS model with entropy weights. The idea of TOPSIS method as applied to the problem of MCGDM is to choose an alternative with the nearest distance from the so-called fuzzy positive ideal solution (FPIS) and the farthest distance from the fuzzy negative ideal solution (FNIS). A positive ideal solution is composed of the best performance values for each criterion whereas the negative ideal solution consists of the worst performance values. Fuzzy TOPSIS has been applied to facility location problems by researchers in [5, 21, 25].

An algorithm for the practical solution of the location planning problem for service centers using fuzzy TOPSIS can be formulated as follows:

Step 1: Experts lingual assessments convert into the assessments in a form of trapezoidal fuzzy numbers.

Step 2: Based on the experts' hesitant trapezoidal evaluations construct the aggregate hesitant trapezoidal decision matrix $\widetilde{T} = (\widetilde{t}_{ij})_{m\times n}$.

Step 3: Transform aggregate hesitant trapezoidal decision matrix $\widetilde{T} = (\widetilde{t}_{ij})_{m\times n}$ into aggregate hesitant decision matrix $H = (h_{ij})_{m\times n}$ by using Graded Mean Integration Representation Method.

Step 4: Determine the criteria weights $w = (w_1, w_2, \dots, w_n)$ based on the method given in Section 2 (Subsection 2.3).

Step 5: Determine the corresponding hesitant FPIS A^+ and the hesitant FNIS A^- by formulas:

$$\begin{aligned} A^+ = \Big\{ \Big\langle \max_{i=1,\dots,m} [h_{ij}^{\sigma(\lambda)}],\ \lambda = 1,\dots,l \Big\rangle \mid j \in J';\\ \Big\langle \min_{i=1,\dots,m} [h_{ij}^{\sigma(\lambda)}],\ \lambda = 1,\dots,l \Big\rangle \mid j \in J'' \Big\}, \end{aligned} \tag{3.1}$$

$$\begin{aligned} A^- = \Big\{ \Big\langle \min_{i=1,\dots,m} [h_{ij}^{\sigma(\lambda)}],\ \lambda = 1,\dots,l \Big\rangle \mid j \in J';\\ \Big\langle \max_{i=1,\dots,m} [h_{ij}^{\sigma(\lambda)}],\ \lambda = 1,\dots,l \Big\rangle \mid j \in J'' \Big\}, \end{aligned} \tag{3.2}$$

where l is a length of HFE h_{ij} (quantity of elements in h_{ij}), J' is associated with a benefit criteria, and J'' is associated with a cost criteria.

Step 6: Using (2.2) calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the hesitant FPIS A^+ and the hesitant FNIS A^-, respectively:

$$d_i^+ = \sum_{j=1}^{n} d(h_{ij}, h_j^+) w_j = \sum_{j=1}^{n} w_j \left[\frac{1}{l} \sum_{j=1}^{l} \left| h_{ij}^{\sigma(j)} - (h_{ij}^{\sigma(j)})^+ \right| \right], \quad i = 1,2,\dots,m, \tag{3.3}$$

$$d_i^- = \sum_{j=1}^{n} d(h_{ij}, h_j^-) w_j = \sum_{j=1}^{n} w_j \left[\frac{1}{l} \sum_{j=1}^{l} \left| h_{ij}^{\sigma(j)} - (h_{ij}^{\sigma(j)})^- \right| \right], \quad i = 1,2,\dots,m. \tag{3.4}$$

Step 7: Calculate the relative closeness coefficient δ_i of each alternative A_i to the hesitant FPIS A^+:

$$\delta_i = d_i^- / (d_i^+ + d_i^-). \tag{3.5}$$

Step 8: Perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, m$, according to the relative closeness coefficients δ_i, $i = 1, 2, \ldots, m$, by the rule: for two alternatives A_α and A_β we say that A_α is more preferred than A_β, i.e. $A_\alpha \succeq A_\beta$, if $\delta_\alpha \geq \delta_\beta$, where $\succeq$ is a preference relation on A.

4. An example of the Application of Fuzzy Decision Making Approach

Suppose, in the MCGDM problem for location planning of service centers the following five main criteria have been identified:

x_1-Accessibility;
x_2-Connectivity to multimodal transport;
x_3-Costs;
x_4-Proximity to customers;
x_5-Proximity to suppliers.

From them only x_3 criterion is of a cost type, the other criteria are of a benefit type.

