Seminar of I. Vekua Institute of Applied Mathematics REPORTS, Vol. 47, 2021

ON APPLICATION OF DIRECT COMPUTATIONAL METHODS TO NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS WITH CAUCHY KERNEL

Sanikidze J., Kublashvili M., Mirianashvili M.

Abstract. A number of quadrature processes connected with approximation of Cauchy type singular integrals are considered in relation with approximate solution of boundary problems of certain type. Namely, significant attention is paid to problems of unique solvability of approximating scheme, accuracy and similar questions related with boundary integral problems based on corresponding approximation.

Keywords and phrases: Singular integral, approximation, solvability, accuracy, estimation, numerical experiment.

AMS subject classification (2010): 30E20, 97N40.

The boundary integral equations method is one of the effective methods for solution of some boundary value problems. Unlike the known grid method, application of the mentioned method decreases the dimension of the initial problem by one unit. In classical mathematical literature, Fredholm type integral equations method is usually meant under integral equations method. But it turns out that in many cases, application of Cauchy type singular integrals approximation method is more effective: the kernel and the right-hand sides usually more simply depend on initial data. These circumstances are more important especially when the mentioned data is defined experimentally. On the other hand, it is also known that approximation schemes, based on the singular integrals approximation methods, may not always be uniquely solvable.

As it is known [1], the determinant of the linear algebraic system, based on the appropriate singular integral approximation scheme, may equal zero. In such cases, individual consideration of the approximating structures is important, which in many cases leads to positive results. As an example, some of works in this direction [2] may be pointed out.

From this and some other points of view, we may say that in many cases, differently from Fredholm integral approximation method, the singular integral approximation schemes lead us to sufficiently effective computational schemes for numerical solution of some boundary value problems. In particular,some problems of elasticity and construction theory, nuclear physics, and other neighboring fields belong to the mentioned problems.

Starting from the above said, classes of the quadrature formulas for singular integrals represent a significant interest. These formulas lead to a priori uniquely solvable linear algebraic approximating schemes for one or another class of singular integral equations. In this direction, the result indicated in [3], which is related to the mentioned above question, should be noted.

The corresponding result consists in construction of effectively realizable computational scheme which approximates a certain class of one-dimensional singular integral equations. It can be seen, that in corresponding illustrations, one of the existing factor is the structure of numerical coefficients of the approximating quadrature scheme. In this connection, it should be noted that in paper [4], a quadrature scheme is constructed, which is analogous to the previous, but is definitely a very little more precise (by order).

It should be noted that the mentioned calculating scheme is constructed on the basis of a singular analog of ordinary (regular) quadrature formulas with equal coefficients (see, e.g.,[5], [6]) which is derived in the same paper.

In the given case, in [4], a quadrature formula with four knots was considered in the case of constant weight [1]

Further, corresponding considerations reveal that the linear algebraic system, obtained by the indicated way, is uniquely solvable with certain accuracy. The above considered material mainly refer to a question of solvability of computational schemes constructed on the basis of approximation of singular integrals. Surely, at this, one of further main questions is a question of accuracy estimation of calculation schemes obtained by the considered way.

In this connection, various attitudes may be used depending on any particular structures of both approximating and approximated expressions. At the some time we note, that as in the case of regular integrals, solution of question of construction and application of certain quadrature formulas to singular integrals is possible.

Here we consider singular integrals of type

$$\int_{-1}^{+1} \rho(t)\frac{\varphi(t)}{t-t_0}dt, \quad t_0 \in (-1,+1),$$

where $\rho(t)$ is the given summable on [-1,+1] weight function without singularities in the interior of the open interval (-1,+1). For certainty below, in the role of $\rho(t)$ we will consider one of the most frequently occurring functions $(\sqrt{1-t^2})^{-1}$ which leads to investigation of the following singular integral

$$\int_{-1}^{+1} \frac{\varphi(t)}{\sqrt{1-t^2}(t-t_0)}dt, \quad t_0 \in (-1,+1). \tag{1}$$

