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ON THE EXISTENCE OF AN OPTIMAL ELEMENT FOR THE NEUTRAL OPTIMAL PROBLEM WITH DELAY IN CONTROLS

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Abstract. For an optimal control problem involving neutral differential equation

$$\dot{x}(t) = A(t)\dot{x}(t-\sigma) + f(t, x(t), x(t-\tau), u(t))$$

$$+g(t, x(t), x(t-\tau), u(t-\theta)), t \in [t_0, t_1]$$

existence theorems of an optimal element are provided. Under element we imply the collection of delay parameters σ and τ, the initial moment and vector, control and finally moment.

Keywords and phrases: Neutral optimal problem, optimal element, existence, delay in controls.

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Let R_x^n be the n-dimensional vector space of points $x = (x^1, \dots, x^n)^T$, where T means transpose, let

$$a < t_{01} < t_{02} < t_{11} < t_{12} < b, 0 < \tau_1 < \tau_2, 0 < \sigma_1 < \sigma_2$$

and let $\theta > 0$ be given numbers with $t_{11} - t_{02} > max\{\tau_2, \sigma_2\}$; suppose that $O \subset R_x^n$ is an open set and $U \subset R_u^r$ is a compact set, the functions

$$f(t, x, y, u) = (f^1(t, x, y, u), ..., f^n(t, x, y, u))^T$$

$$g(t, x, y, u) = (g^1(t, x, y, u), ..., g^n(t, x, y, u))^T$$

are continuous on the set $I \times O^2 \times U$ and continuously differentiable with respect to x and y, where $I = [a, b]$.

let Ω be a set of measurable control functions $u(t) \in U, t \in [a-\theta, b]$ and let

$$q^i(t_0, t_1, \tau, \sigma, x_0, x_1), i = \overline{0, l}$$

be continuous scalar functions on the set

$$[t_{01}, t_{02}] \times [t_{11}, t_{12}] \times [\tau_1, \tau_2] \times [\sigma_1, \sigma_2] \times X_0 \times O,$$

where $X_0 \subset O$ is a compact set.

To each element

$$w = (t_0, t_1, \tau, \sigma, x_0, u(\cdot)) \in W = [t_{01}, t_{02}] \times [t_{11}, t_{12}] \times [\tau_1, \tau_2]$$

$$\times [\sigma_1, \sigma_2] \times X_0 \times \Omega$$

we assign the neutral differential equation

$$\dot{x}(t) = A(t)\dot{x}(t-\sigma) + f(t, x(t), x(t-\tau), u(t)) \\ +g(t, x(t), x(t-\tau), u(t-\theta)), t \in [t_0, t_1] \tag{1}$$

with the initial condition

$$x(t) = \varphi(t), t \in [\hat{\tau}, t_0), x(t_0) = x_0, \tag{2}$$

where $A(t) = (a^i_j(t)), i, j = \overline{1, n}, t \in I$ is a given $n \times n$ -dimensional continuous matrix function; $\varphi(t), t \in [\hat{\tau}, t_{02}]$ is a given continuous differentiable initial function, $\hat{\tau} = a - \max\{\tau_2, \sigma_2\}$.

Definition 1. Let $w = (t_0, t_1, \tau, \sigma, x_0, u(\cdot)) \in W$. A function $x(t) = x(t; w) \in O, t \in [\hat{\tau}, t_1]$, is called a solution corresponding to the element w, if it satisfies condition (2) and is absolutely continuous on the interval $[t_0, t_1]$ and satisfies Eq. (1) almost everywhere on $[t_0, t_1]$.

Definition 2. An element $w = (t_0, t_1, \tau, \sigma, x_0, u(\cdot)) \in W$ is said to be admissible if there exists the corresponding solution $x(t) = x(t; w)$ satisfying the condition

$$q(t_0, t_1, \tau, \sigma, x_0, x(t_1)) = 0, \tag{3}$$

where $q = (q^1, ..., q^l)$.

We denote the set of admissible elements by W_0.

Definition 3. An element $w_0 = (t_{00}, t_{10}, \tau_0, \sigma_0, x_{00}, u_0(\cdot)) \in W_0$ is said to be optimal if

$$q^0(t_0, t_{10}, \tau_0, \sigma_0, x_{00}, x(t_{10})) \leq q^0(t_0, t_1, \tau, \sigma, x_0, x(t_1)), \tag{4}$$

where $x(t) = x(t; w)$ and $x_0(t) = x(t; w_0)$.

Problem (1)-(4) is called the neutral optimal problem with θ delay in controls.

Theorem 1. *For problem (1)-(4) there exists an optimal element w_0 if the following conditions hold:*

1) $W_0 \neq \emptyset$;

2) there exists a compact set $K \subset O$ such that for an arbitrary $w \in W_0$

$$x(t; w) \in K, t \in [t_0, t_1];$$

3) for each fixed $s_1, s_2 \in I$, $x_1, y_1 \in O$ and $x_2, y_2 \in O$ the set

$$\left\{ \Big(f(s_1, x_1, y_1, u), g(s_2, x_2, y_2, u) \Big)^T : u \in U \right\}$$

is convex.

Theorem 1 is proved by the scheme given in [1, 2], where the case is considered when $g = 0$.

Now we consider the following problem

$$\begin{cases} \dot{x}(t) = A(t)\dot{x}(t-\sigma) + B(t, x(t), x(t-\tau))u(t) \\ +C(t, x(t), x(t-\tau))u(t-\theta), t \in [t_0, t_1], \\ x(t) = \varphi(t), t \in [\hat{\tau}, t_0), x(t_0) = x_0, \\ q(t_0, t_1, \tau, \sigma, x_0, x(t_1)) = 0, \\ q^0(t_0, t_1, \tau, \sigma, x_0, x(t_1)) \to \min . \end{cases} \tag{5}$$

Theorem 2. *For the problem (5) there exists an optimal element w_0 if the following conditions hold:*

4) $W_0 \neq \emptyset$;

5) there exists a compact set $K \subset O$ such that for an arbitrary $w \in W_0$

$$x(t; w) \in K, t \in [t_0, t_1];$$

6) the set U is convex.

Theorem 3. *For the problem*

$$\begin{cases} \dot{x}(t) = A(t)\dot{x}(t-\sigma) + B(t)x(t) + C(t)x(t-\tau)) \\ +D(t)u(t) + E(t)u(t-\theta), t \in [t_0, t_1], \\ x(t) = \varphi(t), t \in [\hat{\tau}, t_0), x(t_0) = x_0, \\ q(t_0, t_1, \tau, \sigma, x_0, x(t_1)) = 0, \\ q^0(t_0, t_1, \tau, \sigma, x_0, x(t_1)) \to \min \end{cases}$$

there exists an optimal element w_0 if the following conditions hold:

7) $W_0 \neq \emptyset$;

8) the set U is convex.

R E F E R E N C E S

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