Assume that there are four decision making alternatives - potential locations for candidate centers, and the group of the experts consists of four members. They evaluate each possible location regarding all criteria and give assessments in linguistic terms as follows (see Table 3).

Table 3. Experts initial assessments - ratings of alternatives

	Criteria				
	x_1	x_2	x_3	x_4	x_5
A_1	{H,VH,VL,VH}	{H,H,M,VH}	{L,H,M,VL }	{VH,VH,M,VM}	{VL,L,VL,VL}
A_2	{L,L,VL,VL}	{L,M,VL,M}	{L,M,VL,M}	{VL,L,VL,VL}	{L,VL,L,L}
A_3	{M,VH,H,H}	{L,H,M,H}	{L,VL,M,H}	{M,VH,VH,M}	{M,H,M,M}
A_4	{H,VL,M,M}	{L,VL,H,L}	{L,M,VL,L}	{H,H,H,H}	{VH,H,VH,M}

These initial assessments are transformed into HTF matrix by assigning for each lingual assessment the appropriate TrFN as given in Table 2. Thus, we obtained the following hesitant trapezoidal fuzzy decision matrix (see Table 4).

Then we transform the constructed matrix into the hesitant fuzzy decision matrix (see equation (2.1)). If the evaluation values of any criterion given by experts are coincident, then such values are included in HFE only once. We assume that the experts are pessimistic, and the hesitant fuzzy data in HFEs are changed by adding the minimal values. Hence, the hesitant fuzzy decision matrix H looks like Table 5.

According to the method of determining the criteria weights given in Subsection 2.3, we first calculate the score matrix S of hesitant decision matrix H based on equation (2.3):

Table 4. The Hesitant Trapezoidal Fuzzy Decision Matrix

	Criteria				
	x_1	x_2	x_3	x_4	x_5
A_1	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)	(0.1,0.2,0.3,0.4)	(0.7,0.8,0.9,1.0)	(0.0,0.1,0.2,0.3)
	(0.7,0.8,0.9,1.0)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)	(0.7,0.8,0.9,1.0)	(0.1,0.2,0.3,0.4)
	(0.0,0.1,0.2,0.3)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)	(0, 0.1, 0.2, 0.3)
	(0.7,0.8,0.9,1.0)	(0.7,0.8,0.9,1.0)	(0,0.1,0.2,0.3)	(0.7,0.8,0.9,1.0)	(0,0.1,0.2,0.3)
A_2	(0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4)	(0, 0.1,0.2,0.3)	(0.1,0.2,0.3,0.4)
	(0.1,0.2,0.3,0.4)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)	(0.1,0.2,0.3,0.4)	(0, 0.1,0.2,0.3)
	(0,0.1,0.2,0.3)	(0,0.1,0.2,0.3)	(0,0.1,0.2,0.3)	(0,0.1,0.2,0.3)	(0.1,0.2,0.3,0.4)
	(0, 0.1, 0.2, 0.3)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)	(0, 0.1, 0.2, 0.3)	(0, 0.1, 0.2, 0.3)
A_3	(0.3,0.4,0.5,0.6)	(0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)
	(0.7,0.8,0.9,1.0)	(0.5,0.6,0.7,0.8)	(0,0.1,0.2,0.3)	(0.7,0.8,0.9,1.0)	(0.5,0.6,0.7,0.8)
	(0.5,0.6,0.7,0.8)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)	(0.7,0.8,0.9,1.0)	(0.3,0.4,0.5,0.6)
	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)	(0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6)
A_4	(0.5,0.6,0.7,0.8)	(0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)
	(0, 0.1, 0.2, 0.3)	(0, 0.1, 0.2, 0.3)	(0.3,0.4,0.5,0.6)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)
	(0.3,0.4,0.5,0.6)	(0.5,0.6,0.7,0.8)	(0,0.1,0.2,0.3)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)
	(0.3,0.4,0.5,0.6)	(0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4)	(0.5,0.6,0.7,0.8)	(0.5,0.6,0.7,0.8)