On the basis of (1) and the known quadrature formula [6], we obtain the following approximate formula

$$\int_{-1}^{+1} \frac{\varphi(t)-\varphi(t_0)}{\sqrt{1-t^2}(t-t_0)}dt \approx \frac{\pi}{n}\sum_{k=1}^{n} \frac{\varphi(x_{kn})-\varphi(t_0)}{x_{kn}-t_0}, \tag{2}$$

where $x_{kn} = \cos\frac{2k-1}{2n}\pi$ $(k = 1, 2, ..., n)$ are zeros of the Chebyshev polynomial of order n. The corresponding remainder term at appropriate smoothness of function $\varphi(t)$ has the following form (see in the same [6])

$$\frac{\pi}{2^{2n-1}(2n)!}\varphi^{(2n-1)}(\eta) \quad (-1 < \eta < 1). \tag{3}$$

By this, (2) represents a formula of highest algebraic accuracy [6]. Defining in (1) values of $\{t_0\}$ by conditions

[1]It should be noted that the corresponding formulas exist for $n = 1, .., 7$ and $n = 9$. For the sake of completeness we also note that in [7] expressions of the remainder terms of the Chebyshev formulas for the corresponding values are indicated.

$$\sum_{k=1}^{n} \frac{1}{x_{kn} - t_0} = 0, \quad t_0 \in (-1, +1) \tag{4}$$

for various values of n, finally, taking into account (2), we come to concrete singular integral approximation process:

$$\int_{-1}^{+1} \frac{\varphi(t)}{\sqrt{1-t^2}(t-t_0)} dt \approx \frac{\pi}{n} \sum_{k=1}^{n} \frac{\varphi(x_{kn})}{x_{kn} - t_0} \quad t_0 \in (-1, +1).$$

As the corresponding practical calculations show, more or less acceptable results appear at relatively not large values of n in some cases under certain additional supposition regarding location of the singularity point t_0 in interval (-1,+1). At this, numerical experiment turns out to be notably effective tool in corresponding considerations.

R E F E R E N C E S

1. Ivanov V. V. Theory of approximation methods and its application to numerical solution of singular integral equations (Russian). *Naukova Dumka, Kiev*, 1968.

2. Sanikidze J., Khubedjashvili S., Mirianashvili M., Machavariani A., Emelyanenko G. On numerical solution of Lippman-Schwinger singular integral equations. *Bull Academy of Sciences of GSSR*, **140**, 2 (1990), 261-263.

3. Belotserkovski S., Lifanov I. Numerical methods in singular integral equations and their application to aerodynamics, theory of elasticity, electrodynamics (Russian). *Nauka, Moscow*, 1985.

4. Sanikidze J., Ninidze K. On discrete vortices type computational schemes with higher accuracy for numerical solution of some classes of singular integral equations. *Tbilisi International Conference on Computer Sciences and Applied Mathematics,* 2015, 271-273.

5. Mikeladze S. Numerical methods of mathematical analysis (Russian). *Gostekhizdat, Moscow*, 1967.

6. Krylov V. I. Approximate Calculation of integrals (Russian). *Nauka, Moscow,* 1967.

7. Berezin I. S. and Zhidkov N. P. Calculation methods (Russian). *Fizmatgiz, Moscow*, **1** (1959).

Received 19.07.2021; revised 13.09.2021; accepted 15.09.2021

Authors' addresses:

J. Sanikidze
N. Muskhelishvili Institute of Computational Mathematics
of Georgian Technical University
4, Grigol Peradze St., Tbilisi 0131
Georgia

M. Kublashvili
N. Muskhelishvili Institute of Computational Mathematics
of Georgian Technical University
4, Grigol Peradze St., Tbilisi 0131
Georgia
E-mail: murman.kublashvili@gtu.ge

M. Mirianashvilli
N. Muskhelishvili Institute of Computational Mathematics
of Georgian Technical University
4, Grigol Peradze St., Tbilisi 0131
Georgia
E-mail: pikriag@yahoo.com