Table 5. The Hesitant Fuzzy Decision Matrix H

	Criteria				
	x_1	x_2	x_3	x_4	x_5
A_1	(0.65,0.85, 0.15,0.15)	(0.65,0.45, 0.85,0.45)	(0.25,0.65, 0.45,0.15)	(0.45,0.85, 0.45,0.45)	(0.15,0.25, 0.15,0.15)
A_2	(0.25,0.15, 0.15,0.15)	(0.25,0.15, 0.45,0.15)	(0.15,0.25, 0.45, 0.15)	(0.15,0.25 0.15,0.15)	(0.25,0.15, 0.15,0.15)
A_3	(0.45,0.85 0.65,0.45)	(0.25,0.65 0.45,0.25)	(0.25,0.15, 0.45,0.65)	(0.45,0.85, 0.45,0.45)	(0.45,0.65, 0.45,0.45)
A_4	(0.65,0.15, 0.45,0.15)	(0.15,0.25, 0.65,0.15)	(0.25,0.45, 0.15,0.15)	(0.65,0.65, 0.65,0.65)	(0.85,0.65, 0.45,0.45)

$$S = \begin{bmatrix} 0.45 & 0.6 & 0.375 & 0.55 & 0.175 \\ 0.175 & 0.25 & 0.25 & 0.175 & 0.175 \\ 0.6 & 0.4 & 0.375 & 0.55 & 0.5 \\ 0.35 & 0.3 & 0.25 & 0.65 & 0.6 \end{bmatrix}$$

Secondly, we obtain the normalized score matrix S' using equation (2.4):

$$S' = \begin{bmatrix} 0.2857 & 0.3871 & 0.3 & 0.2857 & 0.1207 \\ 0.1111 & 0.1613 & 0.2 & 0.0909 & 0.1207 \\ 0.3809 & 0.2581 & 0.3 & 0.2857 & 0.3448 \\ 0.2222 & 0.1935 & 0.2 & 0.3376 & 0.4138 \end{bmatrix}$$

Then the weighting vector of criteria is determined using equations (2.5) and (2.6):

$$w = (0.201817,\ 0.192146,\ 0.175857,\ 0.202036,\ 0.228144).$$

Following the hesitant fuzzy TOPSIS method, we determine the hesitant FPIS and the hesitant FNIS by equations (3.1) and (3.2), respectively:

$$\begin{aligned} A^+ = \{&(0.65, 0.85, 0.65, 0.45),\ \ (0.65, 0.65, 0.85, 0.45),\ \ (0.15, 0.15, 0.15, 0.15),\\ &(0.65, 0.85, 0.65, 0.65),\ \ (0.85, 0.65, 0.45, 0.45)\};\\ A^- = \{&(0.25, 0.15, 0.15, 0.15),\ \ (0.15, 0.15, 0.45, 0.15),\ \ (0.25, 0.65, 0.45, 0.65),\\ &(0.15, 0.25, 0.15, 0.15),\ \ (0.15, 0.15, 0.15, 0.15)\}. \end{aligned}$$

Then we calculate the distances and of each alternative from the hesitant FPIS and the hesitant FNIS by equations (3.3) and (3.4), respectively:

$$d_1^+ = 0.216805,\ \ d_2^+ = 0.393337,\ \ d_3^+ = 0.150815,\ \ d_4^+ = 0.155484;$$
$$d_1^- = 0.231004,\ \ d_2^- = 0.0544715,\ \ d_3^- = 0.297994,\ \ d_4^- = 0.292325.$$

Using equation (3.5) to calculate the relative closeness coefficient δ_i of each alternative A_i to the hesitant FPIS A^+ we obtain:

$$\delta_1 = 0.515853,\ \ \delta_2 = 0.12164,\ \ \delta_3 = 0.663216,\ \ \delta_4 = 0.65279.$$

Finally, we perform the ranking of the alternatives A_i, $i = 1, 2, \dots, 4$, according to the relative closeness coefficients δ_i and obtain:

$$A_3 \succ A_4 \succ A_1 \succ A_2.$$

This means that in accordance with the common opinion of the experts TOPSIS method prefers to the alternative A_3 , i.e., A_3 is the best location for service centers.

5. Conclusion

In this paper the novel approach for solving MCGDM problem based on hesitant fuzzy TOPSIS method with entropy weights is developed. Our methodology provides experts with the opportunity to manifest intellectual activity of a high level. Securing the freedom of experts' subjective evaluations, the methodology, however, allows for developing experts' joint decision, for instance, on selection of the best location for service centers among a set of possible alternatives.

The new aspects in the TOPSIS approach have been used. We proposed a new criteria weighting method based on De Luca-Termini information entropy to express the relative intensities of criterion importance and determine the criteria weights. The latter distinguishes our methodology from the others.

It should also be noted that in the real problem of location planning for service centers, practically have been processed the criteria of both benefit and cost types. Based on proposed methodology we have developed software package, the results of which are illustrated in considered example.

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